

# Graphs of trigonometric functions

## Lesson #18

MAT 1375 Precalculus

New York City College of Technology CUNY



# Graphs of sin, cos, and tan

- Graph the function  $y = \sin(x)$

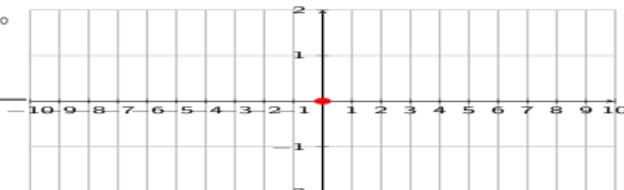
x	0°	30°	45°	60°	90°	120°	135°	150°	180°
y	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$



# Graphs of sin, cos, and tan

- Graph the function  $y = \sin(x)$

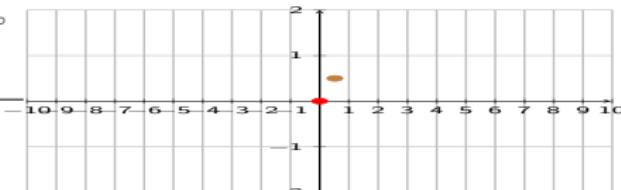
x	0°	30°	45°	60°	90°	120°	135°	150°	180°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$
y	0								
≈	0								



# Graphs of sin, cos, and tan

- Graph the function  $y = \sin(x)$

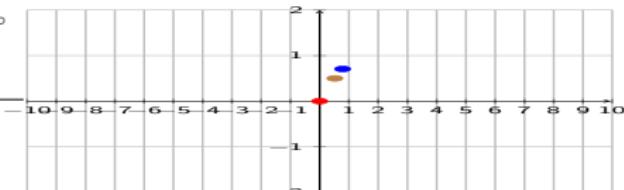
x	0°	30°	45°	60°	90°	120°	135°	150°	180°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$
y	0	$\frac{1}{2}$							
≈	0	.5							



# Graphs of sin, cos, and tan

- Graph the function  $y = \sin(x)$

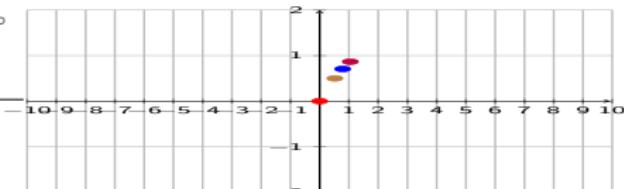
x	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$	$120^\circ$	$135^\circ$	$150^\circ$	$180^\circ$
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$
y	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$						
$\approx$	0	.5	.707						



# Graphs of sin, cos, and tan

- Graph the function  $y = \sin(x)$

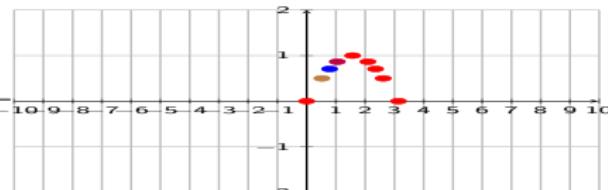
x	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$	$120^\circ$	$135^\circ$	$150^\circ$	$180^\circ$
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$
y	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$					
$\approx$	0	.5	.707	.866					



# Graphs of sin, cos, and tan

- Graph the function  $y = \sin(x)$

x	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$	$120^\circ$	$135^\circ$	$150^\circ$	$180^\circ$
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$
y	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\approx$	0	.5	.707	.866	1	.866	.707	.5	0

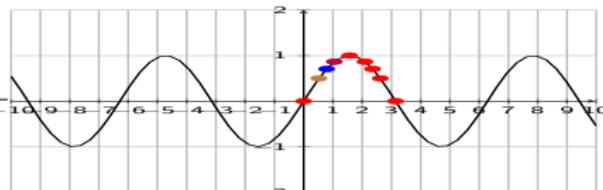


# Graphs of sin, cos, and tan

- Graph the function  $y = \sin(x)$

domain  $D = \mathbb{R}$  range  $R = [-1, 1]$

x	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$	$120^\circ$	$135^\circ$	$150^\circ$	$180^\circ$	$\pi$
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$		
y	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	
$\approx$	0	.5	.707	.866	1	.866	.707	.5	0	

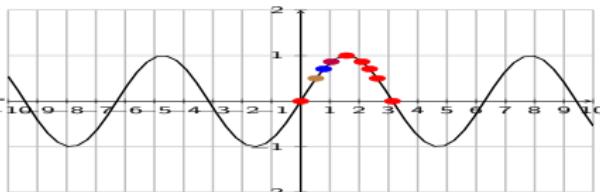


# Graphs of sin, cos, and tan

- Graph the function  $y = \sin(x)$

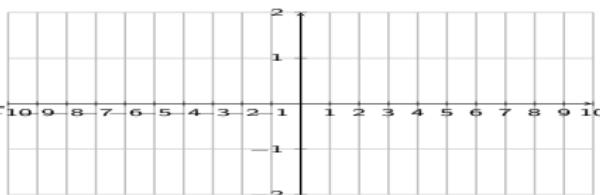
domain  $D = \mathbb{R}$  range  $R = [-1, 1]$

x	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$	$120^\circ$	$135^\circ$	$150^\circ$	$180^\circ$
y	0	$\frac{\pi}{6}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\approx$	0	.5	.707	.866	1	.866	.707	.5	0



- Graph the function  $y = \cos(x)$

x	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$	$120^\circ$	$135^\circ$	$150^\circ$	$180^\circ$
y	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$

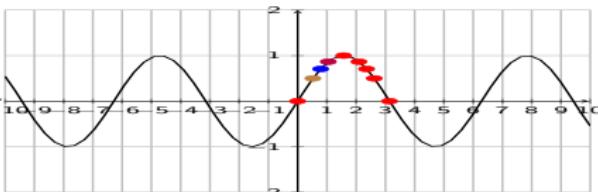


# Graphs of sin, cos, and tan

- Graph the function  $y = \sin(x)$

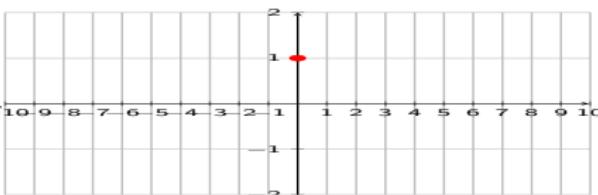
domain  $D = \mathbb{R}$  range  $R = [-1, 1]$

x	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$	$120^\circ$	$135^\circ$	$150^\circ$	$180^\circ$
y	0	$\frac{\pi}{6}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\approx$	0	.5	.707	.866	1	.866	.707	.5	0



- Graph the function  $y = \cos(x)$

x	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$	$120^\circ$	$135^\circ$	$150^\circ$	$180^\circ$
y	1								
$\approx$	1								

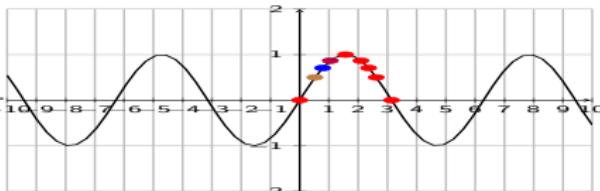


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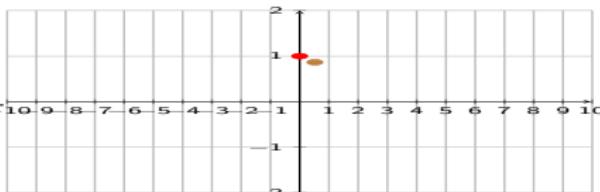
domain  $D = \mathbb{R}$  range  $R = [-1, 1]$

x	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$	$120^\circ$	$135^\circ$	$150^\circ$	$180^\circ$
y	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$
$\approx$	0	.5	.707	.866	1	.866	.707	.5	0



- Graph the function  $y = \cos(x)$

x	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$	$120^\circ$	$135^\circ$	$150^\circ$	$180^\circ$
y	1	$\frac{\sqrt{3}}{2}$		$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
$\approx$	1	.866		.5	0	-.5	-.866	-.866	-1

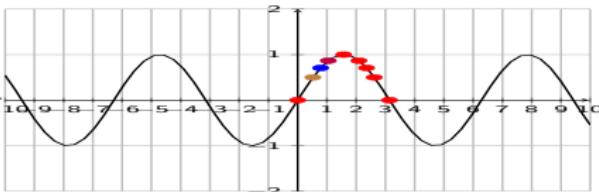


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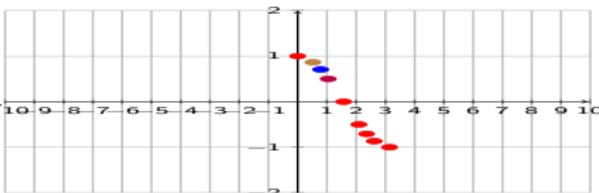
x	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$	$120^\circ$	$135^\circ$	$150^\circ$	$180^\circ$
y	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$
$\approx$	0	.5	.707	.866	1	.866	.707	.5	0

domain  $D = \mathbb{R}$  range  $R = [-1, 1]$



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x	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$	$120^\circ$	$135^\circ$	$150^\circ$	$180^\circ$
y	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
$\approx$	1	.866	.707	.5	0	-.5	-.707	-.866	-1

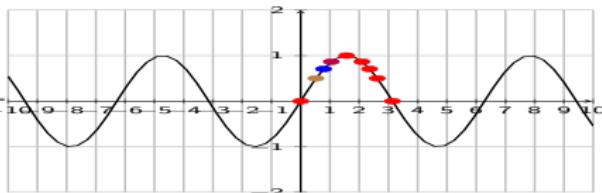


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x	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$	$120^\circ$	$135^\circ$	$150^\circ$	$180^\circ$
x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$
y	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\approx$	0	.5	.707	.866	1	.866	.707	.5	0

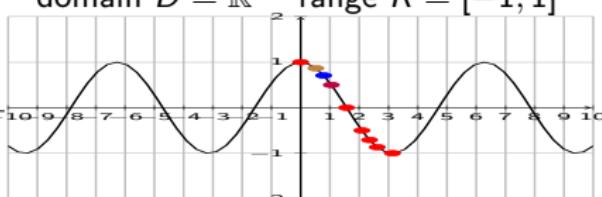
domain  $D = \mathbb{R}$  range  $R = [-1, 1]$



- Graph the function  $y = \cos(x)$

x	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$	$120^\circ$	$135^\circ$	$150^\circ$	$180^\circ$
x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$
y	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
$\approx$	1	.866	.707	.5	0	-.5	-.707	-.866	-1

domain  $D = \mathbb{R}$  range  $R = [-1, 1]$

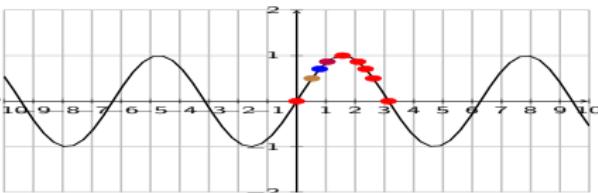


# Graphs of sin, cos, and tan

- Graph the function  $y = \sin(x)$

x	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$	$120^\circ$	$135^\circ$	$150^\circ$	$180^\circ$
x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$
y	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\approx$	0	.5	.707	.866	1	.866	.707	.5	0

