Precalculus

Third Edition (3.0)

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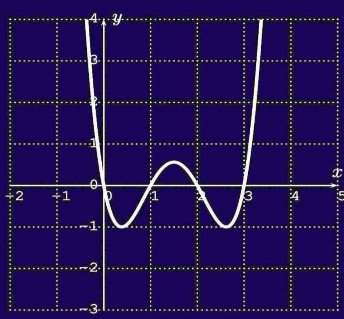
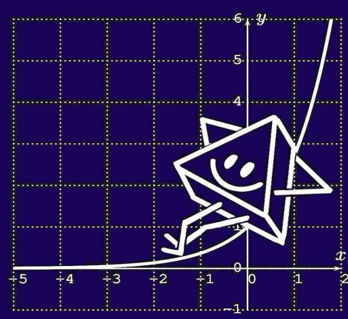
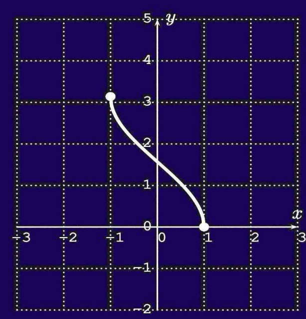
The pages below contain a single chapter from the Precalculus textbook. The full textbook can be downloaded here:

Precalculus Textbook Download Page







This document was created with LaTeX.

The images of the Desmos graphing calculator were generated from the Desmos website at **https://www.desmos.com/calculator**

Chapter 4

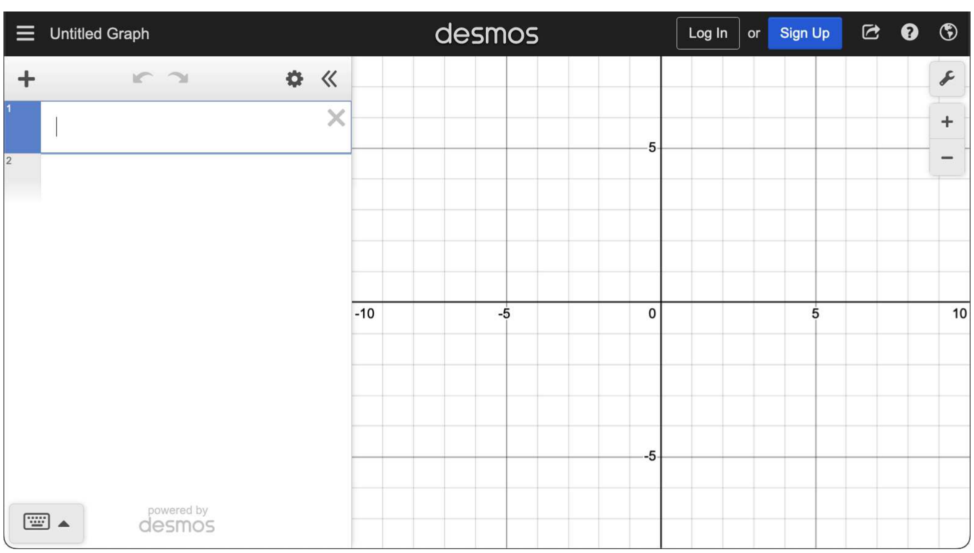
Basic functions and transformations

We now give an introduction to the Desmos graphing calculator that can be used to graph functions given by formulas. We also graph a list of basic functions and observe how these graphs change when adding or multiplying constants to their inputs or outputs.

