Complex numbers

Lesson #23

MAT 1375 Precalculus

New York City College of Technology CUNY

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$$a + b \cdot i, \quad \text{where} \quad i^2 = -1$$

and a =real part, b =imaginary part

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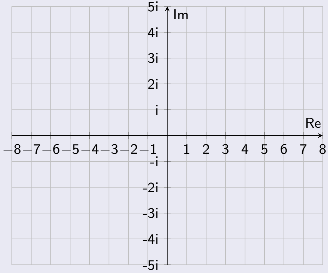

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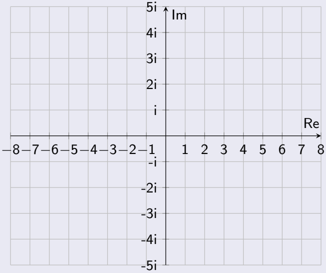

Plot in the complex plane:

$2 + 3i$, $-5 + 4i$, $7 - 3i$, $-4 - 2i$, 4

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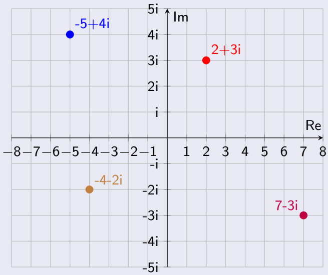

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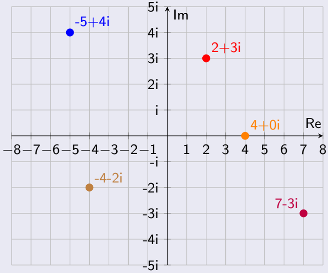

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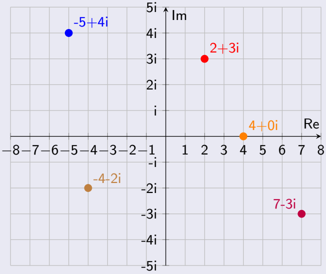

Plot in the complex plane:

$2 + 3i$, $-5 + 4i$, $7 - 3i$, $-4 - 2i$, $4 = 4 + 0i$

Perform the indicated operation:

1. $(2 + 3i) + (-6 + 5i)$
2. $(2 + 3i) - (-6 + 5i)$
3. $(2 + 3i) \cdot (-6 + 5i)$
4. $\dfrac{2+3i}{-6+5i}$

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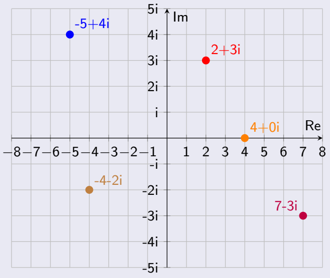

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$2 + 3i$, $-5 + 4i$, $7 - 3i$, $-4 - 2i$, $4 = 4 + 0i$

Perform the indicated operation:

1. $(2 + 3i) + (-6 + 5i) = -4 + 8i$
2. $(2 + 3i) - (-6 + 5i)$
3. $(2 + 3i) \cdot (-6 + 5i)$
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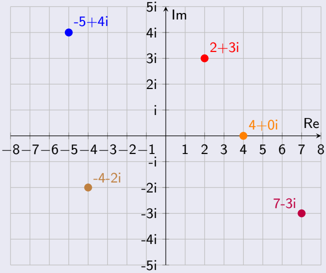

Plot in the complex plane:

$2+3i$, $-5+4i$, $7-3i$, $-4-2i$, $4 = 4+0i$

Perform the indicated operation:

1. $(2+3i)+(-6+5i) = -4+8i$
2. $(2+3i)-(-6+5i)$
   $= 2+3i+6-5i$
   $= 8-2i$
3. $(2+3i)\cdot(-6+5i)$
4. $\frac{2+3i}{-6+5i}$

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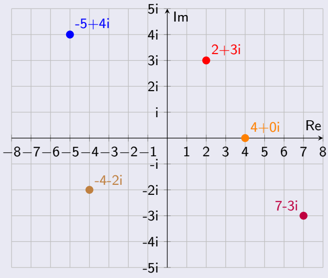

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2. $(2 + 3i) - (-6 + 5i)$
   $= 2 + 3i + 6 - 5i$
   $= 8 - 2i$
3. $(2 + 3i) \cdot (-6 + 5i)$
   $= -12 + 10i - 18i + 15i^2$
   $= -12 - 8i - 15$
   $= -27 - 8i$
4. $\frac{2+3i}{-6+5i}$

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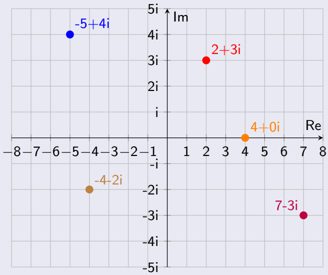

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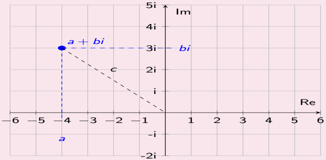
3. $(2+3i)\cdot(-6+5i)$
   $= -12+10i-18i+15i^2$
   $= -12-8i-15$
   $= -27-8i$
4. $\frac{2+3i}{-6+5i}$
   $= \frac{(2+3i)(-6-5i)}{(-6+5i)(-6-5i)}$
   $= \frac{-12-10i-18i-15i^2}{36+30i-30i-25i^2}$
   $= \frac{-12-28i+15}{36+25}$
   $= \frac{3-28i}{61}$
   $= \frac{3}{61} - \frac{28}{61}i$

Modulus of a complex numbers

The **modulus** or **absolute value** of $a + bi$ is



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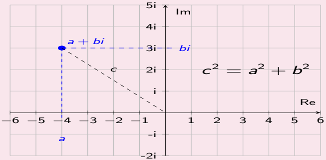

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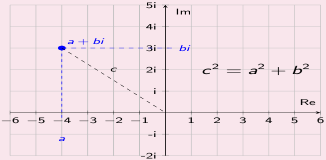

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Find the modulus of the complex number.

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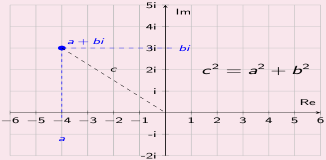

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Find the modulus of the complex number.

