Precalculus

Third Edition (3.0)

Thomas Tradler

Holly Carley

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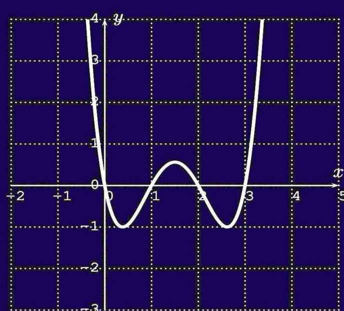

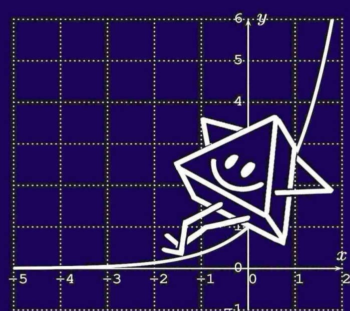

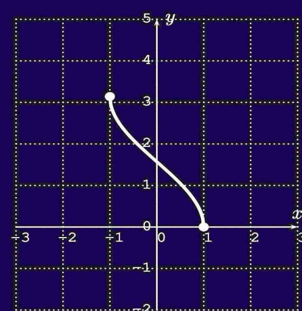

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Chapter 8

Graphing polynomials

We now discuss the features of graphs of polynomial functions.

8.1 Graphs of polynomials

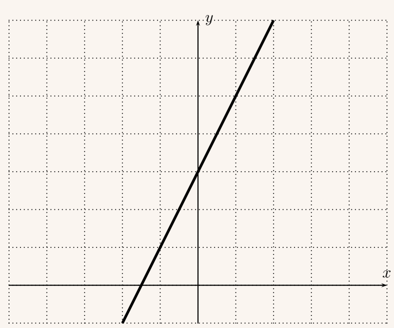
We study graphs of polynomials of various degrees. Recall from definition 7.3 that a polynomial function f of degree n is a function of the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0, \quad \text{with } a_n \neq 0.$$

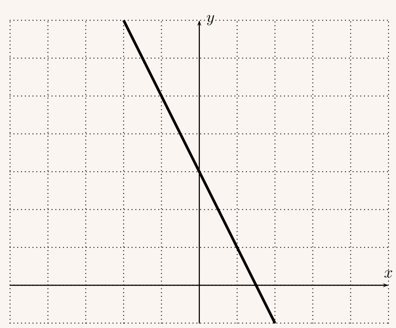
Note 8.1: Polynomial of degree 1

We already know from Section 3.1 that the graphs of polynomials of degree 1, that is, $f(x) = ax + b$, are straight lines.

$y = 2x + 3$



$y = -2x + 3$

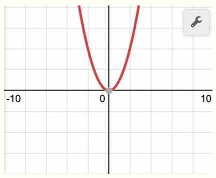


Polynomials of degree 1 have only one root.

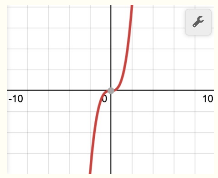
We can also easily sketch the graphs of the functions $f(x) = x^n$.

Observation 8.2: $f(x) = x^n$

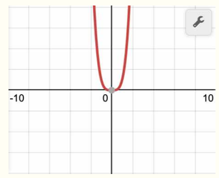
Graphing $y = x^2$, $y = x^3$, $y = x^4$, $y = x^5$, we obtain:



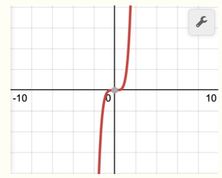
$y = x^2$



$y = x^3$

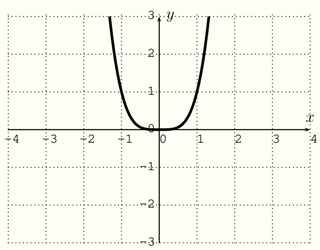


$y = x^4$

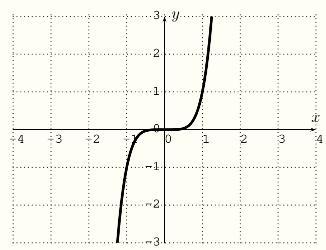


$y = x^5$

From this, we see that the shape of the graph of $f(x) = x^n$ depends on n being even or odd.



$y = x^n$, for n even
If x approaches $\pm\infty$,
$\Longrightarrow$ y approaches $+\infty$.



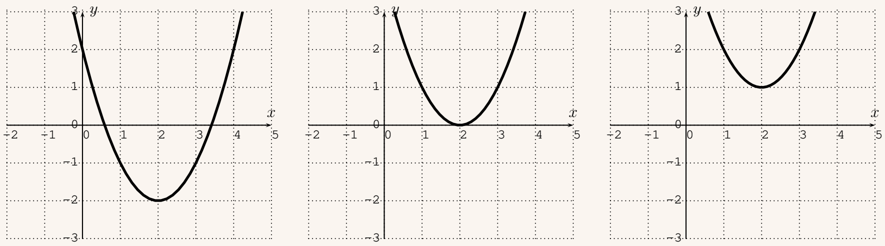
$y = x^n$, for n odd
If x approaches $\pm\infty$,
$\Longrightarrow$ y approaches $\pm\infty$.

Next, we look at graphs of general polynomials of degrees 2, 3, 4, 5, and more generally, of any degree n. In particular, we will be interested in the number of real roots (which are shown at the x-intercepts in the graph of f) and the number of extrema (that is the number of maxima or minima) of a polynomial f.

Note 8.3: Polynomial of degree 2

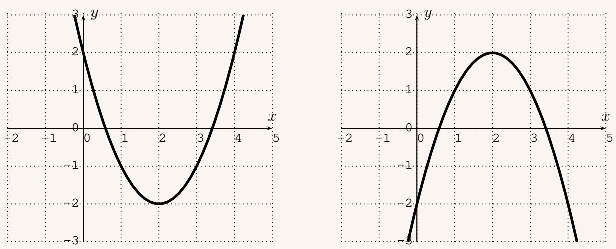
Let $f(x) = ax^2 + bx + c$ be a polynomial of degree 2. The graph of f is a parabola.

- f has at most 2 real roots (displayed at the x-intercepts). f has one extremum (that is one maximum or minimum).



- If $a > 0$ then f opens upward; if $a < 0$ then f opens downward.