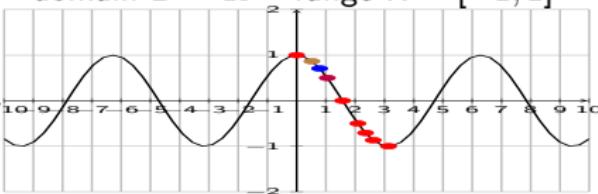
domain  $D = \mathbb{R}$  range  $R = [-1, 1]$



- Graph the function  $y = \cos(x)$

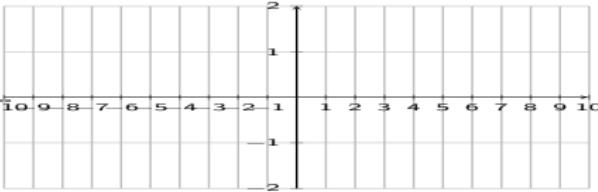
x	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$	$120^\circ$	$135^\circ$	$150^\circ$	$180^\circ$
x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$
y	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
$\approx$	1	.866	.707	.5	0	-.5	-.707	-.866	-1

domain  $D = \mathbb{R}$  range  $R = [-1, 1]$



- Graph the function  $y = \tan(x)$

x	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$	$120^\circ$	$135^\circ$	$150^\circ$	$180^\circ$
x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$
y									

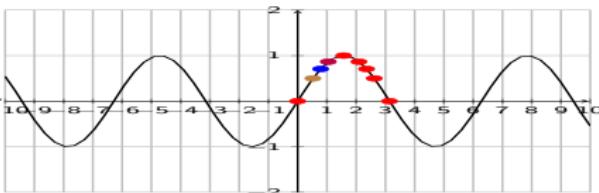


# Graphs of sin, cos, and tan

- Graph the function  $y = \sin(x)$

x	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$	$120^\circ$	$135^\circ$	$150^\circ$	$180^\circ$
x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$
y	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\approx$	0	.5	.707	.866	1	.866	.707	.5	0

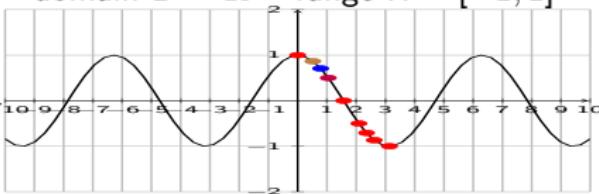
domain  $D = \mathbb{R}$  range  $R = [-1, 1]$



- Graph the function  $y = \cos(x)$

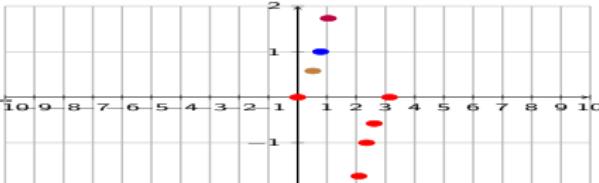
x	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$	$120^\circ$	$135^\circ$	$150^\circ$	$180^\circ$
x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$
y	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
$\approx$	1	.866	.707	.5	0	-.5	-.707	-.866	-1

domain  $D = \mathbb{R}$  range  $R = [-1, 1]$



- Graph the function  $y = \tan(x)$

x	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$	$120^\circ$	$135^\circ$	$150^\circ$	$180^\circ$
x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$
y	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undefined	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0
$\approx$	0	.577	1	1.73		-1.73	-1	-.577	0

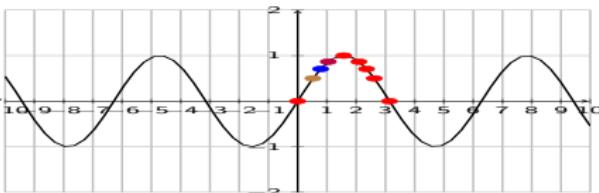


# Graphs of sin, cos, and tan

- Graph the function  $y = \sin(x)$

x	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$	$120^\circ$	$135^\circ$	$150^\circ$	$180^\circ$
x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$
y	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\approx$	0	.5	.707	.866	1	.866	.707	.5	0

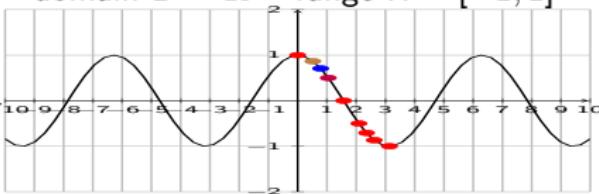
domain  $D = \mathbb{R}$  range  $R = [-1, 1]$



- Graph the function  $y = \cos(x)$

x	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$	$120^\circ$	$135^\circ$	$150^\circ$	$180^\circ$
x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$
y	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
$\approx$	1	.866	.707	.5	0	-.5	-.707	-.866	-1

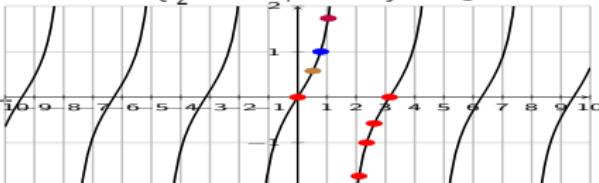
domain  $D = \mathbb{R}$  range  $R = [-1, 1]$



- Graph the function  $y = \tan(x)$

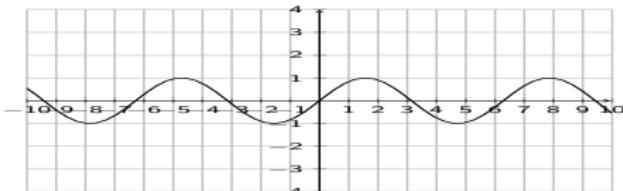
x	$0^\circ$	$30^\circ$	$45^\circ$	$60^\circ$	$90^\circ$	$120^\circ$	$135^\circ$	$150^\circ$	$180^\circ$
x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$
y	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undef	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0
$\approx$	0	.577	1	1.73		-1.73	-1	-.577	0

domain  $D = \mathbb{R} - \{\frac{\pi}{2} + k\pi | k \in \mathbb{Z}\}$  range  $R = \mathbb{R}$



# Amplitude, period, and phase shift for sin, cos

- Amplitude  $A$

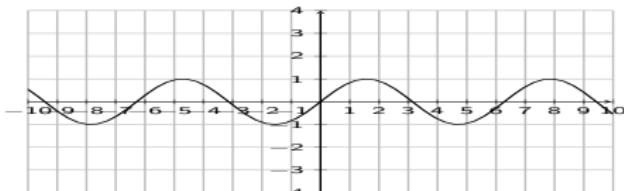


$$y = \sin(x)$$

$$A = 1$$

# Amplitude, period, and phase shift for sin, cos

- Amplitude  $A$

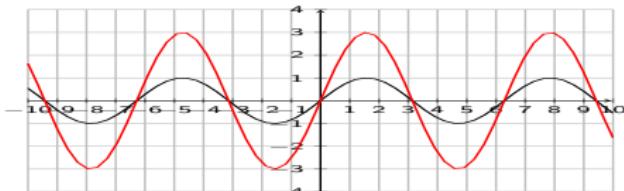


$$y = \sin(x)$$
$$y = 3 \sin(x)$$

$$A = 1$$

# Amplitude, period, and phase shift for sin, cos

- Amplitude  $A$



$$y = \sin(x)$$

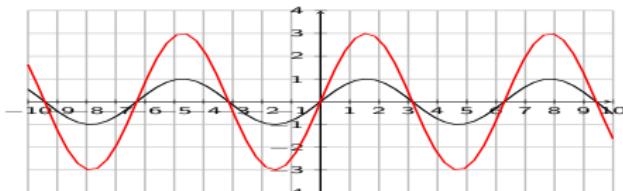
$$y = 3 \sin(x)$$

$$A = 1$$

$$A = 3$$

# Amplitude, period, and phase shift for sin, cos

- Amplitude  $A$



$$y = \sin(x)$$

$$y = 3 \sin(x)$$

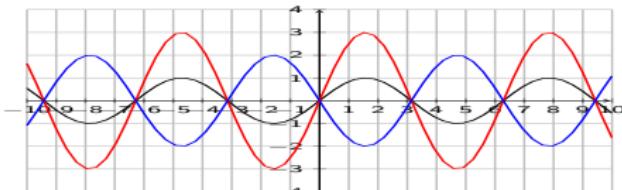
$$y = -2 \sin(x)$$

$$A = 1$$

$$A = 3$$

# Amplitude, period, and phase shift for sin, cos

- Amplitude  $A$



$$y = \sin(x)$$

$$y = 3 \sin(x)$$

$$y = -2 \sin(x)$$

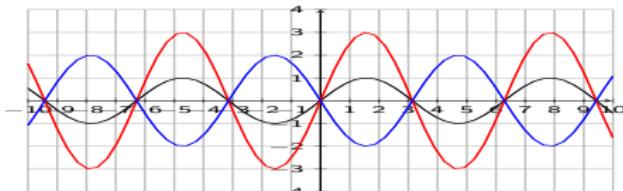
$$A = 1$$

$$A = 3$$

$$A = 2$$

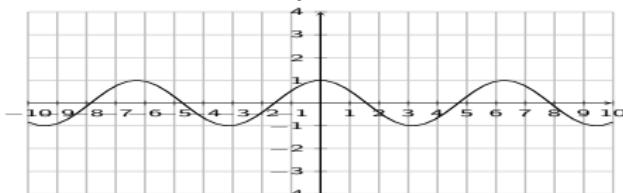
# Amplitude, period, and phase shift for sin, cos