1. $3 + 7i$
   $|3 + 7i| = \sqrt{3^2 + 7^2} = \sqrt{9 + 49} = \sqrt{58}$

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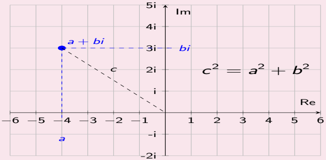

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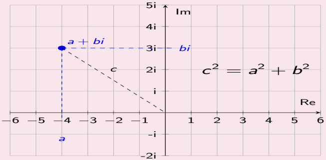

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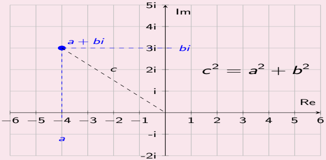

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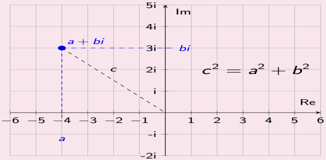

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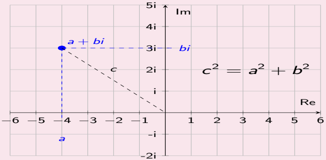

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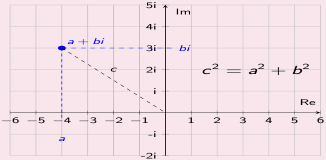

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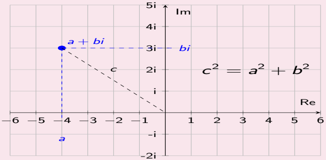

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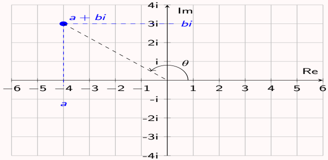
7. $5\sqrt{3} + 5i$
   $|5\sqrt{3} + 5i| = \sqrt{(5\sqrt{3})^2 + 5^2}$
   $= \sqrt{25 \cdot 3 + 25} = \sqrt{100} = 10$

Argument of a complex numbers

The **argument** or **angle** of $a + bi$ is



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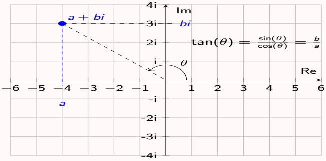

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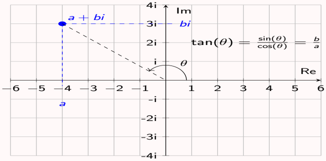

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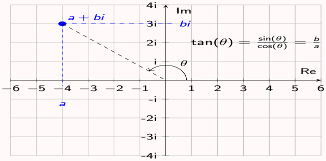

$$\theta = \arctan\left(\frac{b}{a}\right)$$

Find the argument of the complex number.

1. $5 + 2i$
   $\theta = \arctan\left(\frac{2}{5}\right) \approx 21.8^\circ$

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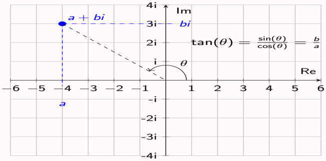

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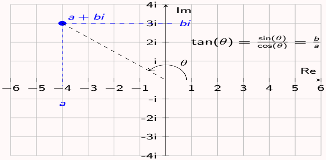

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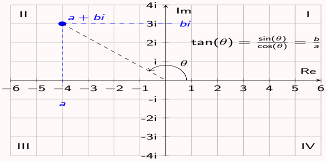

$\theta = \arctan(\frac{b}{a})$	if in I or IV
$\theta = \arctan(\frac{b}{a}) + 180^\circ$	if in II or III

Find the argument of the complex number.

1. $5 + 2i$
   $\theta = \arctan(\frac{2}{5}) \approx 21.8^\circ$
2. $-5 - 2i$ in quadrant III
   $\theta = \arctan(\frac{-2}{-5}) + 180^\circ$
   $\approx 21.8^\circ + 180^\circ \approx 201.8^\circ$

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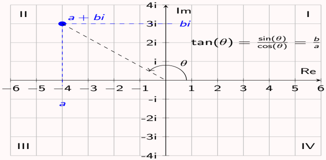

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   $\approx 21.8^\circ + 180^\circ \approx 201.8^\circ$
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4. $-2 + 7i$
5. $4 - 4i$
6. $-2 - 2\sqrt{3}i$
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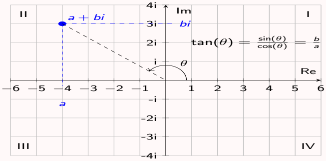

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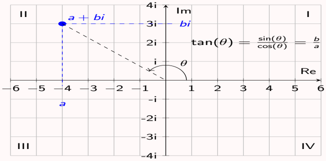
   $\approx 21.8^\circ + 180^\circ \approx 201.8^\circ$
3. $3 + 5i$ in quadrant I
   $\theta = \arctan(\frac{5}{3}) \approx 59.04^\circ$
4. $-2 + 7i$
5. $4 - 4i$
6. $-2 - 2\sqrt{3}i$
7. $-4\sqrt{3} + 4i$

Argument of a complex numbers

The **argument** or **angle** of $a+bi$ is



$$\theta = \arctan\left(\frac{b}{a}\right) \quad \text{if in I or IV}$$
$$\theta = \arctan\left(\frac{b}{a}\right) + 180^\circ \quad \text{if in II or III}$$

Find the argument of the complex number.

1. $5+2i$
   $\theta = \arctan(\frac{2}{5}) \approx 21.8^\circ$
2. $-5-2i$ in quadrant III
   $\theta = \arctan(\frac{-2}{-5}) + 180^\circ$
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3. $3+5i$ in quadrant I
   $\theta = \arctan(\frac{5}{3}) \approx 59.04^\circ$
4. $-2+7i$ in quadrant II
   $\theta = \arctan(\frac{7}{-2}) + 180^\circ$
   $\approx -74.05^\circ + 180^\circ \approx 105.95^\circ$
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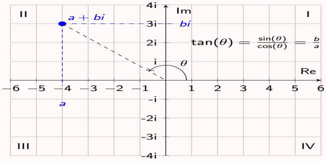

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5. $4 - 4i$ in quadrant IV
   $\theta = \arctan(\frac{-4}{4}) = -45^\circ$
   or: $\theta = -45^\circ + 360^\circ = 315^\circ$
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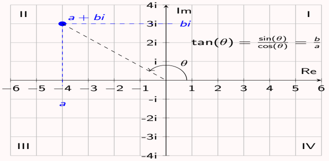
Argument of a complex numbers