4.1 Basics of the Desmos graphing calculator

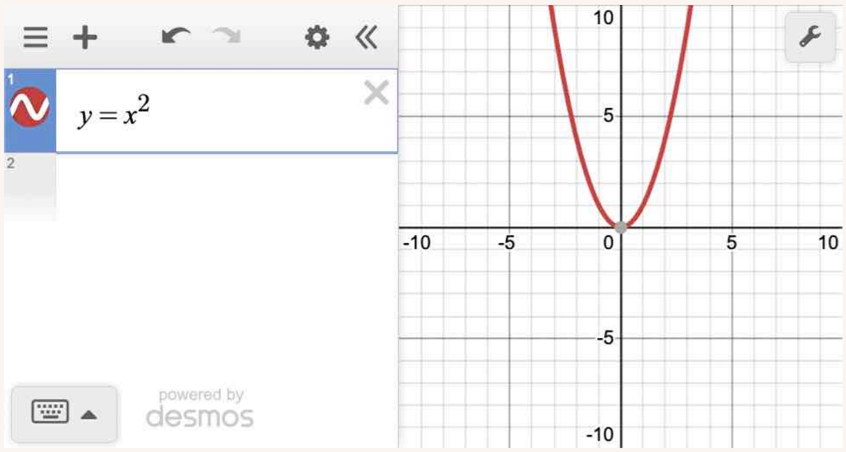
To get started, visit the Desmos graphing calculator at the URL

https://www.desmos.com/calculator



Note 4.1

Graphing a function is straightforward in Desmos, say $y = x^2$. Simply type $y = x^2$ into the input field on the top left.



Moreover, we can easily locate *local maxima* and *local minima* of a function (the peaks and valleys of its graph), as well as its x- and y-intercepts by simply clicking on the graph.
The x-intercepts are also commonly called **zeros** or **roots** of the function. In other words, a root or a zero of a function f is a number c for which $f(c) = 0$.

We demonstrate this in the next example.

Example 4.2

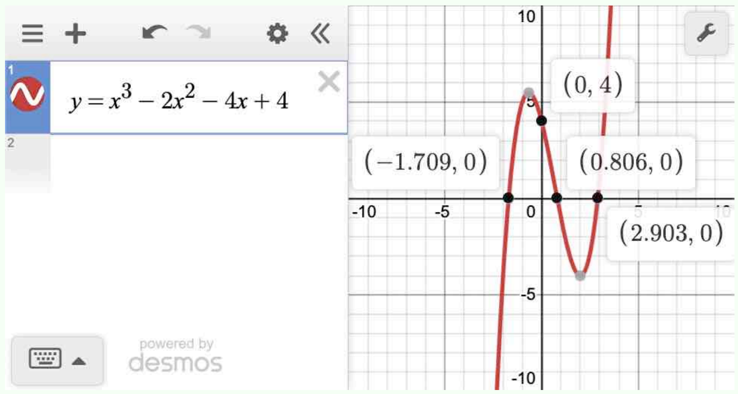
Graph the function $y = x^3 - 2x^2 - 4x + 4$.

a) Approximate the x-intercepts and the y-intercept of the function.

b) Approximate the (local) maximum and minimum. A (local) maximum or minimum is also called a **(local) extremum**.

Solution.

a) Enter the function in Desmos and click on the x-intercepts (points where the graph intersects with the x-axis) and the y-intercept (the point where the graph intersects with the y-axis).

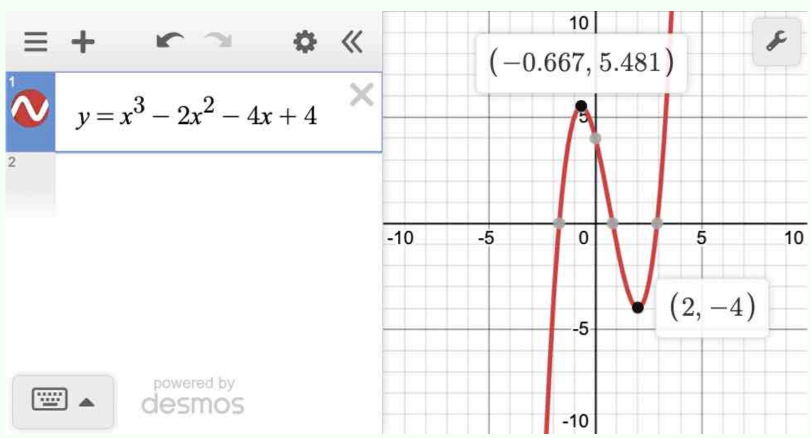


There are three x-intercepts and one y-intercept:

$$
\begin{aligned}
x\text{-intercepts: } & (x, y) \approx (-1.709, 0), \quad (x, y) \approx (0.806, 0) \\
& (x, y) \approx (2.903, 0) \\
y\text{-intercept: } & (x, y) = (0, 4)
\end{aligned}
$$

Note that the y-intercept is precisely at $(0, 4)$, whereas the x-intercepts are only approximated.

b) Similarly, we obtain the local maximum and the local minimum.



local max: $(x, y) \approx (-0.667, 5.481)$, local min: $(x, y) = (2, -4)$

One can check (using methods from calculus) that the local minimum is precisely at $(2, -4)$, whereas the local maximum is only approximated.

