Graphs of trigonometric functions
Lesson #18

MAT 1375 Precalculus

New York City College of Technology CUNY
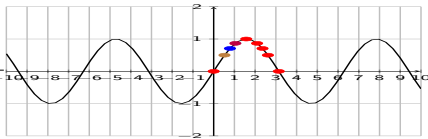
Graphs of sin, cos, and tan

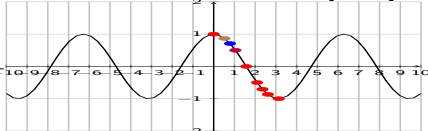- Graph the function $y = \sin(x)$ domain $D = \mathbb{R}$ range $R = [-1, 1]$

x	0°	30°	45°	60°	90°	120°	135°	150°	180°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
y	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\approx$	0	.5	.707	.866	1	.866	.707	.5	0

- Graph the function $y = \cos(x)$ domain $D = \mathbb{R}$ range $R = [-1, 1]$

x	0°	30°	45°	60°	90°	120°	135°	150°	180°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
y	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
$\approx$	1	.866	.707	.5	0	$-.5$	$-.707$	$-.866$	-1

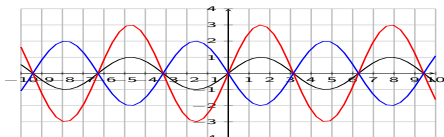- Graph the function $y = \tan(x)$ domain $D = \mathbb{R} - \{\frac{\pi}{2} + k\pi \mid k \in \mathbb{Z}\}$ range $R = \mathbb{R}$

x	0°	30°	45°	60°	90°	120°	135°	150°	180°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
y	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undef	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0
$\approx$	0	.577	1	1.73		-1.73	-1	$-.577$	0

Amplitude, period, and phase shift for sin, cos

- Amplitude A
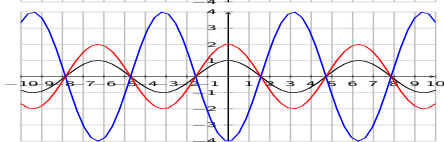
$y = \sin(x)$ $A = 1$
$y = 3\sin(x)$ $A = 3$
$y = -2\sin(x)$ $A = 2$

$y = \cos(x)$ $A = 1$
$y = 2\cos(x)$ $A = 2$
$y = -4\cos(x)$ $A = 4$
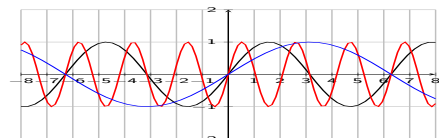
- Period P

$y = \sin(x)$ $P = 2\pi$
$y = \sin(3x)$ $P = \frac{2\pi}{3}$
$y = \sin(\frac{1}{2}x)$ $P = \frac{2\pi}{\frac{1}{2}} = 2\pi \cdot \frac{2}{1}$
$= 4\pi$
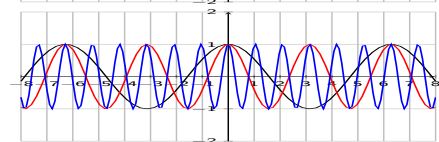
$y = \cos(x)$ $P = 2\pi$
$y = \cos(2x)$ $P = \frac{2\pi}{2} = \pi$
$y = \cos(6x)$ $P = \frac{2\pi}{6} = \frac{\pi}{3}$

Amplitude, period, and phase shift for sin, cos

- Phase shift S
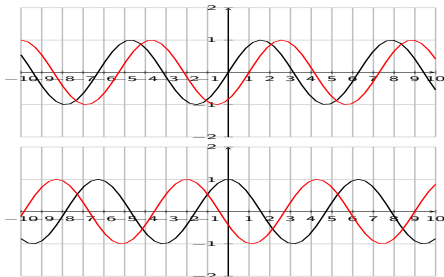
$y = \sin(x)$ $S = 0$
$y = \sin(x - 1)$ $S = 1$