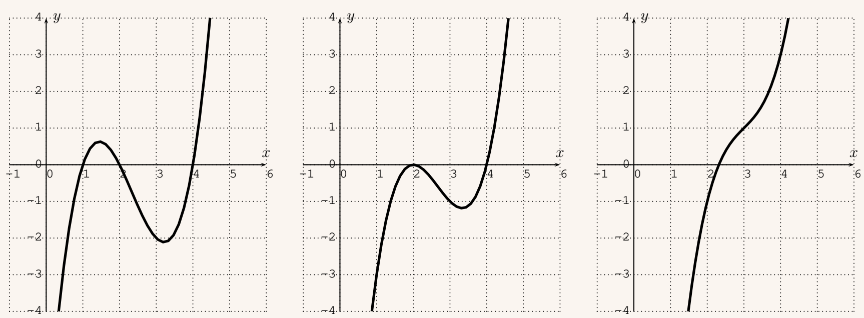
$y = x^2 - 4x + 2$ $\qquad$ $y = -x^2 + 4x - 2$



Note 8.4: Polynomial of degree 3

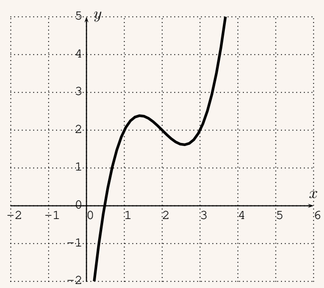
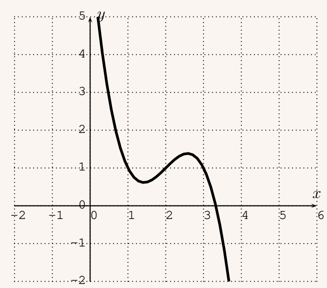
Let $f(x) = ax^3 + bx^2 + cx + d$ be a polynomial of degree 3. The graph may change its direction at most twice.

- f has at most 3 real roots. f has at most 2 extrema.



3 roots, 2 extrema $\qquad$ 2 roots, 2 extrema $\qquad$ 1 root, 0 extrema

- If $a > 0$ then $f(x)$ approaches $+\infty$ when x approaches $+\infty$ (that is, $f(x)$ gets large when x gets large), and $f(x)$ approaches $-\infty$ when x approaches $-\infty$. If $a < 0$ then $f(x)$ approaches $-\infty$ when x approaches $+\infty$, and $f(x)$ approaches $+\infty$ when x approaches $-\infty$.

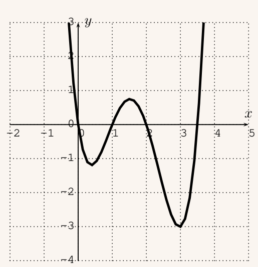
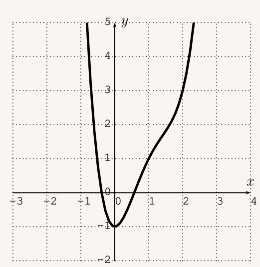
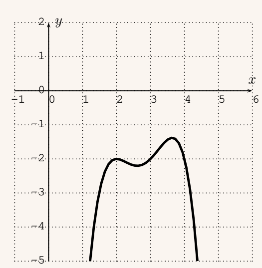
$y = x^3 - 6x^2 + 11x - 4$	$y = -x^3 + 6x^2 - 11x + 7$
	

Above, we have an instance of a polynomial of degree n which "changes its direction" one more time than a polynomial of one lesser degree $n - 1$. This phenomenon happens for higher degrees as well.

Note 8.5: Polynomial of degree 4

Let $f(x) = ax^4 + bx^3 + cx^2 + dx + e$ be a polynomial of degree 4.

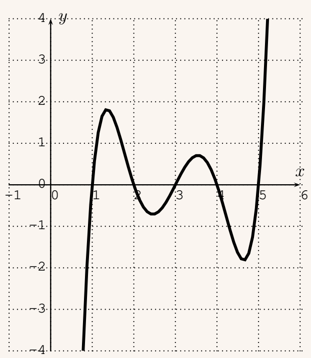
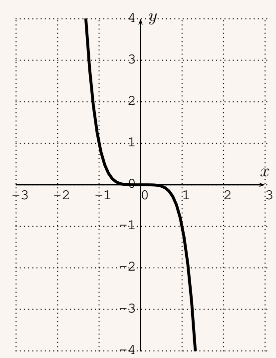
- f has at most 4 real roots. f has at most 3 extrema. If $a > 0$ then f opens upward, if $a < 0$ then f opens downward.

$y = x^4 - 6.5x^3 + 12.5x^2 - 7x$	$y = x^4 - 4x^3 + 5x^2 - 1$	$y = -x^4 + 11x^3 - 44x^2 + 76x - 50$
		
4 roots, 3 extrema	2 roots, 1 extremum	0 roots, 3 extrema

Note 8.6: Polynomial of degree 5

Let f be a polynomial of degree 5.

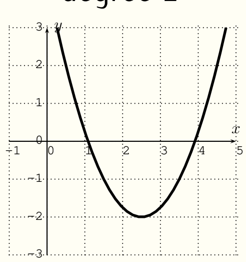
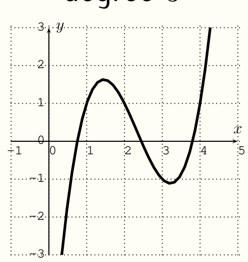
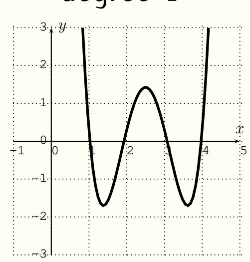
- f has at most 5 real roots. f has at most 4 extrema. If $a > 0$ then $f(x)$ approaches $+\infty$ when x approaches $+\infty$, and $f(x)$ approaches $-\infty$ when x approaches $-\infty$. If $a < 0$ then $f(x)$ approaches $-\infty$ when x approaches $+\infty$, and $f(x)$ approaches $+\infty$ when x approaches $-\infty$.

		$y = -x^5$
		
5 roots, 4 extrema	2 roots, 4 extrema	1 root, 0 extrema

We summarize our findings in the following observation.

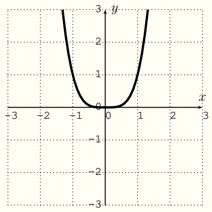
Observation 8.7: Graphs of polynomials

- Let $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2x^2 + a_1x + a_0$ be a polynomial of degree n. Then f has at most n real roots, and at most $n-1$ extrema.

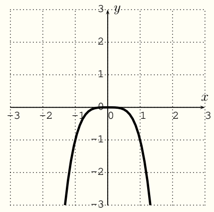
degree 2	degree 3	degree 4
		

- Assume the degree of f is even, $n = 2, 4, 6, \ldots$. If $a_n > 0$, then the polynomial opens upward. If $a_n < 0$ then the polynomial opens downward.