The **argument** or **angle** of $a+bi$ is



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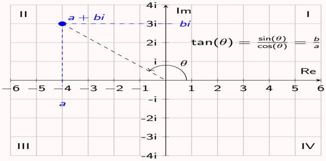
   or: $\theta = -45^\circ + 360^\circ = 315^\circ$
6. $-2-2\sqrt{3}i$ in quadrant III
   $\theta = \arctan(\frac{-2\sqrt{3}}{-2}) + 180^\circ$
   $= 60^\circ + 180^\circ = 240^\circ$
7. $-4\sqrt{3}+4i$

Argument of a complex numbers

The **argument** or **angle** of $a + bi$ is



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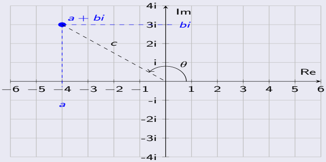
6. $-2 - 2\sqrt{3}i$ in quadrant III
   $\theta = \arctan(\frac{-2\sqrt{3}}{-2}) + 180^\circ$
   $= 60^\circ + 180^\circ = 240^\circ$
7. $-4\sqrt{3} + 4i$ in quadrant II
   $\theta = \arctan(\frac{4}{-4\sqrt{3}}) + 180^\circ$
   $= -30^\circ + 180^\circ = 150^\circ$

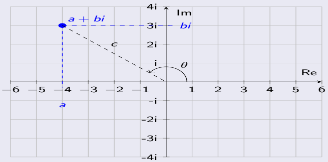
Polar form of a complex number

Polar form and standard form of a complex number



Standard form: $\boxed{a + bi}$

Polar form and standard form of a complex number



Standard form: $\boxed{a + bi}$

Since $\cos(\theta) = \frac{a}{c}$ and $\sin(\theta) = \frac{b}{c}$, we get:
$a = c \cdot \cos(\theta)$ and $b = c \cdot \sin(\theta)$ and thus:

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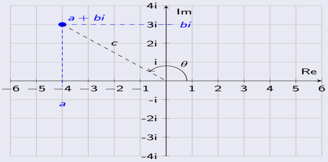

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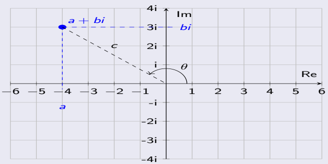
$a = c \cdot \cos(\theta)$ and $b = c \cdot \sin(\theta)$ and thus:

$a + bi = c\cos(\theta) + c\sin(\theta)i = c(\cos(\theta) + i\sin(\theta))$

Polar form: $\boxed{c \cdot (\cos(\theta) + i\sin(\theta))}$

Polar form and standard form of a complex number



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Since $\cos(\theta) = \frac{a}{c}$ and $\sin(\theta) = \frac{b}{c}$, we get:

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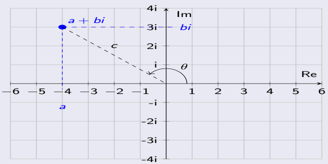
$$a + bi = c\cos(\theta) + c\sin(\theta)i = c(\cos(\theta) + i\sin(\theta))$$

Polar form: $\boxed{c \cdot (\cos(\theta) + i\sin(\theta))}$

Convert to polar form:

1. $-3 - 3i$

Polar form and standard form of a complex number



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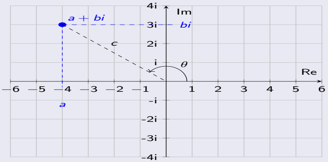
Polar form: $\boxed{c \cdot (\cos(\theta) + i\sin(\theta))}$

Convert to polar form:

1. $-3 - 3i$
   $c = |-3 - 3i| = \sqrt{(-3)^2 + (-3)^2}$
   $= \sqrt{18} = 3\sqrt{2}$

Polar form and standard form of a complex number



Standard form: $\boxed{a+bi}$

Since $\cos(\theta)=\frac{a}{c}$ and $\sin(\theta)=\frac{b}{c}$, we get:

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$$a+bi=c\cos(\theta)+c\sin(\theta)i=c(\cos(\theta)+i\sin(\theta))$$

Polar form: $\boxed{c\cdot(\cos(\theta)+i\sin(\theta))}$

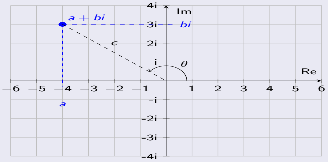
Convert to polar form:

1. $-3-3i$

$$c=|-3-3i|=\sqrt{(-3)^2+(-3)^2}$$
$$=\sqrt{18}=3\sqrt{2}$$
$$\theta=\arctan\left(\frac{-3}{-3}\right)+180^\circ$$
$$=45^\circ+180^\circ=225^\circ$$

Polar form of a complex number

Polar form and standard form of a complex number



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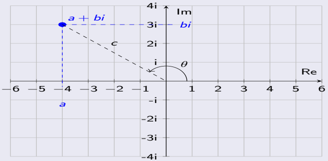
$a+bi=c\cos(\theta)+c\sin(\theta)i=c(\cos(\theta)+i\sin(\theta))$

Polar form: $\boxed{c\cdot(\cos(\theta)+i\sin(\theta))}$

Convert to polar form:

1. $-3-3i$
   $c=|-3-3i|=\sqrt{(-3)^2+(-3)^2}$
   $=\sqrt{18}=3\sqrt{2}$
   $\theta=\arctan(\frac{-3}{-3})+180^\circ$
   $=45^\circ+180^\circ=225^\circ$
   Therefore:
   $-3-3i=3\sqrt{2}\cdot(\cos(225^\circ)+i\sin(225^\circ))$

Polar form and standard form of a complex number



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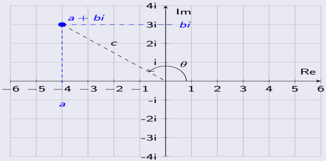
$a+bi=c\cos(\theta)+c\sin(\theta)i=c(\cos(\theta)+i\sin(\theta))$

Polar form: $\boxed{c\cdot(\cos(\theta)+i\sin(\theta))}$

Convert to polar form:

1. $-3-3i$
   $c=|-3-3i|=\sqrt{(-3)^2+(-3)^2}$
   $=\sqrt{18}=3\sqrt{2}$
   $\theta=\arctan(\frac{-3}{-3})+180^\circ$
   $=45^\circ+180^\circ=225^\circ$
   Therefore:
   $-3-3i=3\sqrt{2}\cdot(\cos(225^\circ)+i\sin(225^\circ))$
2. $4\sqrt{3}-4i$