- Amplitude  $A$



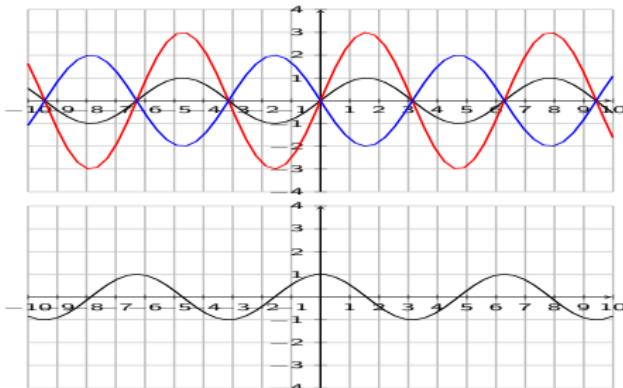
$$\begin{array}{ll} y = \sin(x) & A = 1 \\ y = 3\sin(x) & A = 3 \\ y = -2\sin(x) & A = 2 \end{array}$$

$$y = \cos(x) \quad A = 1$$



# Amplitude, period, and phase shift for sin, cos

- Amplitude  $A$

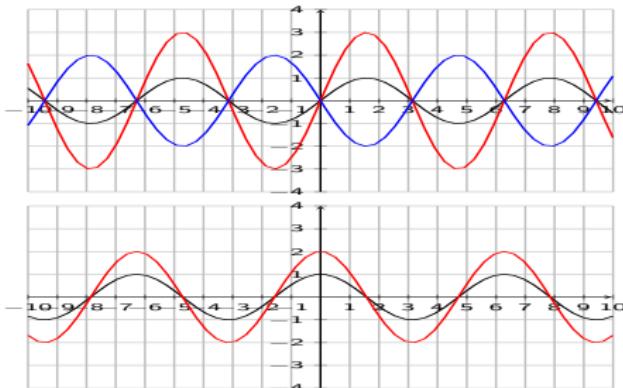


$$\begin{array}{ll} y = \sin(x) & A = 1 \\ y = 3\sin(x) & A = 3 \\ y = -2\sin(x) & A = 2 \end{array}$$

$$\begin{array}{ll} y = \cos(x) & A = 1 \\ y = 2\cos(x) & \end{array}$$

# Amplitude, period, and phase shift for sin, cos

- Amplitude  $A$

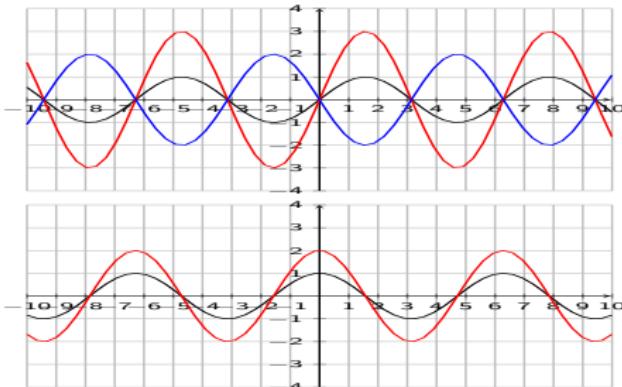


$$\begin{array}{ll} y = \sin(x) & A = 1 \\ y = 3\sin(x) & A = 3 \\ y = -2\sin(x) & A = 2 \end{array}$$

$$\begin{array}{ll} y = \cos(x) & A = 1 \\ y = 2\cos(x) & A = 2 \end{array}$$

# Amplitude, period, and phase shift for sin, cos

- Amplitude  $A$

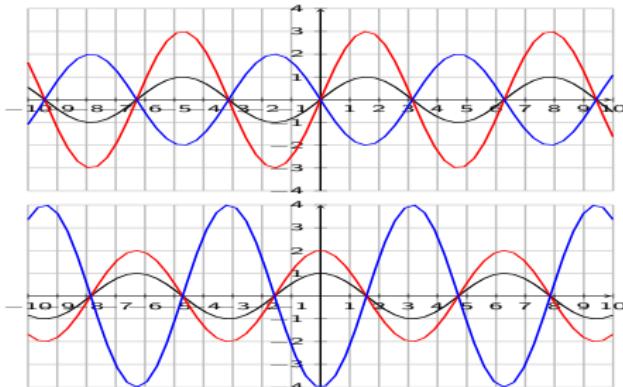


$$\begin{array}{ll} y = \sin(x) & A = 1 \\ y = 3 \sin(x) & A = 3 \\ y = -2 \sin(x) & A = 2 \end{array}$$

$$\begin{array}{ll} y = \cos(x) & A = 1 \\ y = 2 \cos(x) & A = 2 \\ y = -4 \cos(x) & \end{array}$$

# Amplitude, period, and phase shift for sin, cos

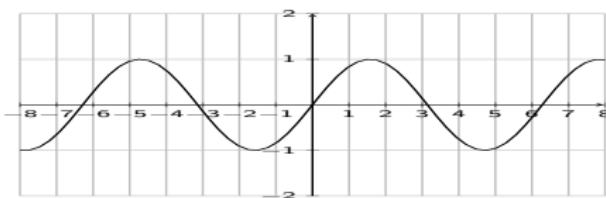
- Amplitude  $A$



$$\begin{array}{ll} y = \sin(x) & A = 1 \\ y = 3\sin(x) & A = 3 \\ y = -2\sin(x) & A = 2 \end{array}$$

$$\begin{array}{ll} y = \cos(x) & A = 1 \\ y = 2\cos(x) & A = 2 \\ y = -4\cos(x) & A = 4 \end{array}$$

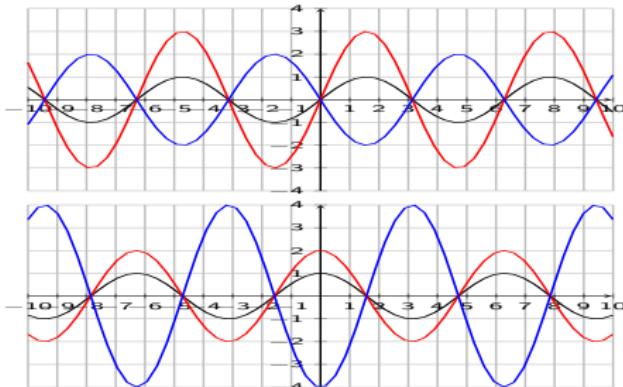
- Period  $P$



$$y = \sin(x) \qquad P = 2\pi$$

# Amplitude, period, and phase shift for sin, cos

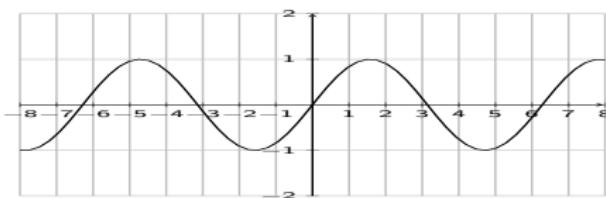
- Amplitude  $A$



$$\begin{array}{ll} y = \sin(x) & A = 1 \\ y = 3\sin(x) & A = 3 \\ y = -2\sin(x) & A = 2 \end{array}$$

$$\begin{array}{ll} y = \cos(x) & A = 1 \\ y = 2\cos(x) & A = 2 \\ y = -4\cos(x) & A = 4 \end{array}$$

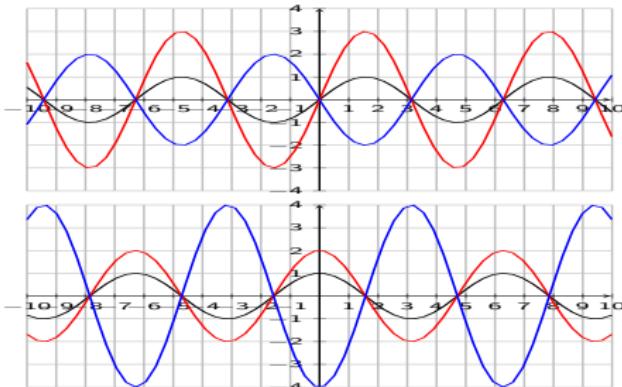
- Period  $P$



$$\begin{array}{l} y = \sin(x) \\ y = \sin(3x) \end{array} \quad P = 2\pi$$

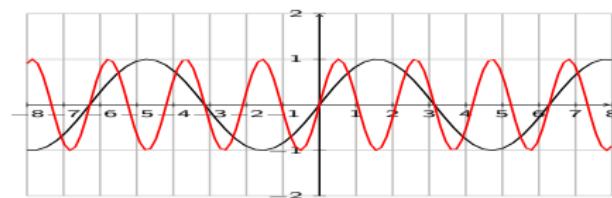
# Amplitude, period, and phase shift for sin, cos

- Amplitude  $A$



$$\begin{array}{ll} y = \sin(x) & A = 1 \\ y = 3\sin(x) & A = 3 \\ y = -2\sin(x) & A = 2 \end{array}$$

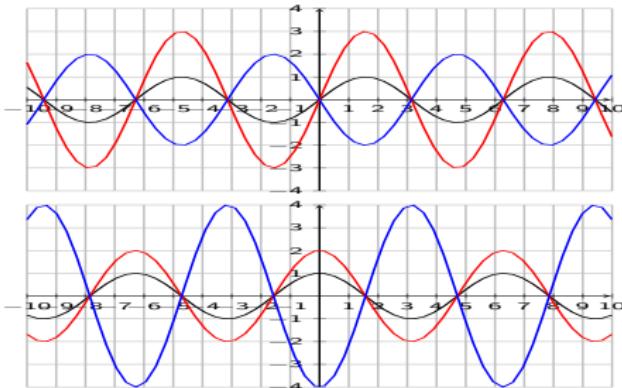
- Period  $P$



$$\begin{array}{ll} y = \sin(x) & P = 2\pi \\ y = \sin(3x) & P = \frac{2\pi}{3} \end{array}$$

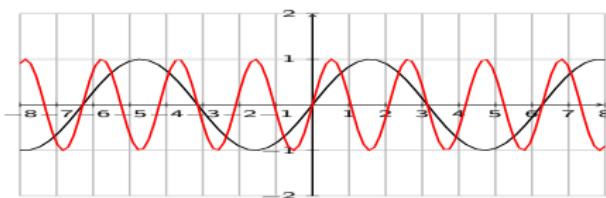
# Amplitude, period, and phase shift for sin, cos

- Amplitude  $A$



$$\begin{array}{ll} y = \sin(x) & A = 1 \\ y = 3 \sin(x) & A = 3 \\ y = -2 \sin(x) & A = 2 \end{array}$$

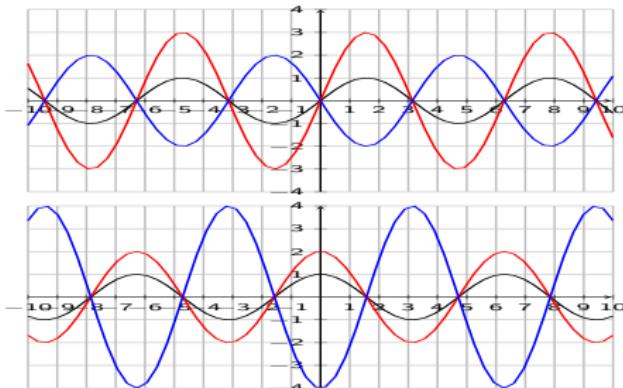
- Period  $P$



$$\begin{array}{ll} y = \sin(x) & P = 2\pi \\ y = \sin(3x) & P = \frac{2\pi}{3} \\ y = \sin(\frac{1}{2}x) & \end{array}$$

# Amplitude, period, and phase shift for sin, cos

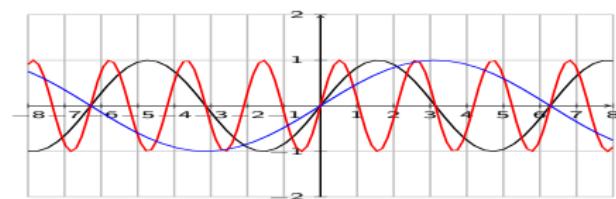
- Amplitude  $A$



$$\begin{array}{ll} y = \sin(x) & A = 1 \\ y = 3 \sin(x) & A = 3 \\ y = -2 \sin(x) & A = 2 \end{array}$$

$$\begin{array}{ll} y = \cos(x) & A = 1 \\ y = 2 \cos(x) & A = 2 \\ y = -4 \cos(x) & A = 4 \end{array}$$