$f(x) = x^4$

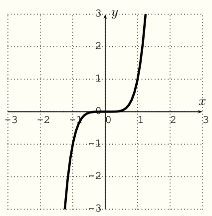


$f(x) = -x^4$

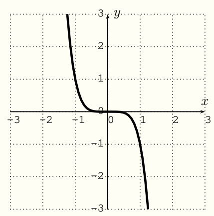


- Assume the degree of f is odd, $n = 1, 3, 5, \ldots$. If $a_n > 0$, then $f(x)$ approaches $+\infty$ when x approaches $+\infty$, and $f(x)$ approaches $-\infty$ as x approaches $-\infty$. If $a_n < 0$, then $f(x)$ approaches $-\infty$ when x approaches $+\infty$, and $f(x)$ approaches $+\infty$ as x approaches $-\infty$.

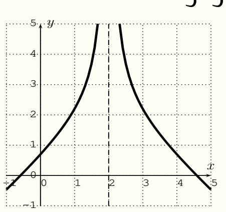
$f(x) = x^5$



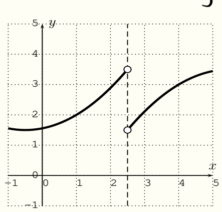
$f(x) = -x^5$



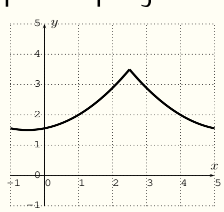
- The domain of a polynomial f is all real numbers, and f is continuous for all real numbers (there are no jumps in the graph). The graph of f has no horizontal or vertical asymptotes, no discontinuities (jumps in the graph), and no corners. Furthermore, $f(x)$ approaches $\pm\infty$ when x approaches $\pm\infty$. Therefore, the following graphs ***cannot*** be graphs of polynomials.



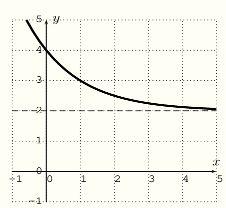
vertical asymptote / pole



discontinuity



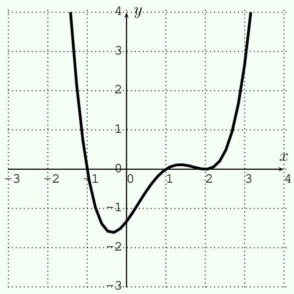
corner

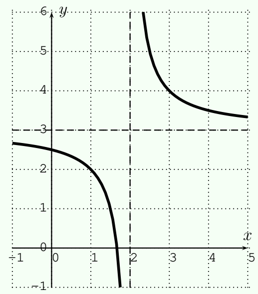


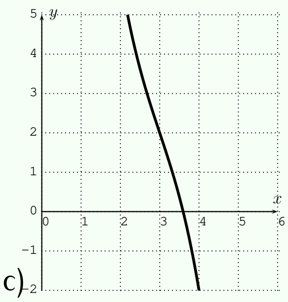
horizontal asymptote

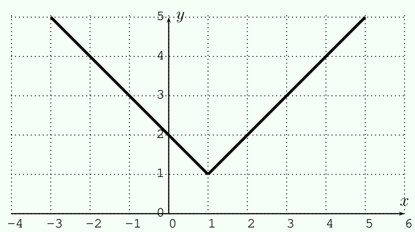
Example 8.8

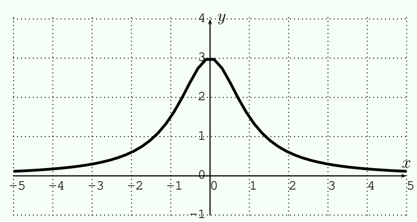
Which of the following graphs could be the graphs of a polynomial? If the graph could indeed be a graph of a polynomial, then determine a possible degree of the polynomial.

a) 

b) 

c) 

d) 

e) 

Solution.

a) Yes, this could be a polynomial. The degree could be, for example, 4.

b) No, since the graph has a pole.

c) Yes, this could be a polynomial. A possible degree would be degree 3.

d) No, since the graph has a corner.

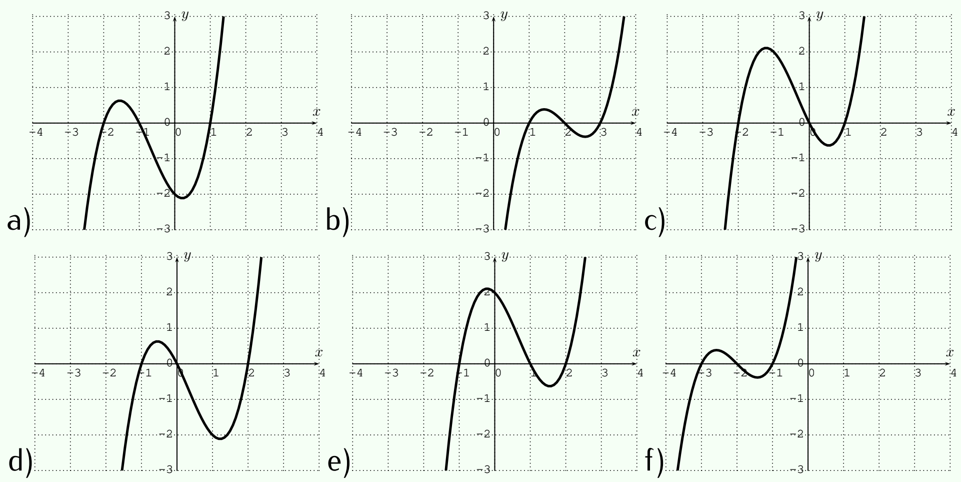
e) No, since $f(x)$ does not approach ∞ or $-\infty$ as x approaches ∞. (In fact, $f(x)$ approaches 0 as x approaches $\pm\infty$ and we say that the function (or graph) has a horizontal asymptote $y = 0$.)