$y = \cos(x)$ $S = 0$
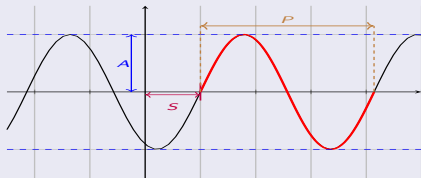$y = \cos(x + 2)$ $S = -2$

Amplitude, period, phase shift

When $y = a \cdot \sin(bx + c)$ or $y = a \cdot \cos(bx + c)$

Amplitude $A = |a|$

Period $P = \left| \frac{2\pi}{b} \right|$
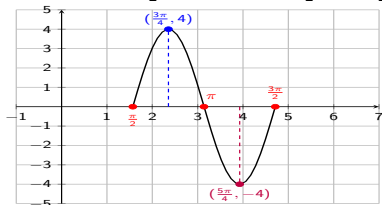
Phase shift $S = \frac{-c}{b}$

Graphing sin and cos - exercises

Find amplitude, period, and phase shift. Graph the function over one period. Label all maxima, minima, and roots.

1 $y = 4\sin(2x - \pi)$

$A = 4$, $P = \frac{2\pi}{2} = \pi$, $S = \frac{-(-\pi)}{2} = \frac{\pi}{2}$

Roots:
- $S = \frac{\pi}{2}$
- $S + P = \frac{\pi}{2} + \pi = \frac{3\pi}{2}$
- $\left(\frac{\pi}{2} + \frac{3\pi}{2}\right) \div 2 = \frac{4\pi}{2} \cdot \frac{1}{2} = \frac{4\pi}{4} = \pi$
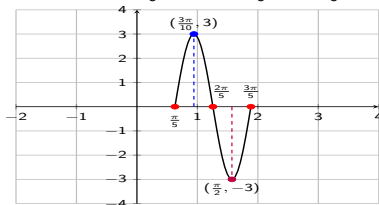
Maximum:
- $\left(\frac{\pi}{2} + \pi\right) \div 2 = \frac{3\pi}{2} \cdot \frac{1}{2} = \frac{3\pi}{4}$

Minimum:
- $\left(\pi + \frac{3\pi}{2}\right) \div 2 = \frac{5\pi}{2} \cdot \frac{1}{2} = \frac{5\pi}{4}$

2 $y = 3\sin(5x - \pi)$

$A = 3$, $P = \frac{2\pi}{5}$, $S = \frac{-(-\pi)}{5} = \frac{\pi}{5}$

Roots:
- $S = \frac{\pi}{5}$
- $S + P = \frac{\pi}{5} + \frac{2\pi}{5} = \frac{3\pi}{5}$
- $\left(\frac{\pi}{5} + \frac{3\pi}{5}\right) \div 2 = \frac{4\pi}{5} \cdot \frac{1}{2} = \frac{2\pi}{5}$

Maximum:
- $\left(\frac{\pi}{5} + \frac{2\pi}{5}\right) \div 2 = \frac{3\pi}{5} \cdot \frac{1}{2} = \frac{3\pi}{10}$
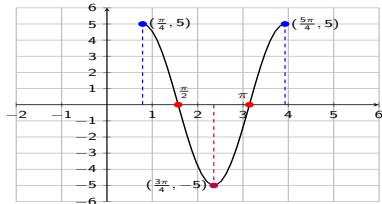
Minimum:
- $\left(\frac{2\pi}{5} + \frac{3\pi}{5}\right) \div 2 = \frac{5\pi}{5} \cdot \frac{1}{2} = \frac{\pi}{2}$

Graphing sin and cos - exercises

Find amplitude, period, and phase shift. Graph the function over one period. Label all maxima, minima, and roots.