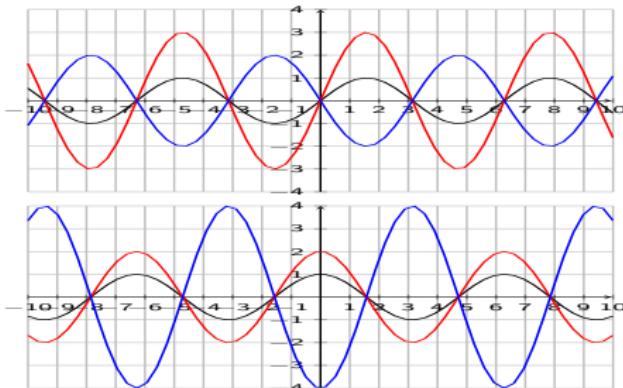
- Period  $P$



$$\begin{array}{ll} y = \sin(x) & P = 2\pi \\ y = \sin(3x) & P = \frac{2\pi}{3} \\ y = \sin(\frac{1}{2}x) & P = \frac{2\pi}{\frac{1}{2}} = 2\pi \cdot \frac{2}{1} \\ & = 4\pi \end{array}$$

# Amplitude, period, and phase shift for sin, cos

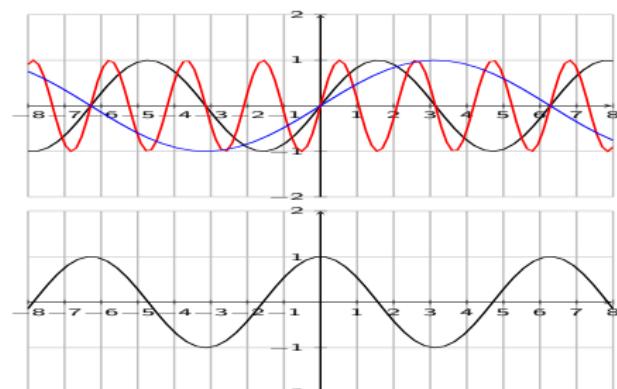
## • Amplitude $A$



$$\begin{array}{ll} y = \sin(x) & A = 1 \\ y = 3 \sin(x) & A = 3 \\ y = -2 \sin(x) & A = 2 \end{array}$$

$$\begin{array}{ll} y = \cos(x) & A = 1 \\ y = 2 \cos(x) & A = 2 \\ y = -4 \cos(x) & A = 4 \end{array}$$

## • Period $P$

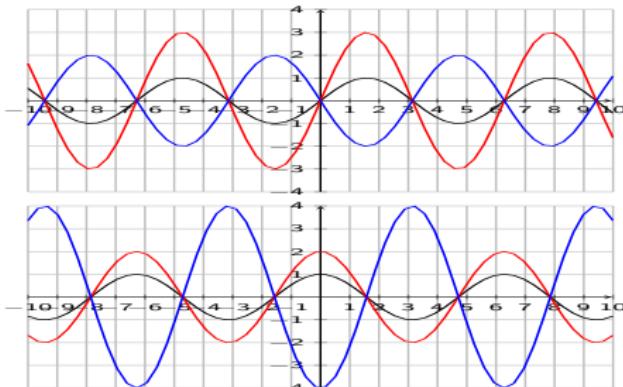


$$\begin{array}{ll} y = \sin(x) & P = 2\pi \\ y = \sin(3x) & P = \frac{2\pi}{3} \\ y = \sin(\frac{1}{2}x) & P = \frac{2\pi}{\frac{1}{2}} = 2\pi \cdot \frac{2}{1} \\ & = 4\pi \end{array}$$

$$\begin{array}{ll} y = \cos(x) & P = 2\pi \\ y = \cos(2x) & \\ y = \cos(6x) & \end{array}$$

# Amplitude, period, and phase shift for sin, cos

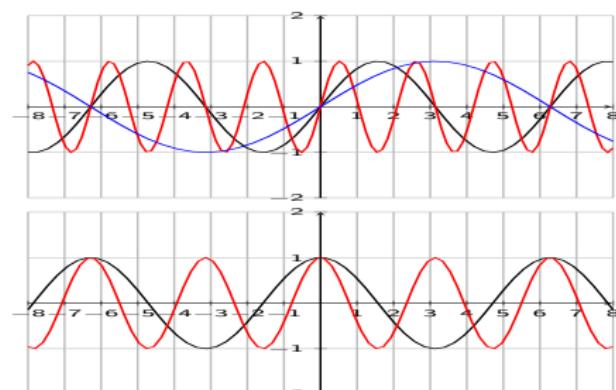
## • Amplitude $A$



$$\begin{array}{ll} y = \sin(x) & A = 1 \\ y = 3 \sin(x) & A = 3 \\ y = -2 \sin(x) & A = 2 \end{array}$$

$$\begin{array}{ll} y = \cos(x) & A = 1 \\ y = 2 \cos(x) & A = 2 \\ y = -4 \cos(x) & A = 4 \end{array}$$

## • Period $P$

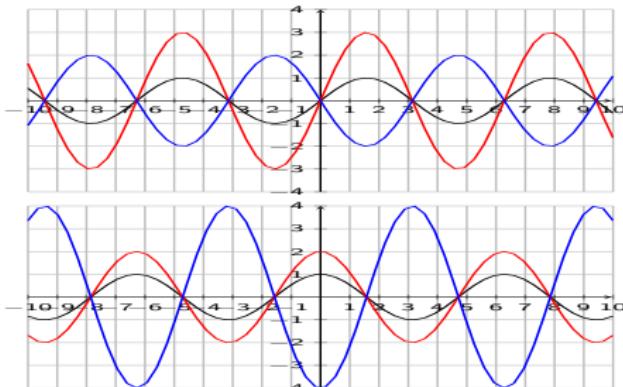


$$\begin{array}{ll} y = \sin(x) & P = 2\pi \\ y = \sin(3x) & P = \frac{2\pi}{3} \\ y = \sin(\frac{1}{2}x) & P = \frac{2\pi}{\frac{1}{2}} = 2\pi \cdot \frac{2}{1} \\ & = 4\pi \end{array}$$

$$\begin{array}{ll} y = \cos(x) & P = 2\pi \\ y = \cos(2x) & P = \frac{2\pi}{2} = \pi \\ y = \cos(6x) & \end{array}$$

# Amplitude, period, and phase shift for sin, cos

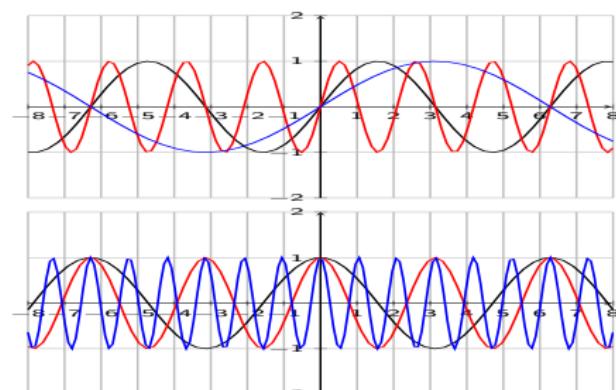
## • Amplitude $A$



$$\begin{array}{ll} y = \sin(x) & A = 1 \\ y = 3 \sin(x) & A = 3 \\ y = -2 \sin(x) & A = 2 \end{array}$$

$$\begin{array}{ll} y = \cos(x) & A = 1 \\ y = 2 \cos(x) & A = 2 \\ y = -4 \cos(x) & A = 4 \end{array}$$

## • Period $P$

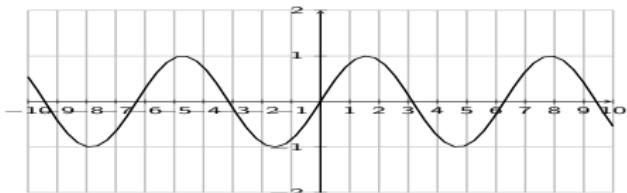


$$\begin{array}{ll} y = \sin(x) & P = 2\pi \\ y = \sin(3x) & P = \frac{2\pi}{3} \\ y = \sin(\frac{1}{2}x) & P = \frac{2\pi}{\frac{1}{2}} = 2\pi \cdot \frac{2}{1} \\ & = 4\pi \end{array}$$

$$\begin{array}{ll} y = \cos(x) & P = 2\pi \\ y = \cos(2x) & P = \frac{2\pi}{2} = \pi \\ y = \cos(6x) & P = \frac{2\pi}{6} = \frac{\pi}{3} \end{array}$$

# Amplitude, period, and phase shift for sin, cos

- Phase shift  $S$

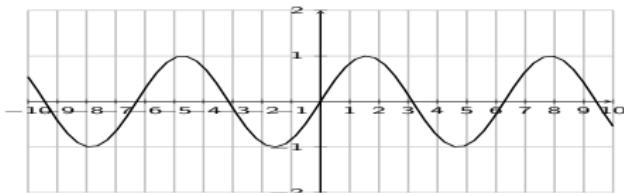


$$y = \sin(x)$$

$$S = 0$$

# Amplitude, period, and phase shift for sin, cos

- Phase shift  $S$



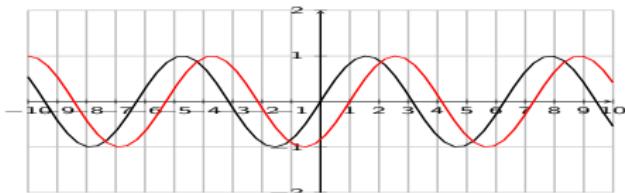
$$y = \sin(x)$$

$$y = \sin(x - 1)$$

$$S = 0$$

# Amplitude, period, and phase shift for sin, cos

- Phase shift  $S$



$$y = \sin(x)$$

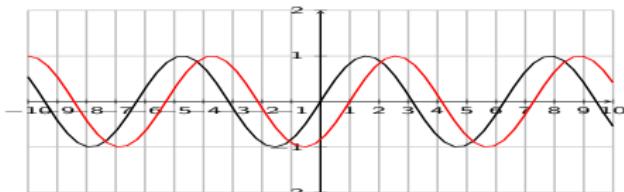
$$y = \sin(x - 1)$$

$$S = 0$$

$$S = 1$$

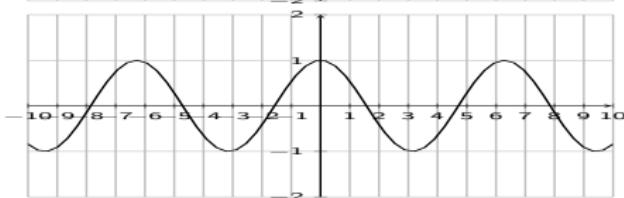
# Amplitude, period, and phase shift for sin, cos

## • Phase shift $S$



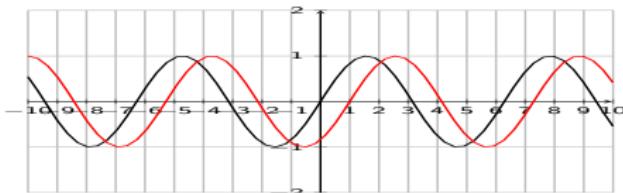
$$y = \sin(x) \quad S = 0$$
$$y = \sin(x - 1) \quad S = 1$$

$$y = \cos(x) \quad S = 0$$

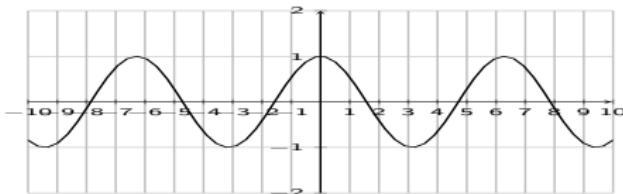


# Amplitude, period, and phase shift for sin, cos

## • Phase shift $S$



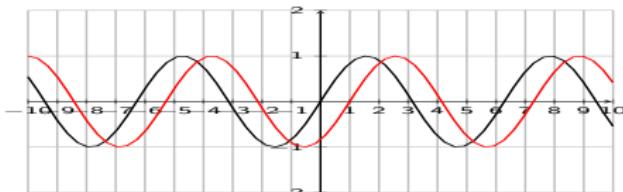
$$y = \sin(x) \quad S = 0$$
$$y = \sin(x - 1) \quad S = 1$$