□

Example 8.9

Identify the graphs of the polynomials f, g, h, and k.

$$\begin{aligned} f(x) &= (x-1)\cdot(x+1)\cdot(x-2) \\ g(x) &= (x+1)\cdot(x+2)\cdot(x+3) \\ h(x) &= (x-1)\cdot(x-2)\cdot(x-3) \\ k(x) &= x\cdot(x+1)\cdot(x-2) \end{aligned}$$



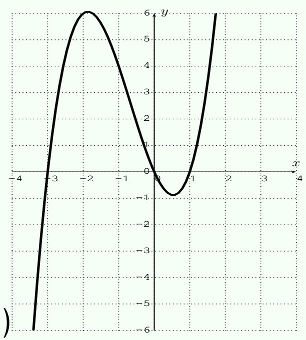
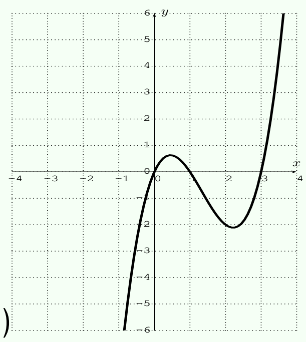
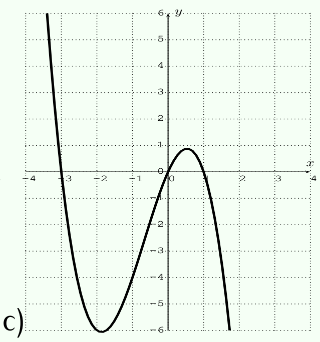
Solution.

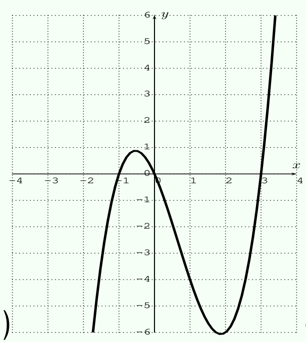
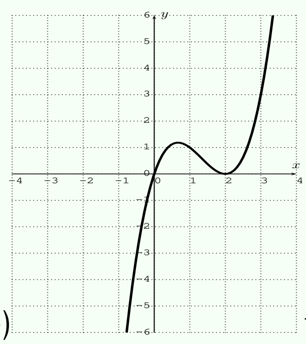
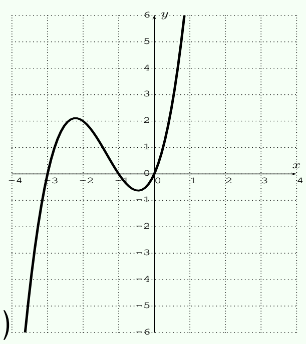
Since $(x-1)$ is a factor of f, the factor theorem (7.4) tells us that $f(1)=0$, that is, 1 is a root of f. Similarly, we see that the function f has roots at 1, -1, and 2. The only graph with roots 1, -1, and 2 is graph (e), so that the graph of f is (e). Similarly, the roots of g are -1, -2, -3, so that it's graph is (f). This should not be confused with the function h, which has roots at 1, 2, 3, and thus has graph (b). To identify the function k, note that the factor x can be expressed as $(x-0)$, so that k can also be written as $k(x)=(x-0)\cdot(x+1)\cdot(x-2)$. The roots of k are 0, -1, 2, and so k has graph (d). □

Example 8.10

Identify the graphs of the polynomials f, g, h, and k.

$$\begin{aligned}
f(x) &= x^3 - 2x^2 - 3x \\
g(x) &= x^3 - 4x^2 + 4x \\
h(x) &= x^3 - 4x^2 + 3x \\
k(x) &= -x^3 - 2x^2 + 3x
\end{aligned}$$

a) 
b) 
c) 

d) 
e) 
f) 

Solution.

Factoring first x, and then factoring again, the functions can be written as

$$\begin{aligned}
f(x) &= x(x^2 - 2x - 3) = x(x+1)(x-3) \\
g(x) &= x(x^2 - 4x + 4) = x(x-2)(x-2) = x \cdot (x-2)^2 \\
h(x) &= x(x^2 - 4x + 3) = x(x-1)(x-3) \\
k(x) &= -x(x^2 + 2x - 3) = -x(x-1)(x+3)
\end{aligned}$$

Note that $x = (x - 0)$, so that a factor x gives a root at 0. Therefore, f has roots 0, -1, 3, and thus has graph (d). For g, note that we have roots 0 and 2, and thus g has graph (e). Note that the factor $(x - 2)$

appears twice in the factored form for g. The number of times a root appears in the factored expression is called the *multiplicity* of the root. Thus, the multiplicity of the root 2 of g is 2. This higher multiplicity can also be observed in the graph of g: the graph of g does not cut through the x-axis at 2, but only touches the x-axis at 2. In fact, the graph resembles a parabola close to the root 2; of course it looks very different than a parabola further away from 2. Next, the function h has roots 0, 1, 3, and thus has graph (b). Finally, the function k has roots 0, 1, -3. There are two graphs with these roots, namely, graph (a) and graph (c). Since the first coefficient is negative, the correct graph has to be (c); see Note 8.4. □

When graphing a function, we want to make sure to draw the function in a window that shows all the interesting properties of the graph.

Note 8.11: A complete graph

Generally, we would like to graph a function in a way that includes all essential parts of the function, such as all *intercepts* (both x-intercepts and y-intercept), all *roots*, all *asymptotes* (as discussed in the following chapters), and the *long-range behavior* of the function (that is how the function behaves when x approaches $\pm\infty$). Moreover, if possible, we also want to include all *extrema* (that is all maxima and minima) of the function. Such a graph is called a **complete graph**.

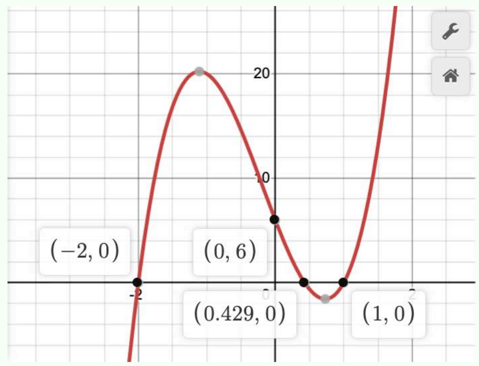
Note that we have a certain amount of choice when graphing a complete graph, as we want to pick a "reasonable" viewing window that displays the wanted features. Depending on the graph, it may be sometimes difficult or even impossible to make a good choice. Moreover, it may not be clear if all of the wanted features (such as all maxima, minima, etc.) have been displayed in the graph. In fact, some of the tools that will be developed in a course in calculus may be needed to ensure that this has indeed been achieved.

Example 8.12

Draw a complete graph of the function below. Label all intercepts and roots.

$$f(x) = 7x^3 + 4x^2 - 17x + 6$$

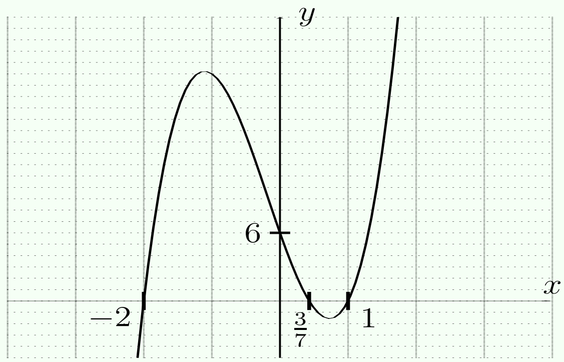
Solution.
We use the graphing calculator to graph $y = f(x)$.