Polar form and standard form of a complex number



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$a + bi = c\cos(\theta) + c\sin(\theta)i = c(\cos(\theta) + i\sin(\theta))$

Polar form: $\boxed{c \cdot (\cos(\theta) + i\sin(\theta))}$

Convert to polar form:

1. $-3 - 3i$
$$c = |-3 - 3i| = \sqrt{(-3)^2 + (-3)^2}$$
$$= \sqrt{18} = 3\sqrt{2}$$
$$\theta = \arctan(\tfrac{-3}{-3}) + 180^\circ$$
$$= 45^\circ + 180^\circ = 225^\circ$$
Therefore:
$$-3 - 3i = 3\sqrt{2} \cdot (\cos(225^\circ) + i\sin(225^\circ))$$

2. $4\sqrt{3} - 4i$
$$c = |4\sqrt{3} - 4i| = \sqrt{(4\sqrt{3})^2 + (-4)^2}$$
$$= \sqrt{16 \cdot 3 + 16} = \sqrt{64} = 8$$
$$\theta = \arctan(\tfrac{-4}{4\sqrt{3}}) = -30^\circ$$
Therefore:
$$4 - 4\sqrt{3}i = 8 \cdot (\cos(-30^\circ) + i\sin(-30^\circ))$$

Converting polar form ↔ standard form - exercises

Convert to polar form:

3. $-7 + 7\sqrt{3}i$

Convert to polar form:

3. $-7+7\sqrt{3}i$
   $c = |-7\sqrt{3}+7i| = \sqrt{(-7)^2+(7\sqrt{3})^2}$
   $= \sqrt{49+49\cdot 3} = \sqrt{196} = 14$
   $\theta = \arctan(\frac{7\sqrt{3}}{-7}) + 180^\circ$
   $= -60^\circ + 180^\circ = 120^\circ$
   Therefore:
   $-7+7\sqrt{3}i$
   $= 14\cdot(\cos(120^\circ) + i\sin(120^\circ))$

Convert to polar form:

3. $-7+7\sqrt{3}i$

   $c = |-7\sqrt{3}+7i| = \sqrt{(-7)^2+(7\sqrt{3})^2}$
   $= \sqrt{49+49\cdot 3} = \sqrt{196} = 14$
   $\theta = \arctan(\frac{7\sqrt{3}}{-7}) + 180^\circ$
   $= -60^\circ + 180^\circ = 120^\circ$

   Therefore:
   $-7+7\sqrt{3}i$
   $= 14\cdot(\cos(120^\circ)+i\sin(120^\circ))$

Convert to standard form:

4. $4\cdot(\cos(210^\circ)+i\sin(210^\circ))$

Converting polar form ↔ standard form - exercises

Convert to polar form:

3. $-7+7\sqrt{3}i$
   $c = |-7\sqrt{3}+7i| = \sqrt{(-7)^2+(7\sqrt{3})^2}$
   $= \sqrt{49+49\cdot 3} = \sqrt{196} = 14$
   $\theta = \arctan(\frac{7\sqrt{3}}{-7}) + 180^\circ$
   $= -60^\circ + 180^\circ = 120^\circ$
   Therefore:
   $-7+7\sqrt{3}i$
   $= 14\cdot(\cos(120^\circ) + i\sin(120^\circ))$

Convert to standard form:

4. $4\cdot(\cos(210^\circ) + i\sin(210^\circ))$
   $= 4\cdot(\frac{-\sqrt{3}}{2} + i\frac{-1}{2})$

Converting polar form $\leftrightarrow$ standard form - exercises

Convert to polar form:

3. $-7+7\sqrt{3}i$
   $c = |-7\sqrt{3}+7i| = \sqrt{(-7)^2+(7\sqrt{3})^2}$
   $= \sqrt{49+49\cdot 3} = \sqrt{196} = 14$
   $\theta = \arctan(\frac{7\sqrt{3}}{-7}) + 180^\circ$
   $= -60^\circ + 180^\circ = 120^\circ$
   Therefore:
   $-7+7\sqrt{3}i$
   $= 14\cdot(\cos(120^\circ) + i\sin(120^\circ))$

Convert to standard form:

4. $4\cdot(\cos(210^\circ) + i\sin(210^\circ))$
   $= 4\cdot(\frac{-\sqrt{3}}{2} + i\frac{-1}{2})$
   $= 4\cdot\frac{-\sqrt{3}}{2} + 4\cdot\frac{-1}{2}\cdot i$
   $= -2\sqrt{3} - 2i$

Convert to polar form:

3. $-7+7\sqrt{3}i$

   $c = |-7\sqrt{3}+7i| = \sqrt{(-7)^2+(7\sqrt{3})^2}$
   $= \sqrt{49+49\cdot 3} = \sqrt{196} = 14$
   $\theta = \arctan(\frac{7\sqrt{3}}{-7}) + 180^\circ$
   $= -60^\circ + 180^\circ = 120^\circ$

   Therefore:
   $-7+7\sqrt{3}i$
   $= 14\cdot(\cos(120^\circ)+i\sin(120^\circ))$

Convert to standard form:

4. $4\cdot(\cos(210^\circ)+i\sin(210^\circ))$

   $= 4\cdot(\frac{-\sqrt{3}}{2}+i\frac{-1}{2})$
   $= 4\cdot\frac{-\sqrt{3}}{2}+4\cdot\frac{-1}{2}\cdot i$
   $= -2\sqrt{3}-2i$

Convert to standard form:

5. $6\cdot(\cos(300^\circ)+i\sin(300^\circ))$

6. $7\cdot(\cos(135^\circ)+i\sin(135^\circ))$

7. $5\cdot(\cos(270^\circ)+i\sin(270^\circ))$

Convert to polar form:

3. $-7+7\sqrt{3}i$
   $c = |-7\sqrt{3}+7i| = \sqrt{(-7)^2+(7\sqrt{3})^2}$
   $= \sqrt{49+49\cdot 3} = \sqrt{196} = 14$
   $\theta = \arctan(\frac{7\sqrt{3}}{-7}) + 180^\circ$
   $= -60^\circ + 180^\circ = 120^\circ$
   Therefore:
   $-7+7\sqrt{3}i$
   $= 14\cdot(\cos(120^\circ) + i\sin(120^\circ))$

Convert to standard form:

4. $4\cdot(\cos(210^\circ) + i\sin(210^\circ))$
   $= 4\cdot(\frac{-\sqrt{3}}{2} + i\frac{-1}{2})$
   $= 4\cdot\frac{-\sqrt{3}}{2} + 4\cdot\frac{-1}{2}\cdot i$
   $= -2\sqrt{3} - 2i$

Convert to standard form:

5. $6\cdot(\cos(300^\circ) + i\sin(300^\circ))$
   $= 6\cdot(\frac{1}{2} + i\frac{-\sqrt{3}}{2})$
   $= 6\cdot\frac{1}{2} + 6\cdot\frac{-\sqrt{3}}{2}\cdot i$
   $= 3 - 3\sqrt{3}i$

6. $7\cdot(\cos(135^\circ) + i\sin(135^\circ))$

7. $5\cdot(\cos(270^\circ) + i\sin(270^\circ))$

Convert to polar form:

3. $-7+7\sqrt{3}i$

$c=|-7\sqrt{3}+7i|=\sqrt{(-7)^2+(7\sqrt{3})^2}$
$=\sqrt{49+49\cdot 3}=\sqrt{196}=14$

$\theta=\arctan(\frac{7\sqrt{3}}{-7})+180^\circ$
$=-60^\circ+180^\circ=120^\circ$

Therefore:
$-7+7\sqrt{3}i$
$=14\cdot(\cos(120^\circ)+i\sin(120^\circ))$

Convert to standard form:

4. $4\cdot(\cos(210^\circ)+i\sin(210^\circ))$
$=4\cdot(\frac{-\sqrt{3}}{2}+i\frac{-1}{2})$
$=4\cdot\frac{-\sqrt{3}}{2}+4\cdot\frac{-1}{2}\cdot i$
$=-2\sqrt{3}-2i$

Convert to standard form:

5. $6\cdot(\cos(300^\circ)+i\sin(300^\circ))$
$=6\cdot(\frac{1}{2}+i\frac{-\sqrt{3}}{2})$
$=6\cdot\frac{1}{2}+6\cdot\frac{-\sqrt{3}}{2}\cdot i$
$=3-3\sqrt{3}i$

6. $7\cdot(\cos(135^\circ)+i\sin(135^\circ))$
$=7\cdot(\frac{-\sqrt{2}}{2}+i\frac{\sqrt{2}}{2})$
$=\frac{-7\sqrt{2}}{2}+\frac{7\sqrt{2}}{2}\cdot i$

7. $5\cdot(\cos(270^\circ)+i\sin(270^\circ))$

Converting polar form ↔ standard form - exercises

Convert to polar form:

3. $-7+7\sqrt{3}i$

   $c = |-7\sqrt{3}+7i| = \sqrt{(-7)^2+(7\sqrt{3})^2}$
   $= \sqrt{49+49\cdot 3} = \sqrt{196} = 14$
   $\theta = \arctan(\frac{7\sqrt{3}}{-7}) + 180^\circ$
   $= -60^\circ + 180^\circ = 120^\circ$
   Therefore:
   $-7+7\sqrt{3}i$
   $= 14\cdot(\cos(120^\circ) + i\sin(120^\circ))$

Convert to standard form:

4. $4\cdot(\cos(210^\circ) + i\sin(210^\circ))$
   $= 4\cdot(\frac{-\sqrt{3}}{2} + i\frac{-1}{2})$
   $= 4\cdot\frac{-\sqrt{3}}{2} + 4\cdot\frac{-1}{2}\cdot i$
   $= -2\sqrt{3} - 2i$

Convert to standard form:

5. $6\cdot(\cos(300^\circ) + i\sin(300^\circ))$
   $= 6\cdot(\frac{1}{2} + i\frac{-\sqrt{3}}{2})$
   $= 6\cdot\frac{1}{2} + 6\cdot\frac{-\sqrt{3}}{2}\cdot i$
   $= 3 - 3\sqrt{3}i$

6. $7\cdot(\cos(135^\circ) + i\sin(135^\circ))$
   $= 7\cdot(\frac{-\sqrt{2}}{2} + i\frac{\sqrt{2}}{2})$
   $= \frac{-7\sqrt{2}}{2} + \frac{7\sqrt{2}}{2}\cdot i$

7. $5\cdot(\cos(270^\circ) + i\sin(270^\circ))$
   $= 5\cdot(0 + i(-1))$
   $= 0 - 5i$
   $= -5i$

Multiplying and dividing complex numbers in polar form

Multiplying and dividing complex numbers in polar form

Multiplication in polar form:

$$\boxed{c_1(\cos\theta_1 + i\sin\theta_1)\cdot c_2(\cos\theta_2 + i\sin\theta_2) = c_1c_2\cdot(\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2))}$$

Division in polar form:

$$\boxed{\frac{c_1(\cos\theta_1 + i\sin\theta_1)}{c_2(\cos\theta_2 + i\sin\theta_2)} = \frac{c_1}{c_2}\cdot(\cos(\theta_1-\theta_2) + i\sin(\theta_1-\theta_2))}$$

Multiplying and dividing complex numbers in polar form

Multiplication in polar form:

$$c_1(\cos\theta_1 + i\sin\theta_1)\cdot c_2(\cos\theta_2 + i\sin\theta_2) = c_1c_2\cdot(\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2))$$

Division in polar form:

$$\frac{c_1(\cos\theta_1 + i\sin\theta_1)}{c_2(\cos\theta_2 + i\sin\theta_2)} = \frac{c_1}{c_2}\cdot(\cos(\theta_1-\theta_2) + i\sin(\theta_1-\theta_2))$$

Perform the multiplication or division in polar form:

1. $$3(\cos 77^\circ + i\sin 77^\circ)\cdot 4(\cos 54^\circ + i\sin 54^\circ)$$
2. $$6(\cos 99^\circ + i\sin 99^\circ)\cdot 10(\cos 183^\circ + i\sin 183^\circ)$$
3. $$\frac{8(\cos 149^\circ + i\sin 149^\circ)}{6(\cos 36^\circ + i\sin 36^\circ)}$$
4. $$\frac{20(\cos 50^\circ + i\sin 50^\circ)}{4(\cos 70^\circ + i\sin 70^\circ)}$$

Multiplying and dividing complex numbers in polar form

Multiplication in polar form:

$$c_1(\cos\theta_1 + i\sin\theta_1)\cdot c_2(\cos\theta_2 + i\sin\theta_2) = c_1c_2\cdot(\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2))$$

