Precalculus

Third Edition (3.0)

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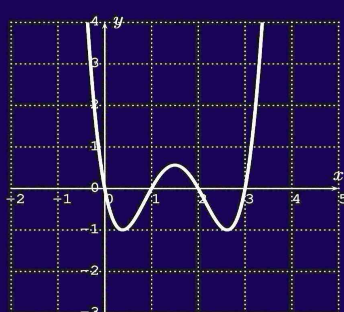

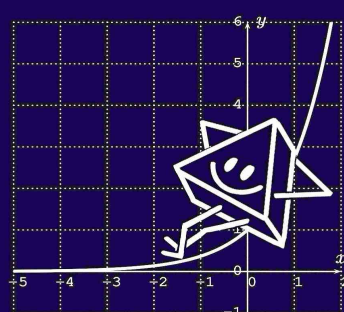

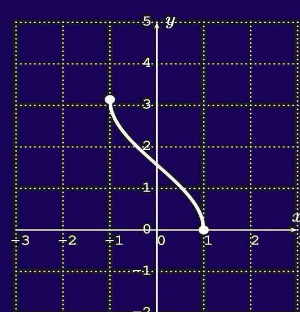

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Chapter 7

Dividing polynomials

We now start our discussion of specific classes of examples of functions. The first class of functions that we discuss are polynomials and rational functions. In this section we discuss an important tool for analyzing these functions, which consists of dividing two polynomials, also known as long division. Before we get to this, let us first recall the definition of polynomials and rational functions.

Definition 7.1: Monomial, polynomial

A **monomial** is a number, a variable, or a product of numbers and variables. A **polynomial** is a sum (or difference) of monomials.

Example 7.2

The following are examples of monomials:

$$5, \quad x, \quad 7x^2y, \quad -12x^3y^2z^4, \quad \sqrt{2}\cdot a^3n^2xy$$

The following are examples of polynomials:

$$x^2+3x-7, \quad 4x^2y^3+2x+z^3+4mn^2, \quad -5x^3-x^2-4x-9, \quad 5x^2y^4$$

In particular, every monomial is also a polynomial.

We are mainly interested in polynomials in one variable x, and consider these as functions. For example, $f(x)=x^2+3x-7$ is such a function.

Definition 7.3: Polynomial in one variable

A **polynomial in one variable** is a function f of the form

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$$

for some constants $a_0, a_1, \ldots, a_n$, where $a_n \neq 0$ and n is a non-negative integer. The numbers $a_0, a_1, \ldots, a_n$ are called **coefficients**. For each k, the number a_k is the **coefficient of** x^k. The number a_n is called the **leading coefficient** and n is the **degree** of the polynomial.
We usually consider polynomials f with real coefficients. In this case, the domain of a f is all real numbers (see our standard convention 2.7).
A **root** or **zero** of a polynomial $f(x)$ is a number c so that $f(c) = 0$.

Definition 7.4: Rational function

A **rational function** is a fraction of two polynomials $f(x) = \frac{g(x)}{h(x)}$ where $g(x)$ and $h(x)$ are both polynomials, and $h \neq 0$ is not the zero function. The domain of f is all real numbers for which the denominator $h(x)$ is not zero:

$$D_f \;=\; \{\ x \mid h(x) \neq 0\ \}.$$

Example 7.5

The following are examples of rational functions:

$$f(x) = \frac{-3x^2 + 7x - 5}{2x^3 + 4x^2 + 3x + 1}, \qquad f(x) = \frac{1}{x}, \qquad f(x) = -x^2 + 3x + 5$$

7.1 Long division

An important tool for analyzing polynomials consists of dividing two polynomials. The method of dividing polynomials that we use is that of *long division*, which is similar to the long division of natural numbers. Our first example shows the procedure in detail.