By clicking on the intercepts, we see their approximate values. From the graph, it appears that there are roots at $x = -2$ and $x = 1$, and there is another root that is not an integer (≈ 0.429). Now to confirm, for example, that $x = -2$ is a root, we could check directly that $f(-2) = 0$. However, to find the other roots, we will need to use the factor theorem (7.4) and divide $f(x)$ by $(x+2)$. So, if we perform the long division and we obtain a remainder of 0, then this also confirms that -2 is indeed a root.

$$
\begin{array}{r|rrrr}
 & & 7x^2 & -10x & +3 \\
\hline
x+2 & 7x^3 & +4x^2 & -17x & +6 \\
 & -(7x^3 & +14^2) & & \\
\hline
 & & -10x^2 & -17x & +6 \\
 & & -(-10x^2 & -20x) & \\
\hline
 & & & 3x & +6 \\
 & & & -(3x & +6) \\
\hline
 & & & & 0
\end{array}
$$

Thus, $f(x) = (x+2) \cdot (7x^2 - 10x + 3)$. To find the other roots of f, we factor the quotient as $7x^2 - 10x + 3 = (x-1) \cdot (7x-3)$, and we get that $f(x) = (x+2) \cdot (x-1) \cdot (7x-3)$. (Note that 1 is a root of f, so it is not surprising that $(x-1)$ appears as a factor of f.) The third root is where $7x - 3 = 0$, i.e., $7x = 3$, or $x = \frac{3}{7}$, which is approximately 0.429. We can now draw a complete graph of f, using a graphing window similar to the one above, labeling all roots and intercepts. We can calculate the y-intercept to be at $y = f(0) = 6$.



□

8.2 Roots and factors of a polynomial

We have seen that the roots are an important feature of a polynomial. Recall that the roots of the polynomial f are those x for which $f(x) = 0$. These are, of course, precisely the x-intercepts of the graph. By the factor theorem (7.4), this is precisely the information needed to factor a polynomial.

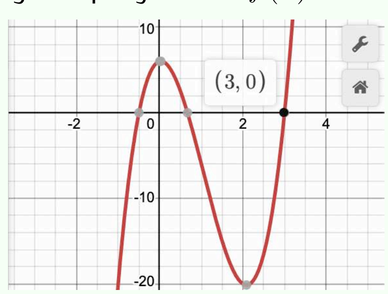
Example 8.13

Find the roots of the polynomial and factor the polynomial completely.

a) $f(x) = 6x^3 - 19x^2 + x + 6$
b) $g(x) = -x^3 - 5x^2 - 3x + 9$
c) $h(x) = 2x^3 + 11x^2 + 11x - 4$

Solution.

a) We start by graphing the polynomial $f(x) = 6x^3 - 19x^2 + x + 6$.



The graph suggests a root at $x = 3$, so that we divide $f(x)$ by $x - 3$.

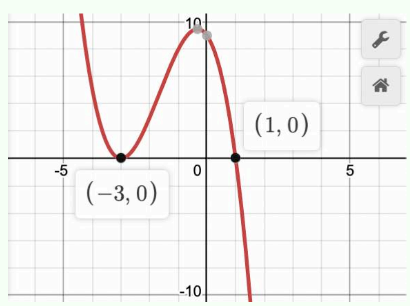
$$
\begin{array}{r|rrrr}
 & & 6x^2 & -x & -2 \\ \hline
x-3 & 6x^3 & -19x^2 & +x & +6 \\
 & -(6x^3 & -18^2) & & \\ \hline
 & & -x^2 & +x & +6 \\
 & & -(-x^2 & +3x) & \\ \hline
 & & & -2x & +6 \\
 & & & -(-2x & +6) \\ \hline
 & & & & 0
\end{array}
$$

We therefore obtain $f(x) = (x - 3) \cdot (6x^2 - x - 2)$. Continuing to factor, we obtain $f(x) = (x - 3) \cdot (3x - 2) \cdot (2x + 1)$. Note that we can factor 3 from $(3x - 2) = 3 \cdot (x - \frac{2}{3})$ and we can factor 2 from $2x + 1 = 2 \cdot (x + \frac{1}{2})$, so that the final factored expression for $f(x)$ is

$$f(x) = (x-3)\cdot 3\cdot \left(x-\frac{2}{3}\right)\cdot 2\cdot \left(x+\frac{1}{2}\right) = 6\cdot (x-3)\cdot \left(x-\frac{2}{3}\right)\cdot \left(x+\frac{1}{2}\right)$$

The roots of f are therefore 3, $\frac{2}{3}$, and $-\frac{1}{2}$.

b) From the graph we see that the roots of $g(x) = -x^3 - 5x^2 - 3x + 9$ appear to be -3 and 1.



Dividing by $x-1$, we obtain

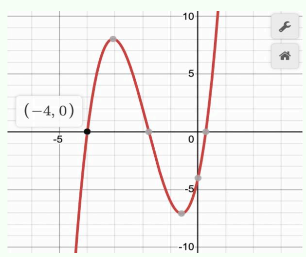
$$\begin{array}{r|rrrr} & & -x^2 & -6x & -9 \\ \hline x-1 & -x^3 & -5x^2 & -3x & +9 \\ & -(-x^3 & +x^2) & & \\ \cline{2-3} & & -6x^2 & -3x & +9 \\ & & -(-6x^2 & +6x) & \\ \cline{3-4} & & & -9x & +9 \\ & & & -(-9x & +9) \\ \cline{4-5} & & & & 0 \end{array}$$

Therefore, $g(x) = (x-1)(-x^2-6x-9)$, and factoring $-x^2-6x-9 = -(x^2+6x+9) = -(x+3)(x+3)$, we obtain

$$g(x) = -(x-1)\cdot(x+3)^2$$

The roots are indeed 1 and -3. Note that -3 is a root of multiplicity 2, as it appears twice in the factored expression for g. The graph of g does not cut the x-axis at -3, but only touches the x-axis.

c) The graph of $h(x) = 2x^3+11x^2+11x-4$ displays an integer root at -4.