3 $y = 5\cos(2x - \frac{\pi}{2})$

$A = 5$, $P = \frac{2\pi}{2} = \pi$, $S = \frac{-(-\frac{\pi}{2})}{2} = \frac{\pi}{4}$

Maxima:
- $S = \frac{\pi}{4}$
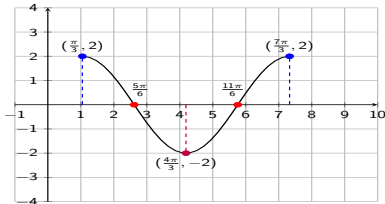- $S + P = \frac{\pi}{4} + \pi = \frac{5\pi}{4}$

Minimum:
- $\left(\frac{\pi}{4} + \frac{5\pi}{4}\right) \div 2 = \frac{6\pi}{4} \cdot \frac{1}{2} = \frac{6\pi}{8} = \frac{3\pi}{4}$

Roots:
- $\left(\frac{\pi}{4} + \frac{3\pi}{4}\right) \div 2 = \frac{4\pi}{4} \cdot \frac{1}{2} = \frac{\pi}{2}$
- $\left(\frac{3\pi}{4} + \frac{5\pi}{4}\right) \div 2 = \frac{8\pi}{4} \cdot \frac{1}{2} = \frac{8\pi}{8} = \pi$

4 $y = 2\cos(x - \frac{\pi}{3})$

$A = 2$, $P = \frac{2\pi}{1} = 2\pi$, $S = \frac{-(-\frac{\pi}{3})}{1} = \frac{\pi}{3}$

Maxima:
- $S = \frac{\pi}{3}$
- $S + P = \frac{\pi}{3} + 2\pi = \frac{7\pi}{3}$

Minimum:
- $\left(\frac{\pi}{3} + \frac{7\pi}{3}\right) \div 2 = \frac{8\pi}{3} \cdot \frac{1}{2} = \frac{8\pi}{6} = \frac{4\pi}{3}$
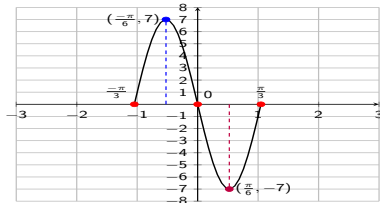
Roots:
- $\left(\frac{\pi}{3} + \frac{4\pi}{3}\right) \div 2 = \frac{5\pi}{3} \cdot \frac{1}{2} = \frac{5\pi}{6}$
- $\left(\frac{4\pi}{3} + \frac{7\pi}{3}\right) \div 2 = \frac{11\pi}{3} \cdot \frac{1}{2} = \frac{11\pi}{6}$

Graphing sin and cos - exercises

Find amplitude, period, and phase shift. Graph the function over one period. Label all maxima, minima, and roots.

5 $y = 7\sin(3x + \pi)$

$A = 7$, $P = \frac{2\pi}{3}$, $S = \frac{-\pi}{3}$

Roots:
- $S = \frac{-\pi}{3}$
- $S + P = \frac{-\pi}{3} + \frac{2\pi}{3} = \frac{\pi}{3}$
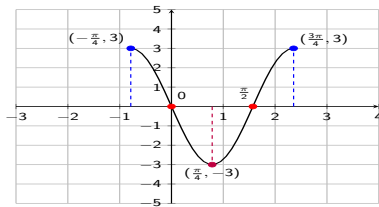- $\left(\frac{-\pi}{3} + \frac{\pi}{3}\right) \div 2 = 0 \cdot \frac{1}{2} = 0$

Maximum:
- $\left(\frac{-\pi}{3} + 0\right) \div 2 = \frac{-\pi}{3} \cdot \frac{1}{2} = \frac{-\pi}{6}$

Minimum:
- $\left(0 + \frac{\pi}{3}\right) \div 2 = \frac{\pi}{3} \cdot \frac{1}{2} = \frac{\pi}{6}$