□

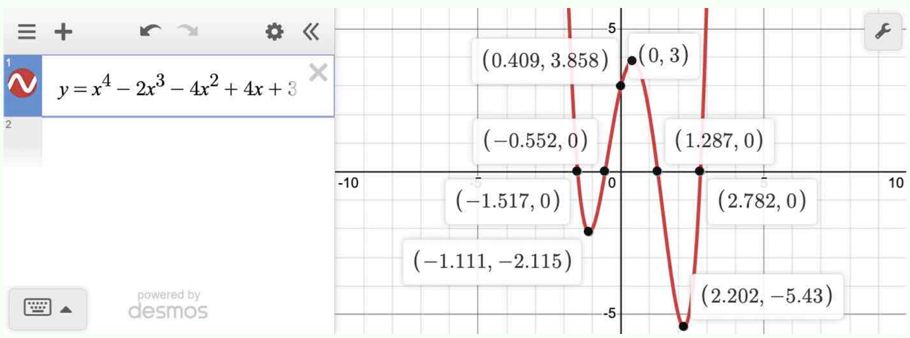
Example 4.3

For the two functions below, find all intercepts and all extrema. Approximate your answer to the nearest thousandth.

a) $f(x) = x^4 - 2x^3 - 4x^2 + 4x + 3$ b) $f(x) = x^3 - 9x^2 - x$

Solution.

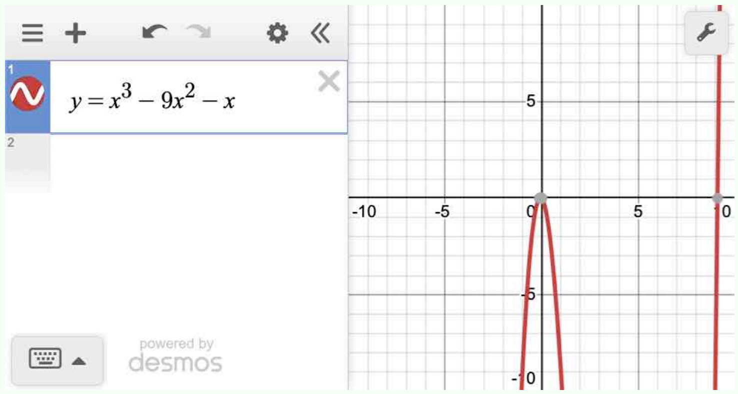
a) Graphing the function in Desmos, we can read off the coordinates of the wanted points (the intercepts and extrema) by clicking on them.



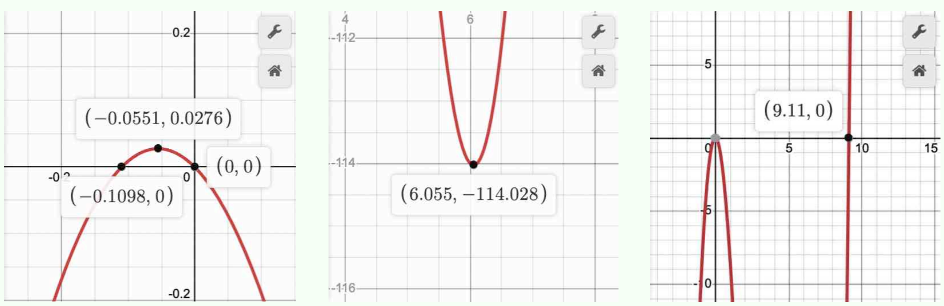
$$\begin{aligned}
x\text{-intercepts: } & (x,y) \approx (-1.517, 0), \quad (x,y) \approx (-0.552, 0), \\
& (x,y) \approx (1.287, 0), \quad (x,y) \approx (2.782, 0) \\
y\text{-intercept: } & (x,y) = (0,3) \\
\text{local maximum: } & (x,y) \approx (0.409, 3.858) \\
\text{local mininima: } & (x,y) \approx (-1.111, -2.115), \quad (x,y) \approx (2.202, -5.430)
\end{aligned}$$