Factoring by $x+4$, we get

$$\begin{array}{r|rrrr} & & 2x^2 & +3x & -1 \\ \hline x+4 & 2x^3 & +11x^2 & +11x & -4 \\ & -(2x^3 & +8^2) & & \\ \cline{2-3} & & 3x^2 & +11x & -4 \\ & & -(3x^2 & +12x) & \\ \cline{3-4} & & & -x & -4 \\ & & & -(-x & -4) \\ \cline{4-5} & & & & 0 \end{array}$$

Therefore, $h(x) = (x+4) \cdot (2x^2 + 3x - 1)$. There does not seem to be an immediate way to factor $2x^2 + 3x - 1$. However, we may use the quadratic formula (reviewed in Proposition 8.14 below) to find the roots of $2x^2 + 3x - 1$. Setting $2x^2 + 3x - 1 = 0$, we get

$$x = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 2 \cdot (-1)}}{2 \cdot 2} = \frac{-3 \pm \sqrt{9+8}}{4} = \frac{-3 \pm \sqrt{17}}{4}$$

These are indeed the remaining two roots of h, and we can write

$$h(x) = 2 \cdot (x+4) \cdot \Big(x - \frac{-3+\sqrt{17}}{4}\Big) \cdot \Big(x - \frac{-3-\sqrt{17}}{4}\Big)$$

Note that there is an overall coefficient 2, which has to appear to obtain the correct leading coefficient for $h(x) = 2x^3 + 11x^2 + 11x - 4$.

□

In the last example we found the roots and factors of a quadratic polynomial via the quadratic formula. We now recall the well-known quadratic formula and state how it can be used to factor any quadratic polynomial.

Proposition 8.14: The quadratic formula

The solutions of the equation $ax^2 + bx + c = 0$ for some real numbers a, b, and c are given by

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}.$$

We may combine the two solutions x_1 and x_2 and simply write this as:

$$\boxed{x_{1/2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}} \tag{8.1}$$

Since we have an explicit formula for the roots of a quadratic polynomial, it is always possible to give an explicit formula of a quadratic polynomial in factored form. We record this in the following note.

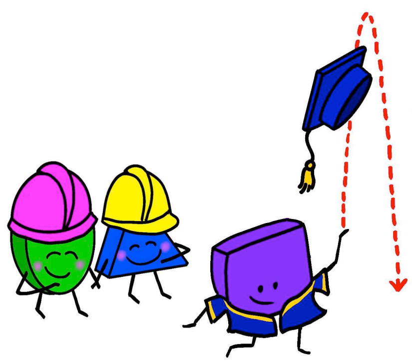
Note 8.15: Factoring a quadratic polynomial

We may always use the roots x_1 and x_2 of a quadratic polynomial $f(x) = ax^2 + bx + c$ from the quadratic formula and rewrite the polynomial as

$$\boxed{ax^2 + bx + c = a \cdot \left(x - \frac{-b + \sqrt{b^2 - 4ac}}{2a}\right)\left(x - \frac{-b - \sqrt{b^2 - 4ac}}{2a}\right)}$$

Application: Vertical position of an object in gravity

An an application, we note that the height $h = h(t)$ of an object thrown into the air as a function of time t will follow a quadratic function. Here, for simplicity, we only consider the effect of the gravitational force and ignore issues such as air resistance and friction, etc.



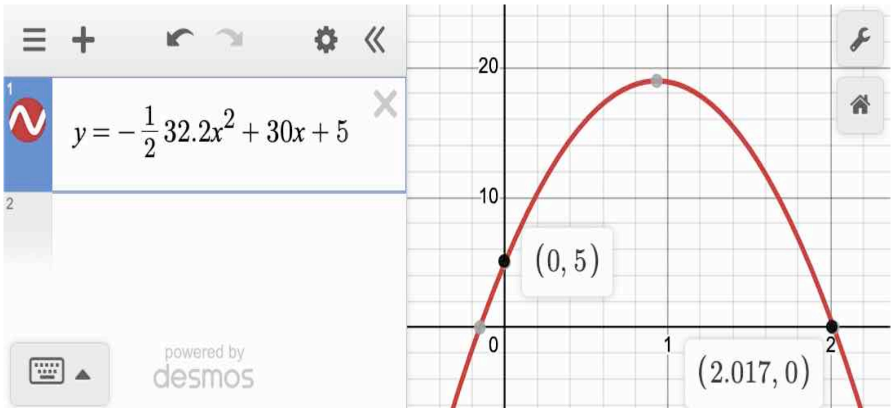
In fact, the vertical position $h(t)$ of an object is a quadratic function in time:[1]

$$\boxed{h(t) = -\frac{1}{2}gt^2 + v_0 t + h_0} \tag{8.2}$$

Here, v_0 is the initial velocity, h_0 is the initial height, and $g = 32.2\frac{\text{ft}}{\text{sec}^2}$ is the acceleration due to the gravitational pull from the Earth.
Therefore, if an object is thrown from an initial height of $h_0 = 5\text{ft}$ with an initial velocity of $v_0 = 30\frac{\text{ft}}{\text{sec}}$, then $h(t)$ follows the formula:

$$h(t) = -\tfrac{1}{2} \cdot 32.2 \cdot t^2 + 30 \cdot t + 5$$

[1] For more information, see https://openstax.org/books/college-physics-2e/pages/3-4-projectile-motion



In the above graph the horizontal axis describes time t in seconds, and the vertical axis describes height $h(t)$ in feet. The graph shows that, in particular, it would take about 2 seconds until the object hits the ground.

8.3 *Optional section:* Graphing polynomials by hand

In this section we will show how to sketch the graph of a factored polynomial without the use of a calculator.

Example 8.16

Sketch the graph of the following polynomial without using the calculator:

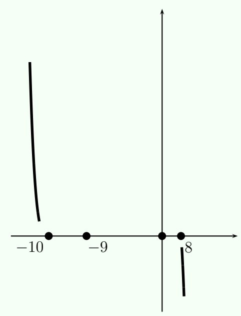
$$p(x) = -2(x+10)^3(x+9)x^2(x-8)$$

Solution.
Note that on the calculator it is impossible to get a window which will give all of the features of the graph (by focusing on a window view that captures the maximum, other features will become invisible). We will sketch the graph by hand so that some of the main features are visible. This will only be a sketch and not the actual graph up to scale. Again, the graph cannot be drawn to scale while being able to see the features.

We first start by putting the x-intercepts on the graph in the right order, but not necessarily to scale. Then note that

$$p(x) = -2x^7 + \ldots \text{(lower terms)} \quad \approx -2x^7 \qquad \text{for large } |x|.$$

This is the leading term of the polynomial (if you expand p it is the term with the largest power) and therefore dominates the polynomial for large $|x|$. So the graph of our polynomial should look something like the graph of $y = -2x^7$ on the extreme left and right side.