Example 7.6

Divide the following fractions via long division:

$$\text{a)}\ \frac{3571}{11} \qquad \text{b)}\ \frac{x^3+5x^2+4x+2}{x+3}$$

Solution.

a) Recall the procedure for long division of natural numbers:

$$\begin{array}{r|l} & 324 \\ \hline 11 & 3571 \\ & -33 \\ \hline & 271 \\ & -22 \\ \hline & 51 \\ & -44 \\ \hline & 7 \quad = \text{remainder} \end{array}$$

The steps above are performed as follows. First, we find the largest multiple of 11 less or equal to 35. The answer 3 is written as the first digit on the top line. Multiply 3 times 11, and subtract the answer 33 from the first two digits 35 of the dividend. The remaining digits 71 are copied below to give 271. Now we repeat the procedure, until we arrive at the remainder 7. In short, what we have shown is that:

$$3571 = 324 \cdot 11 + 7 \quad \text{or alternatively,} \quad \frac{3571}{11} = 324 + \frac{7}{11}.$$

b) We repeat the steps from part (a) as follows. First, write the dividend and divisor as in the format above:

$$x+3 \,\big|\, \overline{x^3 \quad +5x^2 \quad +4x \quad +2}$$

Next, consider the highest term x^3 of the dividend and the highest term x of the divisor. Since $\frac{x^3}{x} = x^2$, we start with the first term x^2 of the quotient:

Step 1:

$$\begin{array}{r|l} & x^2 \\ \hline x+3 & x^3 \quad +5x^2 \quad +4x \quad +2 \end{array}$$

Multiply x^2 by the divisor $x+3$ and write it below the dividend:

Step 2:

$$\begin{array}{r|llll} & x^2 & & & \\ \cline{2-5} x+3 & x^3 & +5x^2 & +4x & +2 \\ & x^3 & +3x^2 & & \end{array}$$

Since we need to subtract x^3+3x^2, so we equivalently add its negative (don't forget to distribute the negative):

Step 3:

$$\begin{array}{r|llll} & & x^2 & & \\ \cline{2-5} x+3 & x^3 & +5x^2 & +4x & +2 \\ & -(x^3 & +3x^2) & & \\ \cline{2-3} & & 2x^2 & & \end{array}$$

Now, carry down the remaining terms of the dividend:

Step 4:

$$\begin{array}{r|llll} & & x^2 & & \\ \cline{2-5} x+3 & x^3 & +5x^2 & +4x & +2 \\ & -(x^3 & +3x^2) & & \\ \cline{2-3} & & 2x^2 & +4x & +2 \end{array}$$

Now, repeat steps 1–4 for the remaining polynomial $2x^2+4x+2$. The outcome after going through steps 1–4 is the following:

$$\begin{array}{r|lllll} & & x^2 & +2x & & \text{(add } \frac{2x^2}{x} = 2x\text{)} \\ \cline{2-5} x+3 & x^3 & +5x^2 & +4x & +2 & \\ & -(x^3 & +3x^2) & & & \\ \cline{2-3} & & 2x^2 & +4x & +2 & \\ & & -(2x^2 & +6x) & & \text{(multiply } 2x \text{ by } (x+3)\text{)} \\ \cline{3-4} & & & -2x & +2 & \text{(subtract from the above)} \end{array}$$

Since x can be divided into $-2x$, we can proceed with the above steps 1–4 one more time. The outcome is this:

$$\begin{array}{r|llll}
 & x^2 & +2x & -2 & \\ \cline{2-5}
x+3 & x^3 \quad +5x^2 & +4x & +2 & \\
 & -(x^3 \quad +3x^2) & & & \\ \cline{2-2}
 & \quad\quad 2x^2 & +4x & +2 & \\
 & \quad -(2x^2 & +6x) & & \\
 & & -2x & +2 & \\
 & & -(-2x & -6) & \\
 & & & 8 & = \text{remainder}
\end{array}$$

Note now that x cannot be divided into 8 so we stop here. The final term 8 is called the remainder. The term $x^2 + 2x - 2$ is called the quotient. In analogy with our result in part (a), we can write our conclusion as:

$$x^3 + 5x^2 + 4x + 2 = (x^2 + 2x - 2) \cdot (x + 3) + 8.$$

Alternatively, we could also divide this by $(x + 3)$ and write it as:

$$\frac{x^3 + 5x^2 + 4x + 2}{x + 3} = x^2 + 2x - 2 + \frac{8}{x + 3}.$$

□

Note 7.7: Dividend, divisor, quotient, remainder

Just as with a division operation involving numbers, when dividing $\frac{f(x)}{g(x)}$, $f(x)$ is called the **dividend** and $g(x)$ is called the **divisor**. As a result of dividing $f(x)$ by $g(x)$ via long division with **quotient** $q(x)$ and **remainder** $r(x)$, we can write

$$\frac{f(x)}{g(x)} = q(x) + \frac{r(x)}{g(x)}. \tag{7.1}$$

If we multiply this equation by $g(x)$, we obtain the following alternative version:

$$f(x) = q(x) \cdot g(x) + r(x) \tag{7.2}$$

Example 7.8

Divide the following fractions via long division.