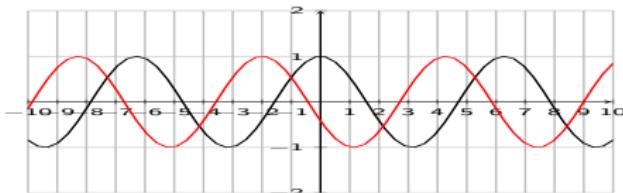
$$y = \cos(x) \quad S = 0$$
$$y = \cos(x + 2) \quad S = 1$$

# Amplitude, period, and phase shift for sin, cos

- Phase shift  $S$



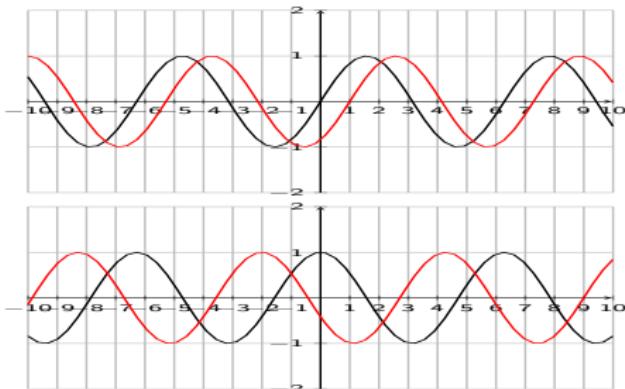
$$y = \sin(x) \quad S = 0$$
$$y = \sin(x - 1) \quad S = 1$$



$$y = \cos(x) \quad S = 0$$
$$y = \cos(x + 2) \quad S = -2$$

# Amplitude, period, and phase shift for sin, cos

## • Phase shift $S$



$$y = \sin(x) \quad S = 0$$

$$y = \sin(x - 1) \quad S = 1$$

$$y = \cos(x) \quad S = 0$$

$$y = \cos(x + 2) \quad S = -2$$

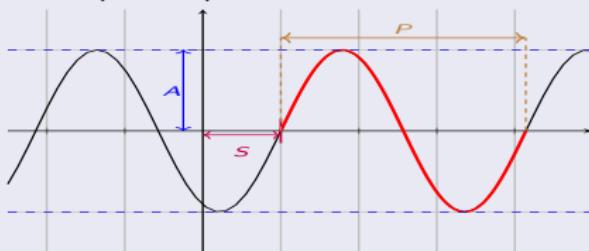
## Amplitude, period, phase shift

When  $y = a \cdot \sin(bx + c)$  or  $y = a \cdot \cos(bx + c)$

Amplitude  $A = |a|$

Period  $P = \left| \frac{2\pi}{b} \right|$

Phase shift  $S = \frac{-c}{b}$



## Graphing sin and cos - exercises

Find amplitude, period, and phase shift. Graph the function over one period. Label all maxima, minima, and roots.

①  $y = 4 \sin(2x - \pi)$

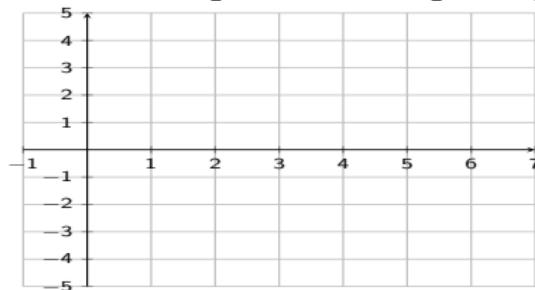
②  $y = 3 \sin(5x - \pi)$

## Graphing sin and cos - exercises

Find amplitude, period, and phase shift. Graph the function over one period. Label all maxima, minima, and roots.

①  $y = 4 \sin(2x - \pi)$

$$A = 4, P = \frac{2\pi}{2} = \pi, S = \frac{-(-\pi)}{2} = \frac{\pi}{2}$$



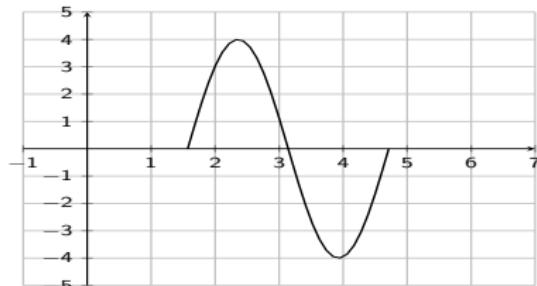
②  $y = 3 \sin(5x - \pi)$

## Graphing sin and cos - exercises

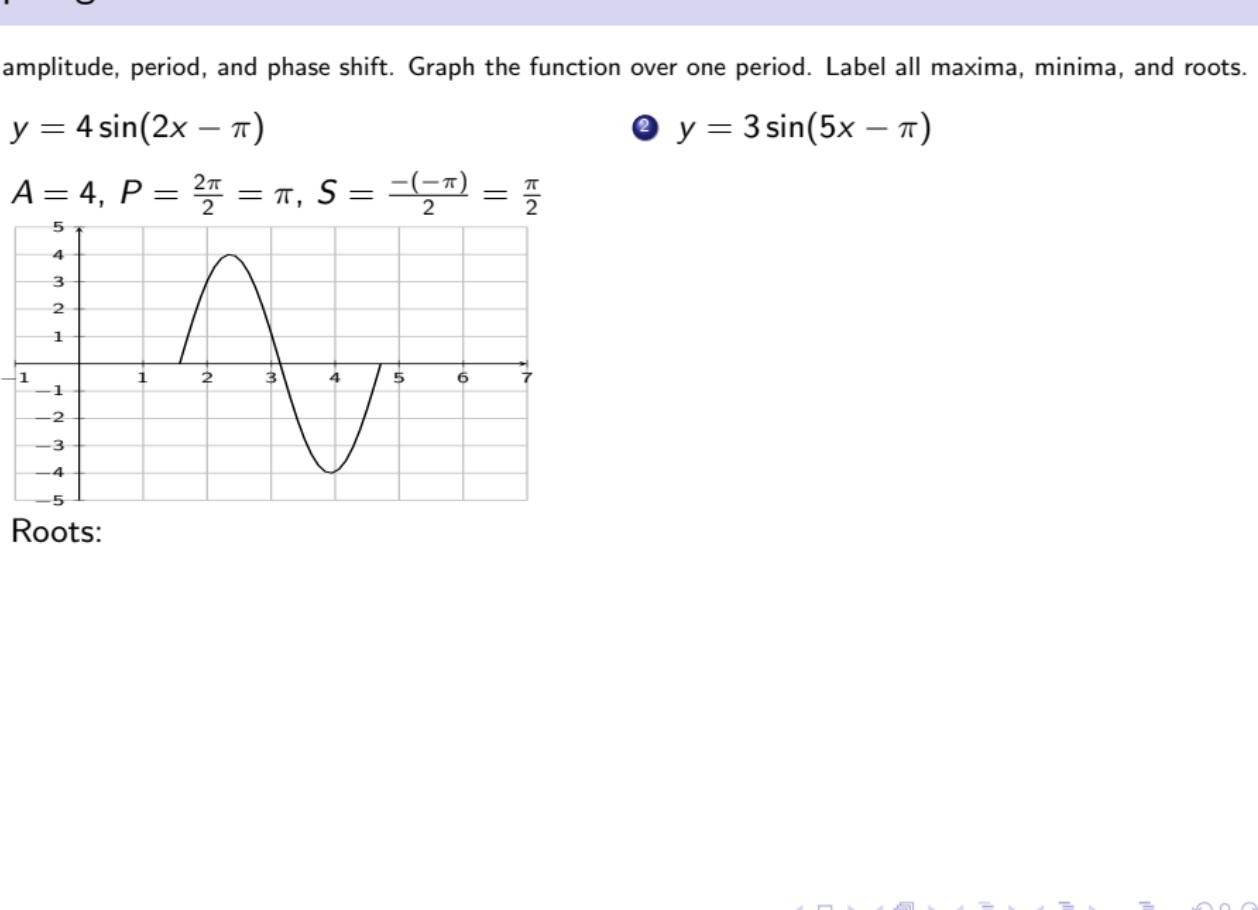
Find amplitude, period, and phase shift. Graph the function over one period. Label all maxima, minima, and roots.

①  $y = 4 \sin(2x - \pi)$

$$A = 4, P = \frac{2\pi}{2} = \pi, S = \frac{-(-\pi)}{2} = \frac{\pi}{2}$$



②  $y = 3 \sin(5x - \pi)$

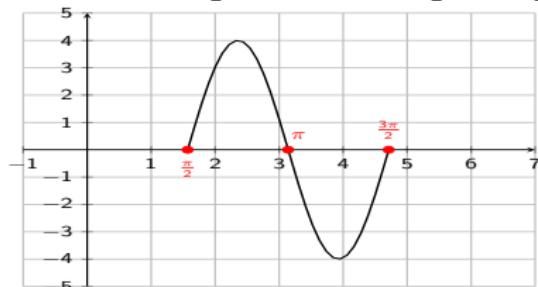


# Graphing sin and cos - exercises

Find amplitude, period, and phase shift. Graph the function over one period. Label all maxima, minima, and roots.

①  $y = 4 \sin(2x - \pi)$

$$A = 4, P = \frac{2\pi}{2} = \pi, S = \frac{-(-\pi)}{2} = \frac{\pi}{2}$$



②  $y = 3 \sin(5x - \pi)$

$$A = 3, P = \frac{2\pi}{5}, S = \frac{-(-\pi)}{2} = \frac{\pi}{2}$$

Roots:

- $S = \frac{\pi}{2}$
- $S + P = \frac{\pi}{2} + \pi = \frac{3\pi}{2}$
- $(\frac{\pi}{2} + \frac{3\pi}{2}) \div 2 = \frac{4\pi}{2} \cdot \frac{1}{2} = \frac{4\pi}{4} = \pi$

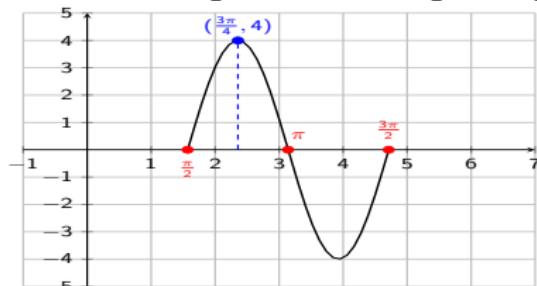
Maximum:

# Graphing sin and cos - exercises

Find amplitude, period, and phase shift. Graph the function over one period. Label all maxima, minima, and roots.

