Explore the Motion of the Earth with Desmos

MAT 1475 Spring 2020

In this project, we will explore the Motion of the earth with the knowledge of Calculus and the graphing tool: Desmos. The project includes:

1. Obtain data of the earth orbit
2. Set up an elliptical equation to describe the trajectory of the earth
3. Using implicit differentiation to compute dy/dx. Discuss if this can represent the velocity of the earth.
4. Set up parametric equation to describe the trajectory of the earth
5. Using the parametric derivative to compute dy/dt and dx/dt. Discuss if this can represent the velocity of the earth.

Part I: Basic Information

Draw (nicely) the earth orbit in the following space. Label the sun, the earth, and the positions of the perihelion and the aphelion.

- What is the shape of the orbit?

- What is the perihelion? what is the distance between the sun and the perihelion?

- What is the aphelion? what is the distance between the sun and the aphelion?

Part II: Mathematical Preparation

Plot the following elliptical equation with Desmos and change a and b respectively to find out how it affects the shape of the ellipse. Show the plot of $a = 5$, and $b = 3$ in the following graphing paper.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

- Where is the center of the ellipse ?

- What distance does a represent? How does a affect the shape of the curve?

- What distance does b represent? How does b affect the shape of the curve?

- Compute c by $c^2 = a^2 - b^2$. Locate the two points $(-c, 0)$ and $(c, 0)$ on the graph.

- Use internet to find out the name of the three parameters a, b, c.

Part III: The Elliptic Equation of the Orbit of the earth

Construct the equation of the orbit and plot it on the graph. To do this we first need to compute the values of a, b and c from aphelion and perihelion.

- How to use a, c to represent aphelion and perihelion, respectively ?

- Solve the system of equations for a and c.

- Find b from a and c.

- Find the equation of the orbit and plot it on the following graph

- Mark the coordinates of the four intercepts on the graph.

Part IV: Implicit Differentiation

1. Take the elliptic equation of the orbit of the earth. Please find the derivative of the function dy/dx implicitly.

2. Compute the dy/dx at the four intercepts. Can we related these values with the velocity of the earth? Explain your thoughts.

Part V: Parametric Equations

1. Plot the following parametric equation by constructing a table and using Desmos.

$$\begin{cases} x(t) = 5\cos t \\ y(t) = 3\sin t \end{cases}$$

t	$x(t)$	$y(t)$	(x, y)
0			
$\frac{\pi}{2}$			
π			
$\frac{3\pi}{2}$			
2π			

2. Plot the graph generated by Desmos on the following graphing paper.

3. What is the shape of the graph? Eliminate variable t from the equations to find an equation in terms of x and y.

Part VI: Velocity Vectors

In a two dimensional space, we need two numbers (x, y) to locate positions of an object. One of the convenient ways to write the location as a function of time is to use parametric equation. For example, as we did in Part V, we can find a parametric equation of the earth orbit.

1. Rewrite the elliptic equation you find in Part III in the following parametric form. What is a, b respectively?

$$\begin{cases} x(t) = a\cos t \\ y(t) = b\sin t \end{cases}$$

2. Find the time derivative of each parametric function and form a vector $(dx/dt, dy/dt)$

3. Find four vectors at the four intercepts.

4. Discuss how these vectors can provide information about the velocity of the earth.