a) $\frac{x^2+4x+5}{x-4}$ b) $\frac{x^4+3x^3-5x+1}{x+1}$

c) $\frac{4x^3+2x^2+6x+18}{2x+3}$ d) $\frac{x^3+x^2+2x+1}{x^2+3x+1}$

Solution.

a) We calculate:

$$\begin{array}{r|rrr} & & x & +8 \\ \hline x-4 & x^2 & +4x & +5 \\ & -(x^2 & -4x) & \\ \cline{2-3} & & 8x & +5 \\ & & -(8x & -32) \\ \cline{3-4} & & & 37 \end{array}$$

Therefore, $x^2 + 4x + 5 = (x + 8) \cdot (x - 4) + 37$.

b) Note that there is no x^2 term in the dividend. This can be resolved by adding $+0\,x^2$ to the dividend:

$$\begin{array}{r|rrrrr} & & x^3 & +2x^2 & -2x & -3 \\ \hline x+1 & x^4 & +3x^3 & +0x^2 & -5x & +1 \\ & -(x^4 & +x^3) & & & \\ \cline{2-3} & & 2x^3 & +0x^2 & -5x & +1 \\ & & -(2x^3 & +2x^2) & & \\ \cline{3-4} & & & -2x^2 & -5x & +1 \\ & & & -(-2x^2 & -2x) & \\ \cline{4-5} & & & & -3x & +1 \\ & & & & -(-3x & -3) \\ \cline{5-6} & & & & & 4 \end{array}$$

Therefore, we showed:

$$\frac{x^4 + 3x^3 - 5x + 1}{x + 1} = x^3 + 2x^2 - 2x - 3 + \frac{4}{x + 1}.$$

c)

$$\begin{array}{r|rrrr} & & 2x^2 & -2x & +6 \\ \hline 2x+3 & 4x^3 & +2x^2 & +6x & +18 \\ & -(4x^3 & +6x^2) & & \\ \hline & & -4x^2 & +6x & +18 \\ & & -(-4x^2 & -6x) & \\ \hline & & & 12x & +18 \\ & & & -(12x & +18) \\ \hline & & & & 0 \end{array}$$

Since the remainder is zero, we succeeded in factoring $4x^3 + 2x^2 + 6x + 18$:

$$4x^3 + 2x^2 + 6x + 18 = (2x^2 - 2x + 6) \cdot (2x + 3)$$

d) The last example has a divisor that is a polynomial of degree 2. Therefore, the remainder is not a number, but a polynomial of degree 1.

$$\begin{array}{r|rrrr} & & & x & -2 \\ \hline x^2+3x+1 & x^3 & +x^2 & +2x & +1 \\ & -(x^3 & +3x^2 & +x) & \\ \hline & & -2x^2 & +x & +1 \\ & & -(-2x^2 & -6x & -2) \\ \hline & & & 7x & +3 \end{array}$$

Here, the remainder is $r(x) = 7x + 3$.

$$\frac{x^3 + x^2 + 2x + 1}{x^2 + 3x + 1} = x - 2 + \frac{7x + 3}{x^2 + 3x + 1}$$

□

Note 7.9: Factoring and zero remainder

The divisor $g(x)$ is a factor of $f(x)$ exactly when the remainder $r(x)$ is zero, that is:

$$f(x) = q(x) \cdot g(x) \quad \Longleftrightarrow \quad r(x) = 0.$$

For example, in the above Example 7.8, only part (c) results in a factorization of the dividend, since this is the only part with remainder zero.

7.2 Dividing by $(x - c)$

We now restrict our attention to the case in which the divisor is $g(x) = x - c$ for some real number c. In this case, the remainder r of the division $f(x)$ by $g(x)$ is a real number. We make the following observations.

Observation 7.10: Remainder theorem, factor theorem

Assume that $g(x) = x - c$, and the long division of $f(x)$ by $g(x)$ has remainder r, that is,

$$\text{Assumption: } f(x) = q(x) \cdot (x - c) + r.$$

When we evaluate both sides in the above equation at $x = c$, we see that $f(c) = q(c) \cdot (c - c) + r = q(c) \cdot 0 + r = r$. In short:

$$\boxed{\text{The remainder when dividing } f(x) \text{ by } (x - c) \text{ is } \quad r = f(c).} \tag{7.3}$$

In particular:

$$\boxed{f(c) = 0 \quad \Longleftrightarrow \quad g(x) = x - c \text{ is a factor of } f(x).} \tag{7.4}$$

The above statement (7.3) is called the **remainder theorem**, and (7.4) is called the **factor theorem**.