Division in polar form:

$$\frac{c_1(\cos\theta_1 + i\sin\theta_1)}{c_2(\cos\theta_2 + i\sin\theta_2)} = \frac{c_1}{c_2}\cdot(\cos(\theta_1-\theta_2) + i\sin(\theta_1-\theta_2))$$

Perform the multiplication or division in polar form:

1. $3(\cos 77^\circ + i\sin 77^\circ)\cdot 4(\cos 54^\circ + i\sin 54^\circ)$
   $= 12\cdot(\cos 131^\circ + i\sin 131^\circ)$
2. $6(\cos 99^\circ + i\sin 99^\circ)\cdot 10(\cos 183^\circ + i\sin 183^\circ)$
3. $\frac{8(\cos 149^\circ + i\sin 149^\circ)}{6(\cos 36^\circ + i\sin 36^\circ)}$
4. $\frac{20(\cos 50^\circ + i\sin 50^\circ)}{4(\cos 70^\circ + i\sin 70^\circ)}$

Multiplying and dividing complex numbers in polar form

Multiplication in polar form:

$$c_1(\cos\theta_1 + i\sin\theta_1)\cdot c_2(\cos\theta_2 + i\sin\theta_2) = c_1c_2\cdot(\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2))$$

Division in polar form:

$$\frac{c_1(\cos\theta_1 + i\sin\theta_1)}{c_2(\cos\theta_2 + i\sin\theta_2)} = \frac{c_1}{c_2}\cdot(\cos(\theta_1-\theta_2) + i\sin(\theta_1-\theta_2))$$

Perform the multiplication or division in polar form:

1. $3(\cos 77^\circ + i\sin 77^\circ)\cdot 4(\cos 54^\circ + i\sin 54^\circ)$
   $= 12\cdot(\cos 131^\circ + i\sin 131^\circ)$
2. $6(\cos 99^\circ + i\sin 99^\circ)\cdot 10(\cos 183^\circ + i\sin 183^\circ)$
   $= 60\cdot(\cos 282^\circ + i\sin 282^\circ)$
3. $\frac{8(\cos 149^\circ + i\sin 149^\circ)}{6(\cos 36^\circ + i\sin 36^\circ)}$
4. $\frac{20(\cos 50^\circ + i\sin 50^\circ)}{4(\cos 70^\circ + i\sin 70^\circ)}$

Multiplying and dividing complex numbers in polar form

Multiplication in polar form:

$$c_1(\cos\theta_1 + i\sin\theta_1)\cdot c_2(\cos\theta_2 + i\sin\theta_2) = c_1c_2\cdot(\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2))$$

Division in polar form:

$$\frac{c_1(\cos\theta_1 + i\sin\theta_1)}{c_2(\cos\theta_2 + i\sin\theta_2)} = \frac{c_1}{c_2}\cdot(\cos(\theta_1-\theta_2) + i\sin(\theta_1-\theta_2))$$

Perform the multiplication or division in polar form:

1. $3(\cos 77^\circ + i\sin 77^\circ)\cdot 4(\cos 54^\circ + i\sin 54^\circ)$
   $= 12\cdot(\cos 131^\circ + i\sin 131^\circ)$
2. $6(\cos 99^\circ + i\sin 99^\circ)\cdot 10(\cos 183^\circ + i\sin 183^\circ)$
   $= 60\cdot(\cos 282^\circ + i\sin 282^\circ)$
3. $\frac{8(\cos 149^\circ + i\sin 149^\circ)}{6(\cos 36^\circ + i\sin 36^\circ)}$
   $= \frac{4}{3}\cdot(\cos 113^\circ + i\sin 113^\circ)$
4. $\frac{20(\cos 50^\circ + i\sin 50^\circ)}{4(\cos 70^\circ + i\sin 70^\circ)}$

Multiplying and dividing complex numbers in polar form

Multiplication in polar form:

$$c_1(\cos\theta_1 + i\sin\theta_1)\cdot c_2(\cos\theta_2 + i\sin\theta_2) = c_1c_2\cdot(\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2))$$

Division in polar form:

$$\frac{c_1(\cos\theta_1 + i\sin\theta_1)}{c_2(\cos\theta_2 + i\sin\theta_2)} = \frac{c_1}{c_2}\cdot(\cos(\theta_1-\theta_2) + i\sin(\theta_1-\theta_2))$$

Perform the multiplication or division in polar form:

1. $$3(\cos 77^\circ + i\sin 77^\circ)\cdot 4(\cos 54^\circ + i\sin 54^\circ)$$
   $$= 12\cdot(\cos 131^\circ + i\sin 131^\circ)$$
2. $$6(\cos 99^\circ + i\sin 99^\circ)\cdot 10(\cos 183^\circ + i\sin 183^\circ)$$
   $$= 60\cdot(\cos 282^\circ + i\sin 282^\circ)$$
3. $$\frac{8(\cos 149^\circ + i\sin 149^\circ)}{6(\cos 36^\circ + i\sin 36^\circ)}$$
   $$= \frac{4}{3}\cdot(\cos 113^\circ + i\sin 113^\circ)$$
4. $$\frac{20(\cos 50^\circ + i\sin 50^\circ)}{4(\cos 70^\circ + i\sin 70^\circ)}$$
   $$= 5\cdot(\cos(-20^\circ) + i\sin(-20^\circ))$$

Multiplying and dividing complex numbers in polar form - exercises

Multiply or divide and write your answer in $a + bi$ form without approximation:

1. $$2(\cos 35^\circ + i \sin 35^\circ) \cdot 7(\cos 100^\circ + i \sin 100^\circ)$$

Multiplying and dividing complex numbers in polar form - exercises

Multiply or divide and write your answer in $a + bi$ form without approximation:

1.
$$2(\cos 35^\circ + i \sin 35^\circ) \cdot 7(\cos 100^\circ + i \sin 100^\circ)$$
$$= 14 \cdot (\cos 135^\circ + i \sin 135^\circ)$$
$$= 14 \cdot (\tfrac{-\sqrt{2}}{2} + i \tfrac{\sqrt{2}}{2})$$
$$= -7\sqrt{2} + 7\sqrt{2}i$$

Multiplying and dividing complex numbers in polar form - exercises

Multiply or divide and write your answer in $a + bi$ form without approximation:

1. $$2(\cos 35^\circ + i \sin 35^\circ) \cdot 7(\cos 100^\circ + i \sin 100^\circ)$$
   $$= 14 \cdot (\cos 135^\circ + i \sin 135^\circ)$$
   $$= 14 \cdot (\tfrac{-\sqrt{2}}{2} + i \tfrac{\sqrt{2}}{2})$$
   $$= -7\sqrt{2} + 7\sqrt{2}i$$
2. $$3(\cos 123^\circ + i \sin 123^\circ) \cdot 5(\cos 177^\circ + i \sin 177^\circ)$$
3. $$8(\cos \tfrac{\pi}{6} + i \sin \tfrac{\pi}{6}) \cdot 4(\cos \tfrac{\pi}{3} + i \sin \tfrac{\pi}{3})$$
4. $$\frac{5(\cos 217^\circ + i \sin 217^\circ)}{9(\cos 67^\circ + i \sin 67^\circ)}$$
5. $$\frac{6(\cos 339^\circ + i \sin 339^\circ)}{7(\cos 114^\circ + i \sin 114^\circ)}$$
6. $$\frac{10(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3})}{25(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3})}$$

Multiplying and dividing complex numbers in polar form - exercises

Multiply or divide and write your answer in $a + bi$ form without approximation:

1. $$\begin{aligned} &2(\cos 35^\circ + i \sin 35^\circ) \cdot 7(\cos 100^\circ + i \sin 100^\circ) \\ &= 14 \cdot (\cos 135^\circ + i \sin 135^\circ) \\ &= 14 \cdot (\tfrac{-\sqrt{2}}{2} + i \tfrac{\sqrt{2}}{2}) \\ &= -7\sqrt{2} + 7\sqrt{2}i \end{aligned}$$

2. $$\begin{aligned} &3(\cos 123^\circ + i \sin 123^\circ) \cdot 5(\cos 177^\circ + i \sin 177^\circ) \\ &= 15 \cdot (\cos 300^\circ + i \sin 300^\circ) \\ &= 15 \cdot (\tfrac{1}{2} + \tfrac{-\sqrt{3}}{2} i) \\ &= \tfrac{15}{2} - \tfrac{15\sqrt{3}}{2} i \end{aligned}$$

3. $$8(\cos \tfrac{\pi}{6} + i \sin \tfrac{\pi}{6}) \cdot 4(\cos \tfrac{\pi}{3} + i \sin \tfrac{\pi}{3})$$

4. $$\frac{5(\cos 217^\circ + i \sin 217^\circ)}{9(\cos 67^\circ + i \sin 67^\circ)}$$

5. $$\frac{6(\cos 339^\circ + i \sin 339^\circ)}{7(\cos 114^\circ + i \sin 114^\circ)}$$

6. $$\frac{10(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3})}{25(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3})}$$

Multiplying and dividing complex numbers in polar form - exercises

Multiply or divide and write your answer in $a + bi$ form without approximation:

1. $$2(\cos 35^\circ + i \sin 35^\circ) \cdot 7(\cos 100^\circ + i \sin 100^\circ)$$
   $$= 14 \cdot (\cos 135^\circ + i \sin 135^\circ)$$
   $$= 14 \cdot \left(\frac{-\sqrt{2}}{2} + i \frac{\sqrt{2}}{2}\right)$$
   $$= -7\sqrt{2} + 7\sqrt{2}i$$

2. $$3(\cos 123^\circ + i \sin 123^\circ) \cdot 5(\cos 177^\circ + i \sin 177^\circ)$$
   $$= 15 \cdot (\cos 300^\circ + i \sin 300^\circ)$$
   $$= 15 \cdot \left(\frac{1}{2} + \frac{-\sqrt{3}}{2} i\right)$$
   $$= \frac{15}{2} - \frac{15\sqrt{3}}{2} i$$

3. $$8\left(\cos \tfrac{\pi}{6} + i \sin \tfrac{\pi}{6}\right) \cdot 4\left(\cos \tfrac{\pi}{3} + i \sin \tfrac{\pi}{3}\right)$$
   $$= 32 \cdot \left(\cos \tfrac{\pi}{2} + i \sin \tfrac{\pi}{2}\right)$$
   $$= 32 \cdot (0 + i \cdot 1)$$
   $$= 0 + 32i$$
   $$= 32i$$

4. $$\frac{5(\cos 217^\circ + i \sin 217^\circ)}{9(\cos 67^\circ + i \sin 67^\circ)}$$

5. $$\frac{6(\cos 339^\circ + i \sin 339^\circ)}{7(\cos 114^\circ + i \sin 114^\circ)}$$

6. $$\frac{10(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3})}{25(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3})}$$

Multiplying and dividing complex numbers in polar form - exercises

Multiply or divide and write your answer in $a + bi$ form without approximation:

1. $$2(\cos 35^\circ + i \sin 35^\circ) \cdot 7(\cos 100^\circ + i \sin 100^\circ)$$
   $$= 14 \cdot (\cos 135^\circ + i \sin 135^\circ)$$
   $$= 14 \cdot \left(\frac{-\sqrt{2}}{2} + i \frac{\sqrt{2}}{2}\right)$$
   $$= -7\sqrt{2} + 7\sqrt{2}i$$

2. $$3(\cos 123^\circ + i \sin 123^\circ) \cdot 5(\cos 177^\circ + i \sin 177^\circ)$$
   $$= 15 \cdot (\cos 300^\circ + i \sin 300^\circ)$$
   $$= 15 \cdot \left(\frac{1}{2} + \frac{-\sqrt{3}}{2} i\right)$$
   $$= \frac{15}{2} - \frac{15\sqrt{3}}{2} i$$

3. $$8(\cos \tfrac{\pi}{6} + i \sin \tfrac{\pi}{6}) \cdot 4(\cos \tfrac{\pi}{3} + i \sin \tfrac{\pi}{3})$$
   $$= 32 \cdot (\cos \tfrac{\pi}{2} + i \sin \tfrac{\pi}{2})$$
   $$= 32 \cdot (0 + i \cdot 1)$$
   $$= 0 + 32i$$
   $$= 32i$$

4. $$\frac{5(\cos 217^\circ + i \sin 217^\circ)}{9(\cos 67^\circ + i \sin 67^\circ)}$$
   $$= \frac{5}{9} \cdot (\cos 150^\circ + i \sin 150^\circ)$$
   $$= \frac{5}{9} \cdot \left(-\frac{\sqrt{3}}{2} + \frac{1}{2} i\right)$$
   $$= -\frac{5\sqrt{3}}{18} + \frac{5}{18} i$$