①  $y = 4 \sin(2x - \pi)$

$$A = 4, P = \frac{2\pi}{2} = \pi, S = \frac{-(-\pi)}{2} = \frac{\pi}{2}$$



②  $y = 3 \sin(5x - \pi)$

$$A = 3, P = \frac{2\pi}{5}, S = \frac{-(-\pi)}{2} = \frac{\pi}{2}$$

Roots:

- $S = \frac{\pi}{2}$
- $S + P = \frac{\pi}{2} + \pi = \frac{3\pi}{2}$
- $(\frac{\pi}{2} + \frac{3\pi}{2}) \div 2 = \frac{4\pi}{2} \cdot \frac{1}{2} = \frac{4\pi}{4} = \pi$

Maximum:

$$\bullet (\frac{\pi}{2} + \pi) \div 2 = \frac{3\pi}{2} \cdot \frac{1}{2} = \frac{3\pi}{4}$$

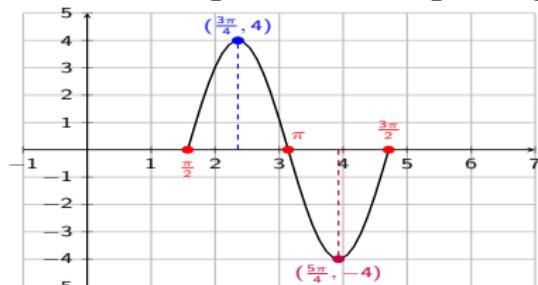
Minimum:

# Graphing sin and cos - exercises

Find amplitude, period, and phase shift. Graph the function over one period. Label all maxima, minima, and roots.

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$$A = 4, P = \frac{2\pi}{2} = \pi, S = \frac{-(-\pi)}{2} = \frac{\pi}{2}$$



②  $y = 3 \sin(5x - \pi)$

$$A = 3, P = \frac{2\pi}{5}, S = \frac{-(-\pi)}{2} = \frac{\pi}{2}$$

Roots:

- $S = \frac{\pi}{2}$
- $S + P = \frac{\pi}{2} + \pi = \frac{3\pi}{2}$
- $(\frac{\pi}{2} + \frac{3\pi}{2}) \div 2 = \frac{4\pi}{2} \cdot \frac{1}{2} = \frac{4\pi}{4} = \pi$

Maximum:

- $(\frac{\pi}{2} + \pi) \div 2 = \frac{3\pi}{2} \cdot \frac{1}{2} = \frac{3\pi}{4}$

Minimum:

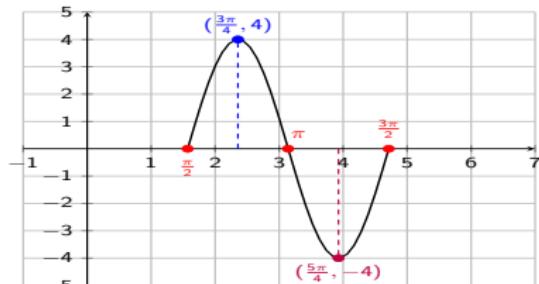
- $(\pi + \frac{3\pi}{2}) \div 2 = \frac{5\pi}{2} \cdot \frac{1}{2} = \frac{5\pi}{4}$

# Graphing sin and cos - exercises

Find amplitude, period, and phase shift. Graph the function over one period. Label all maxima, minima, and roots.

①  $y = 4 \sin(2x - \pi)$

$$A = 4, P = \frac{2\pi}{2} = \pi, S = \frac{-(-\pi)}{2} = \frac{\pi}{2}$$



Roots:

- $S = \frac{\pi}{2}$
- $S + P = \frac{\pi}{2} + \pi = \frac{3\pi}{2}$
- $(\frac{\pi}{2} + \frac{3\pi}{2}) \div 2 = \frac{4\pi}{2} \cdot \frac{1}{2} = \frac{4\pi}{4} = \pi$

Maximum:

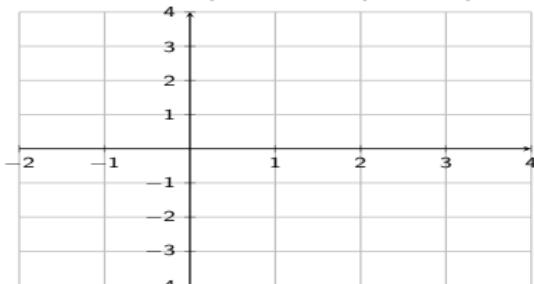
- $(\frac{\pi}{2} + \pi) \div 2 = \frac{3\pi}{2} \cdot \frac{1}{2} = \frac{3\pi}{4}$

Minimum:

- $(\pi + \frac{3\pi}{2}) \div 2 = \frac{5\pi}{2} \cdot \frac{1}{2} = \frac{5\pi}{4}$

②  $y = 3 \sin(5x - \pi)$

$$A = 3, P = \frac{2\pi}{5}, S = \frac{-(-\pi)}{5} = \frac{\pi}{5}$$

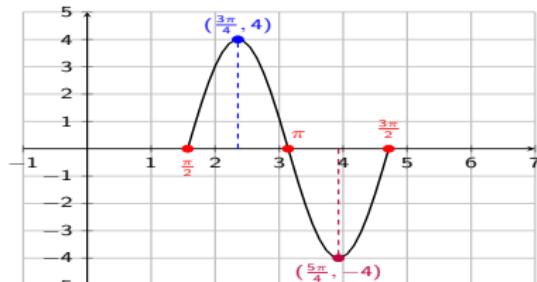


# Graphing sin and cos - exercises

Find amplitude, period, and phase shift. Graph the function over one period. Label all maxima, minima, and roots.

①  $y = 4 \sin(2x - \pi)$

$$A = 4, P = \frac{2\pi}{2} = \pi, S = \frac{-(-\pi)}{2} = \frac{\pi}{2}$$



Roots:

- $S = \frac{\pi}{2}$
- $S + P = \frac{\pi}{2} + \pi = \frac{3\pi}{2}$
- $(\frac{\pi}{2} + \frac{3\pi}{2}) \div 2 = \frac{4\pi}{2} \cdot \frac{1}{2} = \frac{4\pi}{4} = \pi$

Maximum:

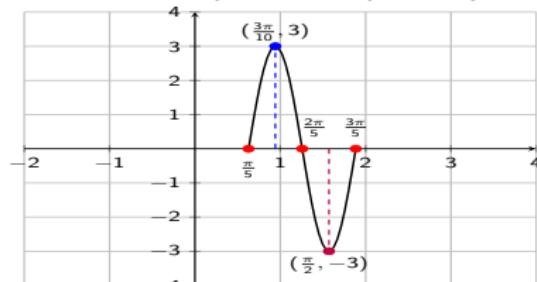
$$\bullet (\frac{\pi}{2} + \pi) \div 2 = \frac{3\pi}{2} \cdot \frac{1}{2} = \frac{3\pi}{4}$$

Minimum:

$$\bullet (\pi + \frac{3\pi}{2}) \div 2 = \frac{5\pi}{2} \cdot \frac{1}{2} = \frac{5\pi}{4}$$

②  $y = 3 \sin(5x - \pi)$

$$A = 3, P = \frac{2\pi}{5}, S = \frac{-(-\pi)}{5} = \frac{\pi}{5}$$



Roots:

- $S = \frac{\pi}{5}$
- $S + P = \frac{\pi}{5} + \frac{2\pi}{5} = \frac{3\pi}{5}$
- $(\frac{\pi}{5} + \frac{3\pi}{5}) \div 2 = \frac{4\pi}{5} \cdot \frac{1}{2} = \frac{2\pi}{5}$

Maximum:

$$\bullet (\frac{\pi}{5} + \frac{2\pi}{5}) \div 2 = \frac{3\pi}{5} \cdot \frac{1}{2} = \frac{3\pi}{10}$$

Minimum:

$$\bullet (\frac{2\pi}{5} + \frac{3\pi}{5}) \div 2 = \frac{5\pi}{5} \cdot \frac{1}{2} = \frac{\pi}{2}$$

## Graphing sin and cos - exercises

Find amplitude, period, and phase shift. Graph the function over one period. Label all maxima, minima, and roots.

③  $y = 5 \cos(2x - \frac{\pi}{2})$

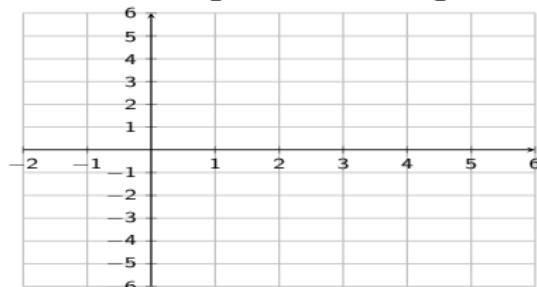
④  $y = 2 \cos(x - \frac{\pi}{3})$

## Graphing sin and cos - exercises

Find amplitude, period, and phase shift. Graph the function over one period. Label all maxima, minima, and roots.

③  $y = 5 \cos(2x - \frac{\pi}{2})$

$$A = 5, P = \frac{2\pi}{2} = \pi, S = \frac{-(-\frac{\pi}{2})}{2} = \frac{\pi}{4}$$



④  $y = 2 \cos(x - \frac{\pi}{3})$

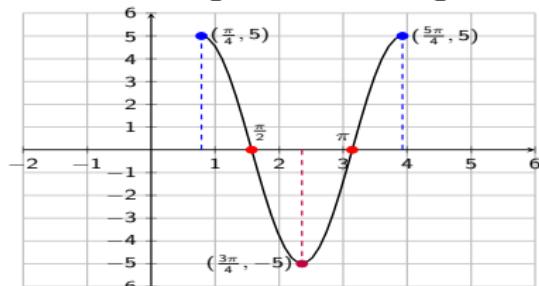
# Graphing sin and cos - exercises

Find amplitude, period, and phase shift. Graph the function over one period. Label all maxima, minima, and roots.

③  $y = 5 \cos(2x - \frac{\pi}{2})$

④  $y = 2 \cos(x - \frac{\pi}{3})$

$$A = 5, P = \frac{2\pi}{2} = \pi, S = \frac{-(-\frac{\pi}{2})}{2} = \frac{\pi}{4}$$



Maxima:

- $S = \frac{\pi}{4}$
- $S + P = \frac{\pi}{4} + \pi = \frac{5\pi}{4}$

Minimum:

- $(\frac{\pi}{4} + \frac{5\pi}{4}) \div 2 = \frac{6\pi}{4} \cdot \frac{1}{2} = \frac{6\pi}{8} = \frac{3\pi}{4}$

Roots:

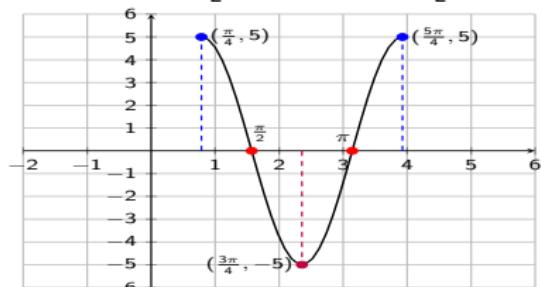
- $(\frac{\pi}{4} + \frac{3\pi}{4}) \div 2 = \frac{4\pi}{4} \cdot \frac{1}{2} = \frac{\pi}{2}$
- $(\frac{3\pi}{4} + \frac{5\pi}{4}) \div 2 = \frac{8\pi}{4} \cdot \frac{1}{2} = \frac{8\pi}{8} = \pi$

# Graphing sin and cos - exercises

Find amplitude, period, and phase shift. Graph the function over one period. Label all maxima, minima, and roots.