Example 7.11

Find the remainder of dividing $f(x) = x^2 + 3x + 2$ by

a) $x - 3$ b) $x + 4$ c) $x + 1$ d) $x - \frac{1}{2}$

Solution.

a) By Observation 7.10, we know that the remainder r of the division by $x - c$ is $f(c)$. Thus, the remainder for part (a), when dividing by

$x - 3$ is

$$r = f(3) = 3^2 + 3 \cdot 3 + 2 = 9 + 9 + 2 = 20.$$

b) For (b), note that $g(x) = x + 4 = x - (-4)$, so that taking $c = -4$ for our input yields a remainder of $r = f(-4) = (-4)^2 + 3 \cdot (-4) + 2 = 16 - 12 + 2 = 6$. Similarly, the other remainders are:

c) $r = f(-1) = (-1)^2 + 3 \cdot (-1) + 2 = 1 - 3 + 2 = 0,$

d) $r = f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 + \frac{1}{2} \cdot 3 + 2 = \frac{1}{4} + \frac{3}{2} + 2 = \frac{1 + 6 + 8}{4} = \frac{15}{4}.$

Note that in part (c), we found a remainder 0, so that $(x + 1)$ is a factor of $f(x)$.

□

Example 7.12

Determine whether $g(x)$ is a factor of $f(x)$.

a) $f(x) = x^3 + 2x^2 + 5x + 1, \quad g(x) = x - 2$
b) $f(x) = x^4 + 4x^3 + x^2 + 18, \quad g(x) = x + 3$
c) $f(x) = x^5 + 3x^2 + 7, \quad g(x) = x + 1$

Solution.

a) We need to determine whether 2 is a root of $f(x) = x^3 + 2x^2 + 5x + 1$, that is, whether $f(2)$ is zero.

$$f(2) = 2^3 + 2 \cdot 2^2 + 5 \cdot 2 + 1 = 8 + 8 + 10 + 1 = 27.$$

Since $f(2) = 27 \neq 0$, we see that $g(x) = x - 2$ is not a factor of $f(x)$.

b) Now, $g(x) = x + 3 = x - (-3)$, so that we calculate:

$$f(-3) = (-3)^4 + 4 \cdot (-3)^3 + (-3)^2 - 18 = 81 - 108 + 9 + 18 = 0.$$

Since the remainder is zero, we see that $x + 3$ is a factor of $x^4 + 4 \cdot x^3 + x^2 + 18$. Therefore, if we wanted to find the other factor, we could use long division to obtain the quotient.

c) Finally, we have:

$$f(-1) = (-1)^5 + 3 \cdot (-1)^2 + 7 = -1 + 3 + 7 = 9.$$

$g(x) = x + 1$ is not a factor of $f(x) = x^5 + 3x^2 + 7$.

□

Example 7.13

a) Show that -2 is a root of $f(x) = x^5 - 3x^3 + 5x^2 - 12$, and use this to factor f.

b) Show that 5 is a root of $f(x) = x^3 - 19x - 30$, and use this to factor f *completely*.

Solution.

a) First, we calculate that -2 is a root.

$$f(-2) = (-2)^5 - 3 \cdot (-2)^3 + 5 \cdot (-2)^2 + 12 = -32 + 24 + 20 - 12 = 0.$$

So we can divide $f(x)$ by $g(x) = x - (-2) = x + 2$:

$$\begin{array}{r|rrrrrr}
 & & x^4 & -2x^3 & +x^2 & +3x & -6 \\ \hline
x+2 & x^5 & +0x^4 & -3x^3 & +5x^2 & +0x & -12 \\
 & -(x^5 & +2x^4) & & & & \\ \cline{2-3}
 & & -2x^4 & -3x^3 & +5x^2 & +0x & -12 \\
 & & -(-2x^4 & -4x^3) & & & \\ \cline{3-4}
 & & & x^3 & +5x^2 & +0x & -12 \\
 & & & -(x^3 & +2x^2) & & \\ \cline{4-5}
 & & & & 3x^2 & +0x & -12 \\
 & & & & -(3x^2 & +6x) & \\ \cline{5-6}
 & & & & & -6x & -12 \\
 & & & & & -(-6x & -12) \\ \cline{6-7}
 & & & & & & 0
\end{array}$$