Here, Desmos already rounded coordinates to the nearest thousandth. For example, the maximum with one more digit is $(x, y) \approx (0.4088, 3.8580)$, which rounds to $(0.409, 3.858)$.

b) Graphing $f(x) = x^3 - 9x^2 - x$ shows that we don't have a complete view of all the points of interest in the initial viewing window.



Zooming in and out (by using the plus and minus symbols on the graph or by scrolling on the screen), we can get a closer view of the wanted points.



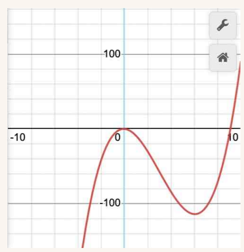
Note that by zooming in, Desmos may display more than three digits after the decimal point. Rounding to the nearest thousandth gives the following answers.

$$
\begin{aligned}
x\text{-intercepts: } & (x, y) \approx (-0.110, 0), \quad (x, y) = (0, 0), \\
& (x, y) \approx (9.110, 0) \\
y\text{-intercept: } & (x, y) = (0, 0) \\
\text{local maximum: } & (x, y) \approx (-0.055, 0.028) \\
\text{local mininimum: } & (x, y) \approx (6.055, -114.028)
\end{aligned}
$$

□

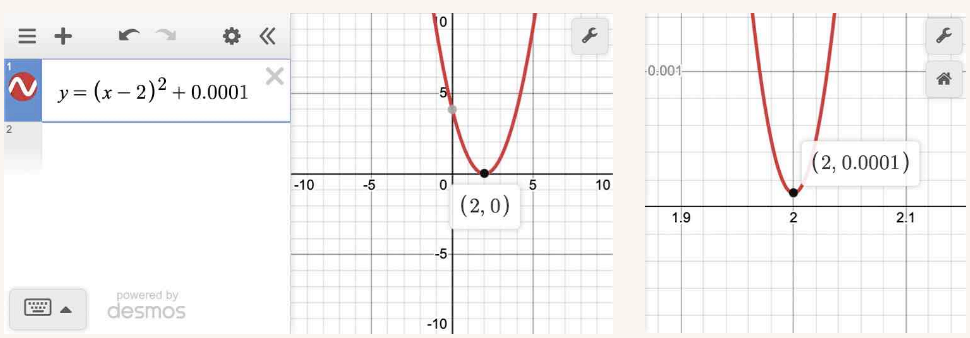
Note 4.4: Zooming

Besides zooming in and out, the display window can also be set manually via the Graph Settings menu (click on the wrench symbol ). The home button  resets the window to a size in which the x is approximately between -10 and 10, and with a matching scale for y. There is also a possibility to rescale each axis individually. To this end, hover the pointer over the axis that needs to be rescaled and press and hold the shift key. The axis will appear in blue, and can then be rescaled (click and move the pointer in the wanted direction). Below is the rescaled graph for $y = x^3 - 9x^2 - x$.



Note 4.5

Desmos only approximates its answers, such as intercepts, maxima, and minima. It is *our* task to correctly interpret and confirm any answers inferred from Desmos.
For example, graphing $y = (x-2)^2 + 0.0001$ appears to show a root at $(2,0)$. Nevertheless, a closer look reveals that this function does *not* have a root at $(2,0)$.



We next show how to graph piecewise defined functions with Desmos.