6 $y = 3\cos(2x + \frac{\pi}{2})$

$A = 3$, $P = \frac{2\pi}{2} = \pi$, $S = \frac{-\frac{\pi}{2}}{2} = -\frac{\pi}{2} \cdot \frac{1}{2} = -\frac{\pi}{4}$

Maxima:
- $S = -\frac{\pi}{4}$
- $S + P = -\frac{\pi}{4} + \pi = -\frac{\pi}{4} + \frac{4\pi}{4} = \frac{3\pi}{4}$

Minimum:
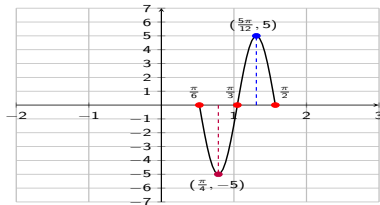- $\left(-\frac{\pi}{4} + \frac{3\pi}{4}\right) \div 2 = \frac{2\pi}{4} \cdot \frac{1}{2} = \frac{\pi}{4}$

Roots:
- $\left(-\frac{\pi}{4} + \frac{\pi}{4}\right) \div 2 = 0 \cdot \frac{1}{2} = 0$
- $\left(\frac{\pi}{4} + \frac{3\pi}{4}\right) \div 2 = \frac{4\pi}{4} \cdot \frac{1}{2} = \frac{\pi}{2}$

Graphing sin and cos - exercises

Find amplitude, period, and phase shift. Graph the function over one period. Label all maxima, minima, and roots.

7 $y = -5\sin(6x - \pi)$

$A = |-5| = 5$, $P = \frac{2\pi}{6} = \frac{\pi}{3}$, $S = \frac{-(-\pi)}{6} = \frac{\pi}{6}$

Roots:
- $S = \frac{\pi}{6}$
- $S + P = \frac{\pi}{6} + \frac{\pi}{3} = \frac{\pi}{6} + \frac{2\pi}{6} = \frac{3\pi}{6} = \frac{\pi}{2}$
- $\left(\frac{\pi}{6} + \frac{\pi}{2}\right) \div 2 = \frac{4\pi}{6} \cdot \frac{1}{2} = \frac{4\pi}{12} = \frac{\pi}{3}$
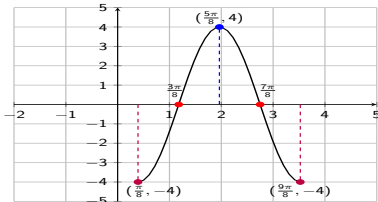
Minimum:
- $\left(\frac{\pi}{6} + \frac{\pi}{3}\right) \div 2 = \frac{3\pi}{6} \cdot \frac{1}{2} = \frac{3\pi}{12} = \frac{\pi}{4}$

Maximum:
- $\left(\frac{\pi}{3} + \frac{\pi}{2}\right) \div 2 = \frac{5\pi}{6} \cdot \frac{1}{2} = \frac{5\pi}{12}$

8 $y = -4\cos(2x - \frac{\pi}{4})$

$A = |-4| = 4$, $P = \frac{2\pi}{2} = \pi$, $S = \frac{-(-\frac{\pi}{4})}{2} = \frac{\pi}{8}$

Minima:
- $S = \frac{\pi}{8}$
- $S + P = \frac{\pi}{8} + \pi = \frac{9\pi}{8}$

Maximum:
- $\left(\frac{\pi}{8} + \frac{9\pi}{8}\right) \div 2 = \frac{10\pi}{8} \cdot \frac{1}{2} = \frac{10\pi}{16} = \frac{5\pi}{8}$

Roots:
- $\left(\frac{\pi}{8} + \frac{5\pi}{8}\right) \div 2 = \frac{6\pi}{8} \cdot \frac{1}{2} = \frac{3\pi}{8}$
- $\left(\frac{5\pi}{8} + \frac{9\pi}{8}\right) \div 2 = \frac{14\pi}{8} \cdot \frac{1}{2} = \frac{7\pi}{8}$