So we factored $f(x)$ as

$$f(x) = (x^4 - 2x^3 + x^2 + 3x - 6) \cdot (x + 2).$$

b) Again, we start by calculating $f(5) = 5^3 - 19 \cdot 5 - 30 = 125 - 95 - 30 = 0$. Long division by $g(x) = x - 5$ gives:

$$\begin{array}{r|rrrr} & & x^2 & +5x & +6 \\ \cline{2-5} x-5 & x^3 & +0x^2 & -19x & -30 \\ & -(x^3 & -5x^2) & & \\ \cline{2-3} & & 5x^2 & -19x & -30 \\ & & -(5x^2 & -25x) & \\ \cline{3-4} & & & 6x & -30 \\ & & & -(6x & -30) \\ \cline{4-5} & & & & 0 \end{array}$$

Thus, $x^3 - 19x - 30 = (x^2 + 5x + 6) \cdot (x - 5)$. To factor f completely, we also factor $x^2 + 5x + 6$.

$$f(x) = (x^2 + 5x + 6) \cdot (x - 5) = (x + 2) \cdot (x + 3) \cdot (x - 5).$$

□

7.3 *Optional section:* Synthetic division

When dividing a polynomial $f(x)$ by $g(x) = x - c$, the actual calculation of the long division has a lot of unnecessary repetitions, and we may want to reduce this redundancy as much as possible. In fact, we can extract the essential part of the long division, the result of which is called **synthetic division**.

Example 7.14

Our first example is the long division of $\frac{5x^3+7x^2+x+4}{x+2}$.

$$\begin{array}{r|rrrr}
 & & 5x^2 & -3x & +7 \\ \hline
x+2 & 5x^3 & +7x^2 & +x & +4 \\
 & -(5x^3 & +10x^2) & & \\ \cline{2-3}
 & & -3x^2 & +x & +4 \\
 & & -(-3x^2 & -6x) & \\ \cline{3-4}
 & & & 7x & +4 \\
 & & & -(7x & +14) \\ \cline{4-5}
 & & & & -10
\end{array}$$

Here, the first term $5x^2$ of the quotient is just copied from the first term of the dividend. We record this together with the coefficients of the dividend $5x^3+7x^2+x+4$ and of the divisor $x+2 = x-(-2)$ as follows:

$$\begin{array}{r|rrrr l}
 & 5 & 7 & 1 & 4 & \text{(dividend } (5x^3+7x^2+x+4)\text{)} \\
-2 & & & & & \text{(divisor } (x-(-2))\text{)} \\ \hline
 & 5 & & & & \text{(quotient)}
\end{array}$$

The first actual calculation is performed when multiplying the $5x^2$ term with 2, and subtracting it from $7x^2$. We record this as follows:

$$\begin{array}{r|rrrr l}
 & 5 & 7 & 1 & 4 & \\
-2 & & -10 & & & (-10 \text{ is the product of } -2\cdot 5) \\ \hline
 & 5 & -3 & & & (-3 \text{ is the sum } 7+(-10))
\end{array}$$

Similarly, we obtain the next step by multiplying the $2x$ by (-3) and subtracting it from $1x$. Therefore, we get

$$\begin{array}{r|rrrr l}
 & 5 & 7 & 1 & 4 & \\
-2 & & -10 & 6 & & (6 \text{ is the product of } -2\cdot(-3)) \\ \hline
 & 5 & -3 & 7 & & (7 \text{ is the sum } 1+(6))
\end{array}$$

The last step multiplies 7 times 2 and subtracts this from 4. In short, we write:

$$\begin{array}{r|rrrr l}
 & 5 & 7 & 1 & 4 & \\
-2 & & -10 & 6 & -14 & (-14 \text{ is the product of } -2\cdot(7)) \\ \hline
 & 5 & -3 & 7 & -10 & (-10 \text{ is the sum } 4+(-14))
\end{array}$$

The answer can be determined from these coefficients. The quotient is $5x^2 - 3x + 7$, and the remainder is -10.