Example 4.6

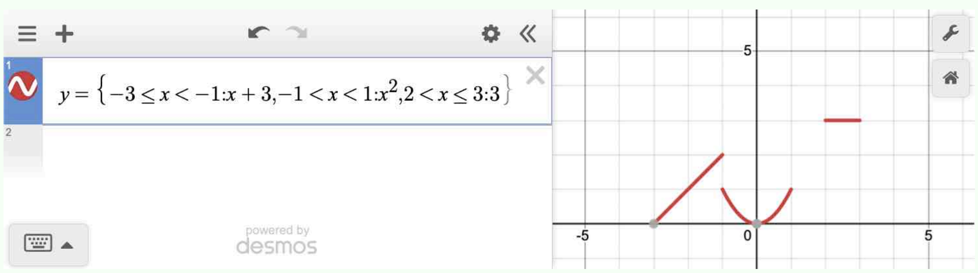
Graph the piecewise defined function from Example 3.20 with Desmos.

$$y = \begin{cases} x+3 & \text{, for } -3 \le x < -1 \\ x^2 & \text{, for } -1 < x < 1 \\ 3 & \text{, for } 2 < x \le 3 \end{cases}$$

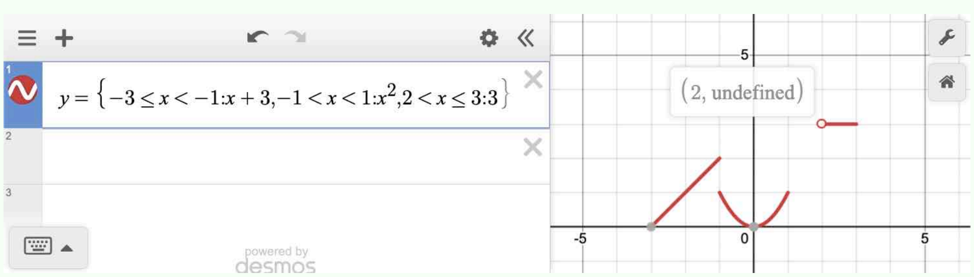
Solution.

A piecewise defined function is entered in Desmos with a set bracket $\{\}$, separating each branch with a comma. Each branch is entered as "condition:function value"; for example, the top branch in our example is entered as $-3 \le x < -1 : x+3$. Combining the three branches, we obtain:

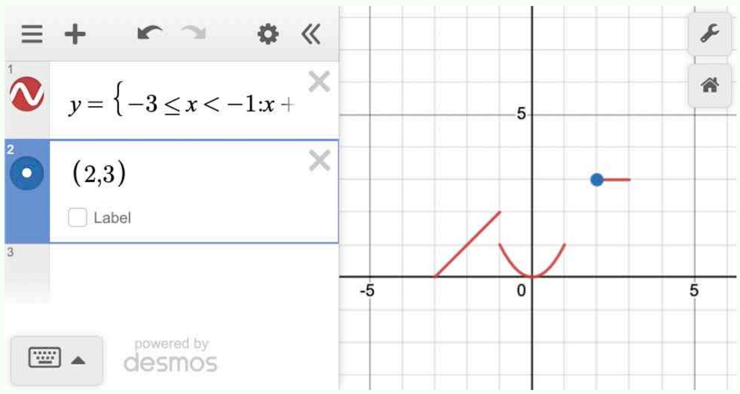
$$y = \{ \quad -3 \le x < -1 : x+3 \quad , \quad -1 < x < 1 : x^2 \quad , \quad 2 < x \le 3 : 3 \quad \}$$



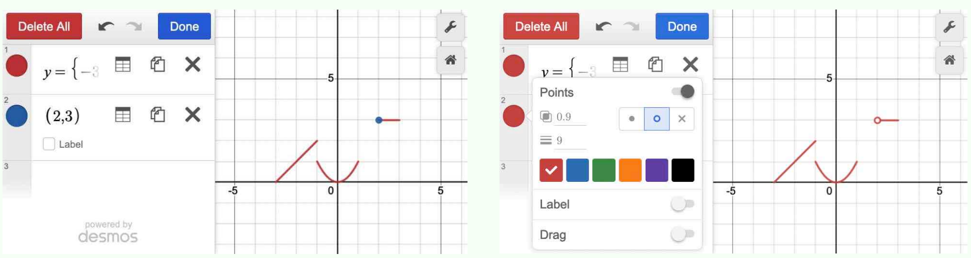
Although there are no open or closed circles at the endpoints of the line segments, Desmos does interpret these endpoints correctly. This can be seen by clicking on the endpoint of a branch.