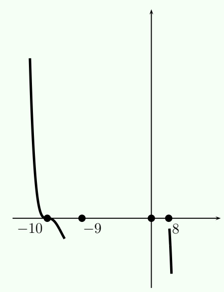


Now we look at what is going on at the roots. Near each root the factor corresponding to that root dominates. So we have

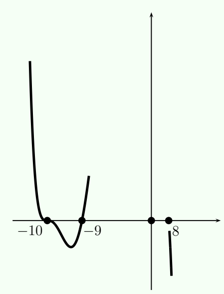
for $x \approx$	$p(x) \approx$	
-10	$C_1(x+10)^3$	cubic
-9	$C_2(x+9)$	line
0	$C_3\ x^2$	parabola
8	$C_4(x-8)$	line

where $C_1, C_2, C_3,$ and C_4 are constants which can, but need not, be calculated. For example, whether or not the parabola near 0 opens up or down will depend on whether the constant $C_3 = -2\cdot(0+10)^3(0+9)(0-8)$ is negative or positive. In this case C_3 is positive, so it opens upward, but we will not use this fact to graph. We will see this independently which is a good check of our work.
Starting from the left of our graph where we had determined the behavior for large negative x, we move toward the left-most zero, -10. Near -10 the graph looks cubic, so we imitate a cubic curve as we pass through

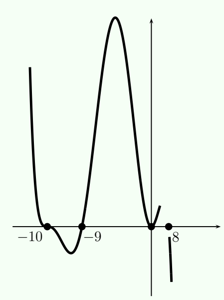
$(-10, 0)$.



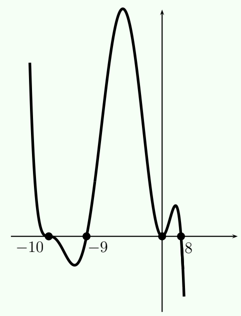
Now we turn and head toward the next zero, -9. Here the graph looks like a line, so we pass through the point $(-9, 0)$ as a line would.



Now we turn and head toward the root 0. Here the graph should look like a parabola. So we form a parabola there. (Note that, as we had said before, the parabola should be opening upward here—and we see that it is).



Now we turn toward the final zero 8. We pass through the point $(8, 0)$ like a line and we join (perhaps with the use of an eraser) to the large x part of the graph. If this does not join nicely (if the graph is going in the wrong direction) then there has been a mistake. This is a check of our work. Here is the final sketch.

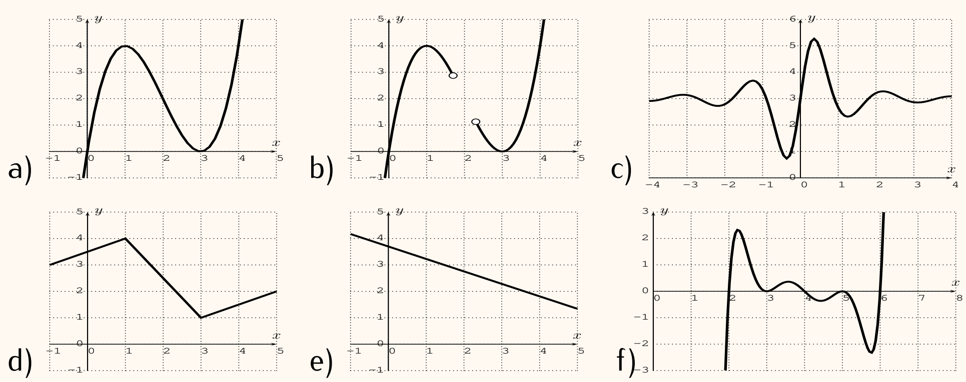


What can be understood from this sketch? Questions like "when is $p(x) > 0$?" can be answered by looking at the sketch. Further, the general shape of the curve is correct so that other properties can be concluded. For example, p has a local minimum between $x = -10$ and $x = -9$ and a local maximum between $x = -9$ and $x = 0$, and between $x = 0$ and $x = 8$. The exact point where the function reaches its maximum or minimum cannot be decided by looking at this sketch. But it will help to decide on an appropriate window so that the minimum or maximum finder on the calculator can be used. □

8.4 Exercises

Exercise 8.1

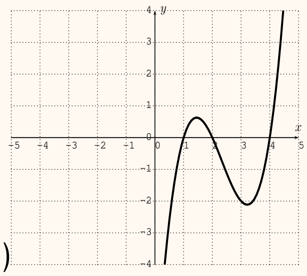
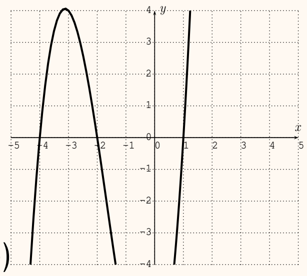
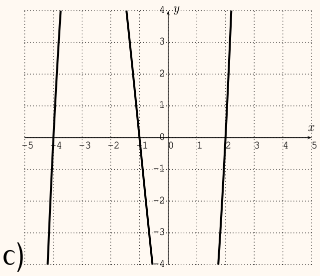
Assuming the graphs below are complete graphs, which of the graphs could be the graphs of a polynomial?

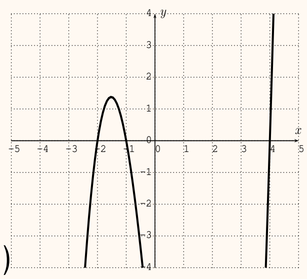
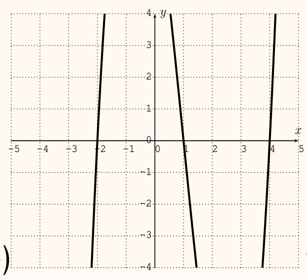
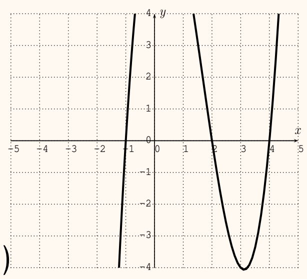


Exercise 8.2

For each of the polynomials f, g, h, and k, find the corresponding graph from (a)–(f) below.

$$f(x) = (x-1)\cdot(x+2)\cdot(x-4) \qquad g(x) = (x+1)\cdot(x-2)\cdot(x+4)$$
$$h(x) = (x-1)\cdot(x-2)\cdot(x-4) \qquad k(x) = (x+1)\cdot(x-2)\cdot(x-4)$$