Example 7.15

Find the following quotients via synthetic division.

$$\text{a)}\ \frac{4x^3 - 7x^2 + 4x - 8}{x - 4} \qquad \text{b)}\ \frac{x^4 - x^2 + 5}{x + 3}$$

Solution.

a) We need to perform the synthetic division.

$$\begin{array}{r|rrrr} & 4 & -7 & 4 & -8 \\ 4 & & 16 & 36 & 160 \\ \hline & 4 & 9 & 40 & 152 \end{array}$$

Therefore we have

$$\frac{4x^3 - 7x^2 + 4x - 8}{x - 4} = 4x^2 + 9x + 40 + \frac{152}{x - 4}.$$

b) Similarly, we calculate part (b). Note that some of the coefficients are now zero.

$$\begin{array}{r|rrrrr} & 1 & 0 & -1 & 0 & 5 \\ -3 & & -3 & 9 & -24 & 72 \\ \hline & 1 & -3 & 8 & -24 & 77 \end{array}$$

We obtain the following result.

$$\frac{x^4 - x^2 + 5}{x + 3} = x^3 - 3x^2 + 8x - 24 + \frac{77}{x + 5}$$

□

Note 7.16

We have only considered synthetic division when dividing by a polynomial of the form $x - c$. The method for dividing by polynomials such as $3x + 7$ or $x^2 + 5x - 4$ is more elaborate.

7.4 Exercises

Exercise 7.1

Divide by long division.

a) $\frac{x^3-4x^2+2x+1}{x-2}$ b) $\frac{x^3+6x^2+7x-2}{x+3}$ c) $\frac{x^2+7x-4}{x+1}$

d) $\frac{x^3+3x^2+2x+5}{x+2}$ e) $\frac{2x^3+x^2+3x+5}{x-1}$ f) $\frac{2x^4+7x^3+x+3}{x+5}$

g) $\frac{2x^4-31x^2-13}{x-4}$ h) $\frac{x^3+27}{x+3}$ i) $\frac{3x^4+7x^3+5x^2+7x+4}{3x+1}$

j) $\frac{8x^3+18x^2+21x+18}{2x+3}$ k) $\frac{x^3+3x^2-4x-5}{x^2+2x+1}$ l) $\frac{x^5+3x^4-20}{x^2+3}$

Exercise 7.2

Find the remainder when dividing $f(x)$ by $g(x)$.

a) $f(x) = x^3 + 2x^2 + x - 3$, $g(x) = x - 2$
b) $f(x) = x^3 - 5x + 8$, $g(x) = x - 3$
c) $f(x) = x^5 - 1$, $g(x) = x + 1$
d) $f(x) = x^5 + 5x^2 - 7x + 10$, $g(x) = x + 2$

Exercise 7.3

Determine whether the given $g(x)$ is a factor of $f(x)$. If so, name the corresponding root of $f(x)$.

a) $f(x) = x^2 + 5x + 6$, $g(x) = x + 3$
b) $f(x) = x^3 - x^2 - 3x + 8$, $g(x) = x - 4$
c) $f(x) = x^4 + 7x^3 + 3x^2 + 29x + 56$, $g(x) = x + 7$
d) $f(x) = x^{999} + 1$, $g(x) = x + 1$

Exercise 7.4

Check that the given numbers for x are roots of $f(x)$ (see Observation 7.10). If the numbers x are indeed roots, then use this information to factor $f(x)$ as much as possible.

a) $f(x) = x^3 - 2x^2 - x + 2$, $x = 1$
b) $f(x) = x^3 - 6x^2 + 11x - 6$, $x = 1, x = 2, x = 3$
c) $f(x) = x^3 - 3x^2 + x - 3$, $x = 3$
d) $f(x) = x^3 + 6x^2 + 12x + 8$, $x = -2$
e) $f(x) = x^3 + 13x^2 + 50x + 56$, $x = -2, x = -4$
f) $f(x) = x^3 + 3x^2 - 16x - 48$, $x = 2, x = -4$
g) $f(x) = x^5 + 5x^4 - 5x^3 - 25x^2 + 4x + 20$, $x = 1, x = -1,$ $x = 2, x = -2$

Exercise 7.5

Divide by using synthetic division.

a) $\frac{2x^3+3x^2-5x+7}{x-2}$ b) $\frac{4x^3+3x^2-15x+18}{x+3}$ c) $\frac{x^3+4x^2-3x+1}{x+2}$

d) $\frac{x^4+x^3+1}{x-1}$ e) $\frac{x^5+32}{x+2}$ f) $\frac{x^3+5x^2-3x-10}{x+5}$