5. $$\frac{6(\cos 339^\circ + i \sin 339^\circ)}{7(\cos 114^\circ + i \sin 114^\circ)}$$

6. $$\frac{10(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3})}{25(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3})}$$

Multiplying and dividing complex numbers in polar form - exercises

Multiply or divide and write your answer in $a + bi$ form without approximation:

1. $$\begin{aligned} &2(\cos 35^\circ + i \sin 35^\circ) \cdot 7(\cos 100^\circ + i \sin 100^\circ) \\ &= 14 \cdot (\cos 135^\circ + i \sin 135^\circ) \\ &= 14 \cdot (\tfrac{-\sqrt{2}}{2} + i \tfrac{\sqrt{2}}{2}) \\ &= -7\sqrt{2} + 7\sqrt{2} i \end{aligned}$$

2. $$\begin{aligned} &3(\cos 123^\circ + i \sin 123^\circ) \cdot 5(\cos 177^\circ + i \sin 177^\circ) \\ &= 15 \cdot (\cos 300^\circ + i \sin 300^\circ) \\ &= 15 \cdot (\tfrac{1}{2} + \tfrac{-\sqrt{3}}{2} i) \\ &= \tfrac{15}{2} - \tfrac{15\sqrt{3}}{2} i \end{aligned}$$

3. $$\begin{aligned} &8(\cos \tfrac{\pi}{6} + i \sin \tfrac{\pi}{6}) \cdot 4(\cos \tfrac{\pi}{3} + i \sin \tfrac{\pi}{3}) \\ &= 32 \cdot (\cos \tfrac{\pi}{2} + i \sin \tfrac{\pi}{2}) \\ &= 32 \cdot (0 + i \cdot 1) \\ &= 0 + 32i \\ &= 32i \end{aligned}$$

4. $$\begin{aligned} &\frac{5(\cos 217^\circ + i \sin 217^\circ)}{9(\cos 67^\circ + i \sin 67^\circ)} \\ &= \tfrac{5}{9} \cdot (\cos 150^\circ + i \sin 150^\circ) \\ &= \tfrac{5}{9} \cdot (-\tfrac{\sqrt{3}}{2} + \tfrac{1}{2} i) \\ &= -\tfrac{5\sqrt{3}}{18} + \tfrac{5}{18} i \end{aligned}$$

5. $$\begin{aligned} &\frac{6(\cos 339^\circ + i \sin 339^\circ)}{7(\cos 114^\circ + i \sin 114^\circ)} \\ &= \tfrac{6}{7} \cdot (\cos 225^\circ + i \sin 225^\circ) \\ &= \tfrac{6}{7} \cdot (-\tfrac{\sqrt{2}}{2} - \tfrac{\sqrt{2}}{2} i) \\ &= -\tfrac{6\sqrt{2}}{14} - \tfrac{6\sqrt{2}}{14} i \\ &= -\tfrac{3\sqrt{2}}{7} - \tfrac{3\sqrt{2}}{7} i \end{aligned}$$

6. $$\frac{10(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3})}{25(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3})}$$

Multiplying and dividing complex numbers in polar form - exercises

Multiply or divide and write your answer in $a + bi$ form without approximation:

1. $$\begin{aligned} &2(\cos 35^\circ + i \sin 35^\circ) \cdot 7(\cos 100^\circ + i \sin 100^\circ) \\ &= 14 \cdot (\cos 135^\circ + i \sin 135^\circ) \\ &= 14 \cdot \left(\tfrac{-\sqrt{2}}{2} + i \tfrac{\sqrt{2}}{2}\right) \\ &= -7\sqrt{2} + 7\sqrt{2}i \end{aligned}$$

2. $$\begin{aligned} &3(\cos 123^\circ + i \sin 123^\circ) \cdot 5(\cos 177^\circ + i \sin 177^\circ) \\ &= 15 \cdot (\cos 300^\circ + i \sin 300^\circ) \\ &= 15 \cdot \left(\tfrac{1}{2} + \tfrac{-\sqrt{3}}{2} i\right) \\ &= \tfrac{15}{2} - \tfrac{15\sqrt{3}}{2} i \end{aligned}$$

3. $$\begin{aligned} &8(\cos \tfrac{\pi}{6} + i \sin \tfrac{\pi}{6}) \cdot 4(\cos \tfrac{\pi}{3} + i \sin \tfrac{\pi}{3}) \\ &= 32 \cdot (\cos \tfrac{\pi}{2} + i \sin \tfrac{\pi}{2}) \\ &= 32 \cdot (0 + i \cdot 1) \\ &= 0 + 32i \\ &= 32i \end{aligned}$$

4. $$\begin{aligned} &\frac{5(\cos 217^\circ + i \sin 217^\circ)}{9(\cos 67^\circ + i \sin 67^\circ)} \\ &= \tfrac{5}{9} \cdot (\cos 150^\circ + i \sin 150^\circ) \\ &= \tfrac{5}{9} \cdot \left(-\tfrac{\sqrt{3}}{2} + \tfrac{1}{2} i\right) \\ &= -\tfrac{5\sqrt{3}}{18} + \tfrac{5}{18} i \end{aligned}$$

5. $$\begin{aligned} &\frac{6(\cos 339^\circ + i \sin 339^\circ)}{7(\cos 114^\circ + i \sin 114^\circ)} \\ &= \tfrac{6}{7} \cdot (\cos 225^\circ + i \sin 225^\circ) \\ &= \tfrac{6}{7} \cdot \left(-\tfrac{\sqrt{2}}{2} - \tfrac{\sqrt{2}}{2} i\right) \\ &= -\tfrac{6\sqrt{2}}{14} - \tfrac{6\sqrt{2}}{14} i \\ &= -\tfrac{3\sqrt{2}}{7} - \tfrac{3\sqrt{2}}{7} i \end{aligned}$$

6. $$\begin{aligned} &\frac{10(\cos \frac{5\pi}{3} + i \sin \frac{5\pi}{3})}{25(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3})} \\ &= \tfrac{2}{5} \cdot \left(\cos(\tfrac{4\pi}{3}) + i \sin(\tfrac{4\pi}{3})\right) \\ &= \tfrac{2}{5} \cdot \left(-\tfrac{1}{2} - \tfrac{\sqrt{3}}{2} i\right) \\ &= -\tfrac{1}{5} - \tfrac{\sqrt{3}}{5} i \end{aligned}$$