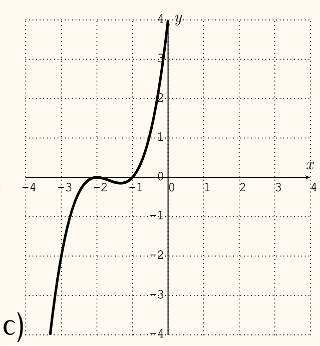
a) 
b) 
c) 

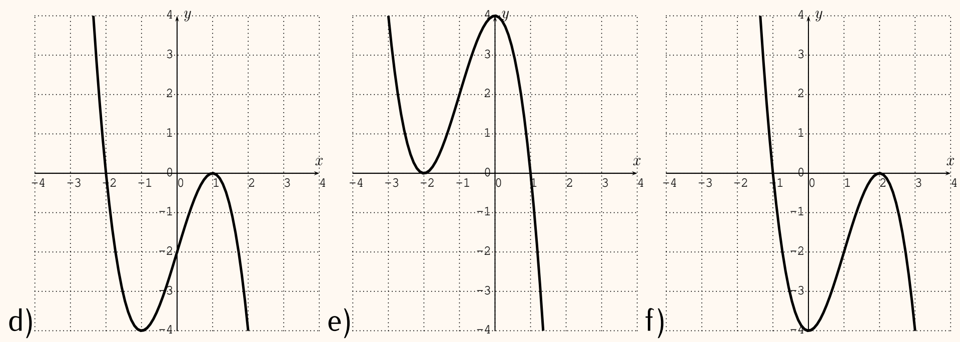
d) 
e) 
f) 

Exercise 8.3

For each of the polynomials f, g, h, and k, find the corresponding graph from (a)–(f) below.

$$f(x) = (x+1)\cdot(x+2)^2 \qquad g(x) = -(x+1)\cdot(x-2)^2$$
$$h(x) = -(x-1)^2\cdot(x+2) \qquad k(x) = (x-1)\cdot(x+2)^2$$

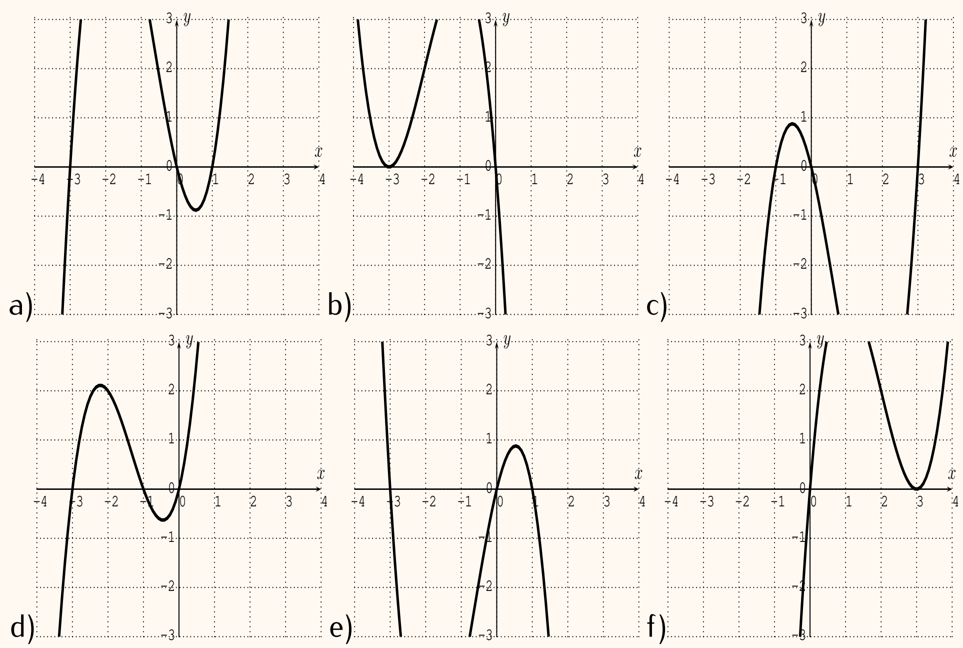
a) 
b) 
c) 



Exercise 8.4

For each of the polynomials f, g, h, and k, find the corresponding graph from (a)–(f) below.

$$f(x) = x^3 + 4x^2 + 3x \qquad g(x) = -x^3 - 2x^2 + 3x$$
$$h(x) = x^2 - 2x^2 - 3x \qquad k(x) = -x^3 - 6x^2 - 9x$$



Exercise 8.5

Sketch a complete the graph of the function. Label all intercepts of the graph.

a) $f(x) = x^3 + 4x^2 + x - 6$
b) $f(x) = 2x^3 - 15x^2 + 34x - 24$
c) $f(x) = x^3 - 16x - 21$
d) $f(x) = -2x^3 - 5x^2 - 2x + 1$
e) $f(x) = x^4 - 7x^3 + 15x^2 - 7x - 6$
f) $f(x) = 3x^4 + 11x^3 - x^2 - 19x + 6$

Exercise 8.6

Find the exact value of at least one root of the given polynomial.

a) $f(x) = x^3 - 10x^2 + 31x - 30$
b) $f(x) = -x^3 - x^2 + 8x + 8$
c) $f(x) = x^3 - 11x^2 - 3x + 33$
d) $f(x) = x^4 + 9x^3 - 6x^2 - 136x - 192$
e) $f(x) = x^2 + 6x + 3$
f) $f(x) = x^4 - 6x^3 + 3x^2 + 5x$

Exercise 8.7

Find all roots and factor the polynomial completely.

a) $f(x) = x^3 - 5x^2 + 2x + 8$
b) $f(x) = x^3 + 7x^2 + 7x - 15$
c) $f(x) = x^3 + 9x^2 + 26x + 24$
d) $f(x) = x^3 + 4x^2 - 11x + 6$
e) $f(x) = 3x^3 + 13x^2 - 52x + 28$
f) $f(x) = 6x^3 - 5x^2 - 13x - 2$
g) $f(x) = 6x^3 - x^2 - 31x - 10$
h) $f(x) = x^3 - 7x^2 + 13x - 3$
i) $f(x) = x^3 + 2x^2 - 11x + 8$
j) $f(x) = 2x^3 + 7x^2 + 5x - 2$
k) $f(x) = 3x^3 - 10x^2 - 4x + 21$

Exercise 8.8

Graph the following polynomials without using the calculator.

a) $f(x) = (x+4)^2(x-5)$
b) $f(x) = -3(x+2)^3x^2(x-4)^5$
c) $f(x) = 2(x-3)^2(x-5)^3(x-7)$
d) $f(x) = -(x+4)(x+3)(x+2)^2(x+1)(x-2)^2$