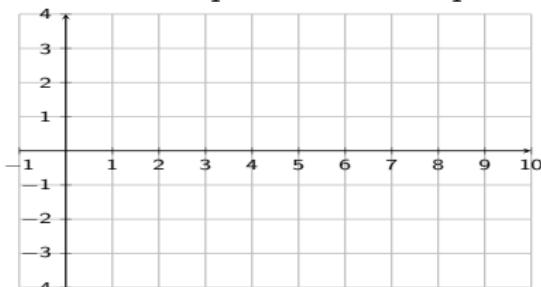
③  $y = 5 \cos(2x - \frac{\pi}{2})$

$$A = 5, P = \frac{2\pi}{2} = \pi, S = \frac{-(-\frac{\pi}{2})}{2} = \frac{\pi}{4}$$



④  $y = 2 \cos(x - \frac{\pi}{3})$

$$A = 2, P = \frac{2\pi}{1} = 2\pi, S = \frac{-(-\frac{\pi}{3})}{1} = \frac{\pi}{3}$$



Maxima:

- $S = \frac{\pi}{4}$
- $S + P = \frac{\pi}{4} + \pi = \frac{5\pi}{4}$

Minimum:

- $(\frac{\pi}{4} + \frac{5\pi}{4}) \div 2 = \frac{6\pi}{4} \cdot \frac{1}{2} = \frac{6\pi}{8} = \frac{3\pi}{4}$

Roots:

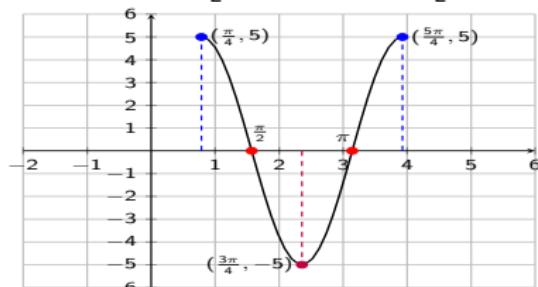
- $(\frac{\pi}{4} + \frac{3\pi}{4}) \div 2 = \frac{4\pi}{4} \cdot \frac{1}{2} = \frac{\pi}{2}$
- $(\frac{3\pi}{4} + \frac{5\pi}{4}) \div 2 = \frac{8\pi}{4} \cdot \frac{1}{2} = \frac{8\pi}{8} = \pi$

# Graphing sin and cos - exercises

Find amplitude, period, and phase shift. Graph the function over one period. Label all maxima, minima, and roots.

③  $y = 5 \cos(2x - \frac{\pi}{2})$

$$A = 5, P = \frac{2\pi}{2} = \pi, S = \frac{-(-\frac{\pi}{2})}{2} = \frac{\pi}{4}$$



Maxima:

- $S = \frac{\pi}{4}$
- $S + P = \frac{\pi}{4} + \pi = \frac{5\pi}{4}$

Minimum:

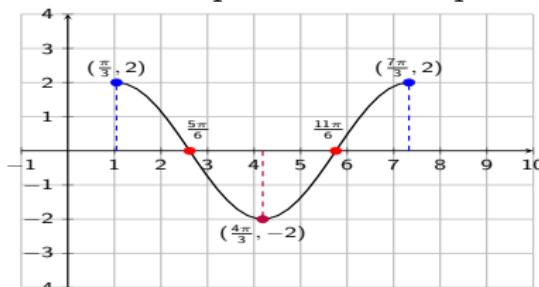
$$\bullet \left(\frac{\pi}{4} + \frac{5\pi}{4}\right) \div 2 = \frac{6\pi}{4} \cdot \frac{1}{2} = \frac{6\pi}{8} = \frac{3\pi}{4}$$

Roots:

- $\left(\frac{\pi}{4} + \frac{3\pi}{4}\right) \div 2 = \frac{4\pi}{4} \cdot \frac{1}{2} = \frac{\pi}{2}$
- $\left(\frac{3\pi}{4} + \frac{5\pi}{4}\right) \div 2 = \frac{8\pi}{4} \cdot \frac{1}{2} = \frac{8\pi}{8} = \pi$

④  $y = 2 \cos(x - \frac{\pi}{3})$

$$A = 2, P = \frac{2\pi}{1} = 2\pi, S = \frac{-(-\frac{\pi}{3})}{1} = \frac{\pi}{3}$$



Maxima:

- $S = \frac{\pi}{3}$
- $S + P = \frac{\pi}{3} + 2\pi = \frac{7\pi}{3}$

Minimum:

$$\bullet \left(\frac{\pi}{3} + \frac{7\pi}{3}\right) \div 2 = \frac{8\pi}{3} \cdot \frac{1}{2} = \frac{8\pi}{6} = \frac{4\pi}{3}$$

Roots:

- $\left(\frac{\pi}{3} + \frac{4\pi}{3}\right) \div 2 = \frac{5\pi}{3} \cdot \frac{1}{2} = \frac{5\pi}{6}$
- $\left(\frac{4\pi}{3} + \frac{7\pi}{3}\right) \div 2 = \frac{11\pi}{3} \cdot \frac{1}{2} = \frac{11\pi}{6}$

## Graphing sin and cos - exercises

Find amplitude, period, and phase shift. Graph the function over one period. Label all maxima, minima, and roots.

5)  $y = 7 \sin(3x + \pi)$

6)  $y = 3 \cos(2x + \frac{\pi}{2})$

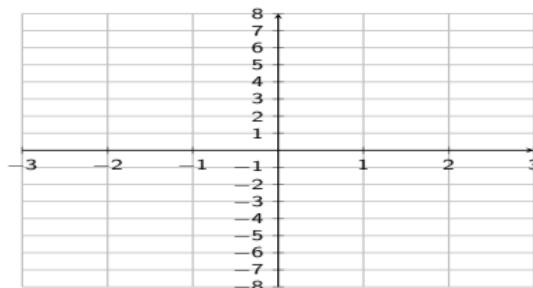
## Graphing sin and cos - exercises

Find amplitude, period, and phase shift. Graph the function over one period. Label all maxima, minima, and roots.

5)  $y = 7 \sin(3x + \pi)$

$$A = 7, P = \frac{2\pi}{3}, S = \frac{-\pi}{3}$$

6)  $y = 3 \cos(2x + \frac{\pi}{2})$

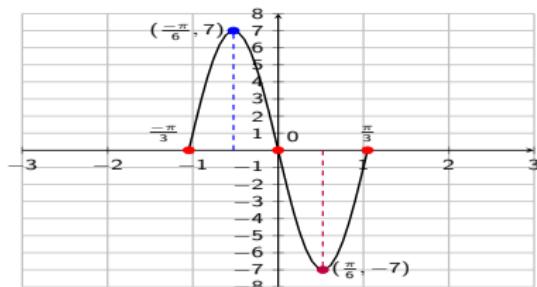


# Graphing sin and cos - exercises

Find amplitude, period, and phase shift. Graph the function over one period. Label all maxima, minima, and roots.

5  $y = 7 \sin(3x + \pi)$

$$A = 7, P = \frac{2\pi}{3}, S = \frac{-\pi}{3}$$



6  $y = 3 \cos(2x + \frac{\pi}{2})$

Roots:

- $S = \frac{-\pi}{3}$
- $S + P = \frac{-\pi}{3} + \frac{2\pi}{3} = \frac{\pi}{3}$
- $(\frac{-\pi}{3} + \frac{\pi}{3}) \div 2 = 0 \cdot \frac{1}{2} = 0$

Maximum:

- $(\frac{-\pi}{3} + 0) \div 2 = \frac{-\pi}{3} \cdot \frac{1}{2} = \frac{-\pi}{6}$

Minimum:

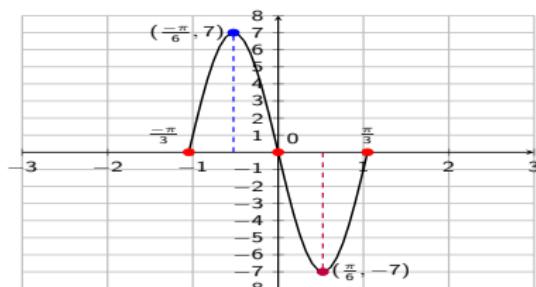
- $(0 + \frac{\pi}{3}) \div 2 = \frac{\pi}{3} \cdot \frac{1}{2} = \frac{\pi}{6}$

# Graphing sin and cos - exercises

Find amplitude, period, and phase shift. Graph the function over one period. Label all maxima, minima, and roots.

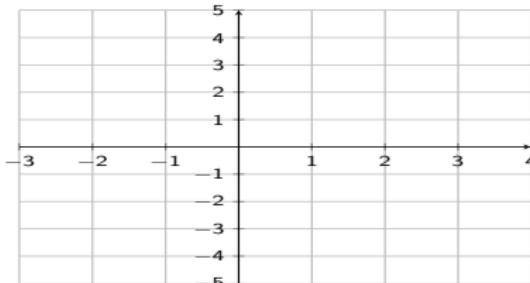
5)  $y = 7 \sin(3x + \pi)$

$$A = 7, P = \frac{2\pi}{3}, S = \frac{-\pi}{3}$$



6)  $y = 3 \cos(2x + \frac{\pi}{2})$

$$A = 3, P = \frac{2\pi}{2} = \pi, S = \frac{-\frac{\pi}{2}}{2} = -\frac{\pi}{2} \cdot \frac{1}{2} = -\frac{\pi}{4}$$



Roots:

- $S = \frac{-\pi}{3}$
- $S + P = \frac{-\pi}{3} + \frac{2\pi}{3} = \frac{\pi}{3}$
- $(\frac{-\pi}{3} + \frac{\pi}{3}) \div 2 = 0 \cdot \frac{1}{2} = 0$

Maximum:

- $(\frac{-\pi}{3} + 0) \div 2 = \frac{-\pi}{3} \cdot \frac{1}{2} = \frac{-\pi}{6}$

Minimum:

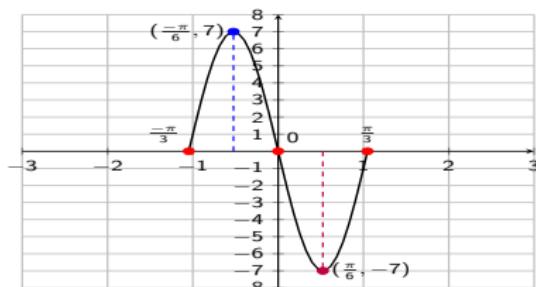
- $(0 + \frac{\pi}{3}) \div 2 = \frac{\pi}{3} \cdot \frac{1}{2} = \frac{\pi}{6}$

# Graphing sin and cos - exercises

Find amplitude, period, and phase shift. Graph the function over one period. Label all maxima, minima, and roots.

5)  $y = 7 \sin(3x + \pi)$

$$A = 7, P = \frac{2\pi}{3}, S = -\frac{\pi}{3}$$



Roots:

- $S = -\frac{\pi}{3}$
- $S + P = -\frac{\pi}{3} + \frac{2\pi}{3} = \frac{\pi}{3}$
- $(-\frac{\pi}{3} + \frac{\pi}{3}) \div 2 = 0 \cdot \frac{1}{2} = 0$

Maximum:

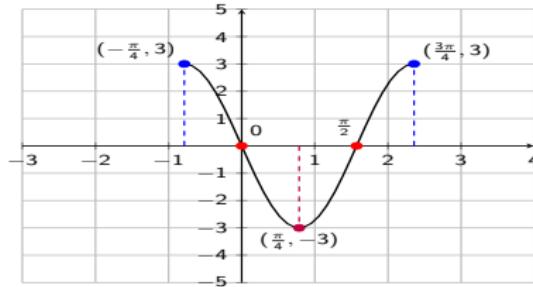
- $(-\frac{\pi}{3} + 0) \div 2 = -\frac{\pi}{3} \cdot \frac{1}{2} = -\frac{\pi}{6}$

Minimum:

- $(0 + \frac{\pi}{3}) \div 2 = \frac{\pi}{3} \cdot \frac{1}{2} = \frac{\pi}{6}$

6)  $y = 3 \cos(2x + \frac{\pi}{2})$

$$A = 3, P = \frac{2\pi}{2} = \pi, S = -\frac{\pi}{2} = -\frac{\pi}{2} \cdot \frac{1}{2} = -\frac{\pi}{4}$$



Maxima:

- $S = -\frac{\pi}{4}$
- $S + P = -\frac{\pi}{4} + \pi = -\frac{\pi}{4} + \frac{4\pi}{4} = \frac{3\pi}{4}$

Minimum:

- $(-\frac{\pi}{4} + \frac{3\pi}{4}) \div 2 = \frac{2\pi}{4} \cdot \frac{1}{2} = \frac{\pi}{4}$

Roots:

- $(-\frac{\pi}{4} + \frac{\pi}{4}) \div 2 = 0 \cdot \frac{1}{2} = 0$
- $(\frac{\pi}{4} + \frac{3\pi}{4}) \div 2 = \frac{4\pi}{4} \cdot \frac{1}{2} = \frac{\pi}{2}$

## Graphing sin and cos - exercises

Find amplitude, period, and phase shift. Graph the function over one period. Label all maxima, minima, and roots.

7)  $y = -5 \sin(6x - \pi)$

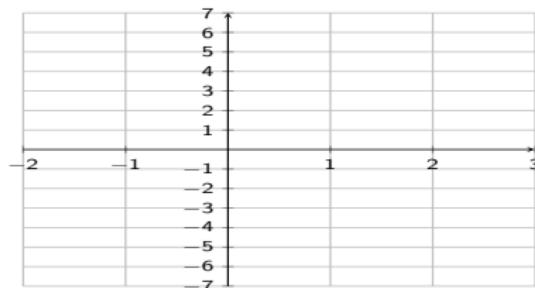
8)  $y = -4 \cos(2x - \frac{\pi}{4})$

## Graphing sin and cos - exercises

Find amplitude, period, and phase shift. Graph the function over one period. Label all maxima, minima, and roots.

7)  $y = -5 \sin(6x - \pi)$

$$A = |-5| = 5, P = \frac{2\pi}{6} = \frac{\pi}{3}, S = \frac{-(-\pi)}{6} = \frac{\pi}{6}$$



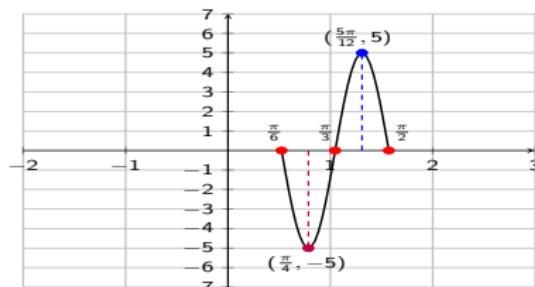
8)  $y = -4 \cos(2x - \frac{\pi}{4})$

# Graphing sin and cos - exercises

Find amplitude, period, and phase shift. Graph the function over one period. Label all maxima, minima, and roots.

7)  $y = -5 \sin(6x - \pi)$

$$A = |-5| = 5, P = \frac{2\pi}{6} = \frac{\pi}{3}, S = \frac{-(-\pi)}{6} = \frac{\pi}{6}$$



8)  $y = -4 \cos(2x - \frac{\pi}{4})$

Roots:

- $S = \frac{\pi}{6}$
- $S + P = \frac{\pi}{6} + \frac{\pi}{3} = \frac{\pi}{6} + \frac{2\pi}{6} = \frac{3\pi}{6} = \frac{\pi}{2}$
- $(\frac{\pi}{6} + \frac{\pi}{2}) \div 2 = \frac{4\pi}{6} \cdot \frac{1}{2} = \frac{4\pi}{12} = \frac{\pi}{3}$

Minimum:

$$\bullet (\frac{\pi}{6} + \frac{\pi}{3}) \div 2 = \frac{3\pi}{6} \cdot \frac{1}{2} = \frac{3\pi}{12} = \frac{\pi}{4}$$

Maximum:

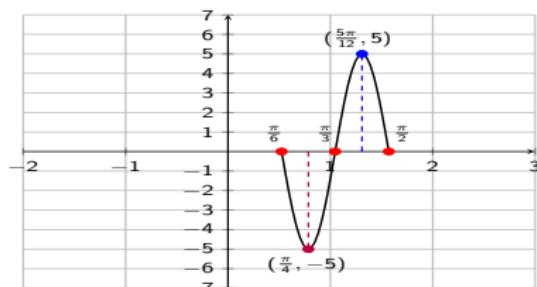
$$\bullet (\frac{\pi}{3} + \frac{\pi}{2}) \div 2 = \frac{5\pi}{6} \cdot \frac{1}{2} = \frac{5\pi}{12}$$

# Graphing sin and cos - exercises

Find amplitude, period, and phase shift. Graph the function over one period. Label all maxima, minima, and roots.

7)  $y = -5 \sin(6x - \pi)$

$$A = |-5| = 5, P = \frac{2\pi}{6} = \frac{\pi}{3}, S = \frac{-(-\pi)}{6} = \frac{\pi}{6}$$



Roots:

- $S = \frac{\pi}{6}$
- $S + P = \frac{\pi}{6} + \frac{\pi}{3} = \frac{\pi}{6} + \frac{2\pi}{6} = \frac{3\pi}{6} = \frac{\pi}{2}$
- $(\frac{\pi}{6} + \frac{\pi}{2}) \div 2 = \frac{4\pi}{6} \cdot \frac{1}{2} = \frac{4\pi}{12} = \frac{\pi}{3}$

Minimum:

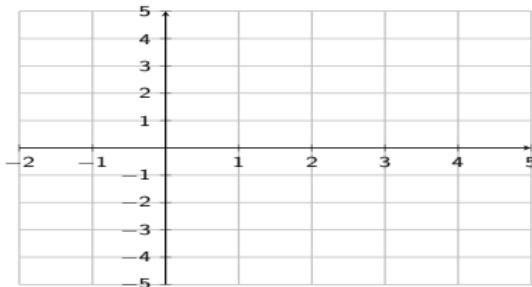
$$\bullet (\frac{\pi}{6} + \frac{\pi}{3}) \div 2 = \frac{3\pi}{6} \cdot \frac{1}{2} = \frac{3\pi}{12} = \frac{\pi}{4}$$

Maximum:

$$\bullet (\frac{\pi}{3} + \frac{\pi}{2}) \div 2 = \frac{5\pi}{6} \cdot \frac{1}{2} = \frac{5\pi}{12}$$

8)  $y = -4 \cos(2x - \frac{\pi}{4})$

$$A = |-4| = 4, P = \frac{2\pi}{2} = \pi, S = \frac{-(-\frac{\pi}{4})}{2} = \frac{\pi}{8}$$

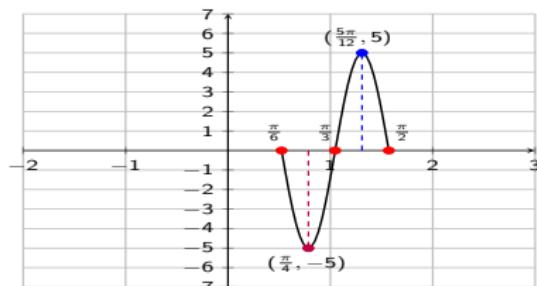


# Graphing sin and cos - exercises

Find amplitude, period, and phase shift. Graph the function over one period. Label all maxima, minima, and roots.

7)  $y = -5 \sin(6x - \pi)$

$$A = |-5| = 5, P = \frac{2\pi}{6} = \frac{\pi}{3}, S = \frac{-(-\pi)}{6} = \frac{\pi}{6}$$



Roots:

- $S = \frac{\pi}{6}$
- $S + P = \frac{\pi}{6} + \frac{\pi}{3} = \frac{\pi}{6} + \frac{2\pi}{6} = \frac{3\pi}{6} = \frac{\pi}{2}$
- $(\frac{\pi}{6} + \frac{\pi}{2}) \div 2 = \frac{4\pi}{6} \cdot \frac{1}{2} = \frac{4\pi}{12} = \frac{\pi}{3}$

Minimum:

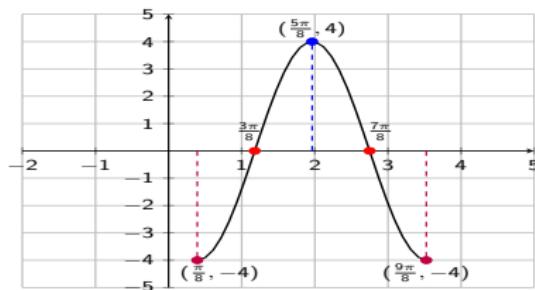
$$\bullet (\frac{\pi}{6} + \frac{\pi}{3}) \div 2 = \frac{3\pi}{6} \cdot \frac{1}{2} = \frac{3\pi}{12} = \frac{\pi}{4}$$

Maximum:

$$\bullet (\frac{\pi}{3} + \frac{\pi}{2}) \div 2 = \frac{5\pi}{6} \cdot \frac{1}{2} = \frac{5\pi}{12}$$

8)  $y = -4 \cos(2x - \frac{\pi}{4})$

$$A = |-4| = 4, P = \frac{2\pi}{2} = \pi, S = \frac{-(-\frac{\pi}{4})}{2} = \frac{\pi}{8}$$



Minima:

- $S = \frac{\pi}{8}$
- $S + P = \frac{\pi}{8} + \pi = \frac{9\pi}{8}$

Maximum:

$$\bullet (\frac{\pi}{8} + \frac{9\pi}{8}) \div 2 = \frac{10\pi}{8} \cdot \frac{1}{2} = \frac{10\pi}{16} = \frac{5\pi}{8}$$

Roots:

- $(\frac{\pi}{8} + \frac{5\pi}{8}) \div 2 = \frac{6\pi}{8} \cdot \frac{1}{2} = \frac{3\pi}{8}$
- $(\frac{5\pi}{8} + \frac{9\pi}{8}) \div 2 = \frac{14\pi}{8} \cdot \frac{1}{2} = \frac{7\pi}{8}$