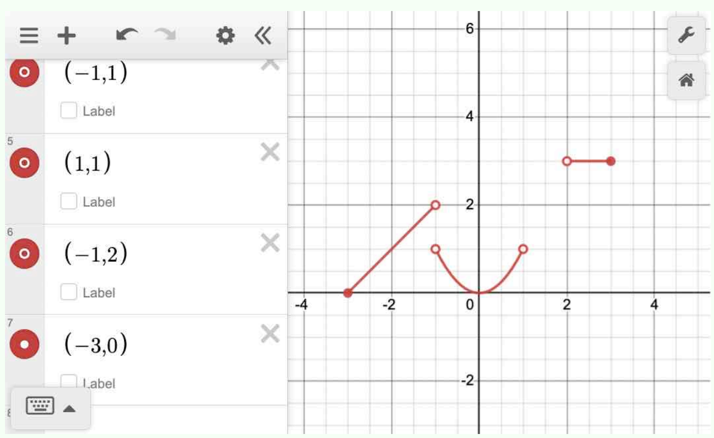
We can add the missing endpoints manually by entering the coordinates for each point.



To adjust properties of the point such as its color or it being depicted as open or closed, we click the Edit List button ⚙ on top of the panel, then click on the big (in this case blue) circle to the left of the entered point. We can then set the style and color of the point.



Performing this for each endpoint, we obtain the following graph.



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Our last example in this section shows how to easily compute function values and how to create tables in Desmos.

Example 4.7

Consider the functions

$$f(x) = x^3 - 4x^2 + 5$$

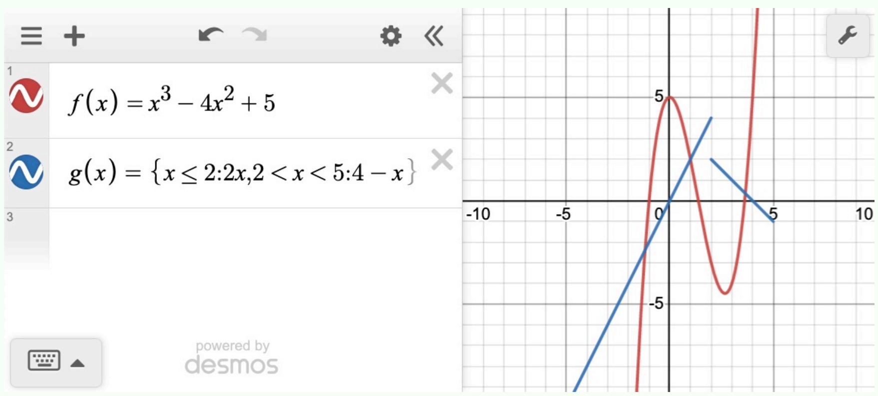
$$\text{and} \quad g(x) = \begin{cases} 2x & \text{, for} \quad x \le 2 \\ 4 - x & \text{, for} \quad 2 < x < 5 \end{cases}$$

Compute the function values

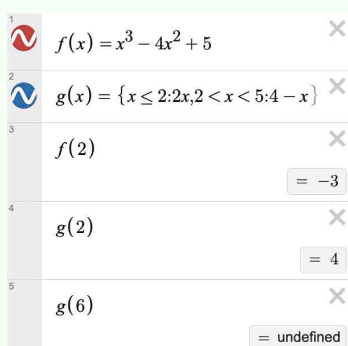
$$f(2), \qquad f(4), \qquad g(1), \qquad g(2), \qquad g(6).$$

Solution.

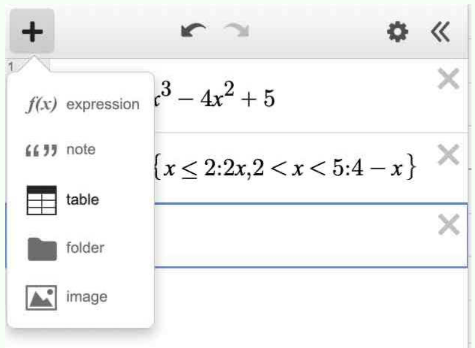
First, graph both functions f and g.



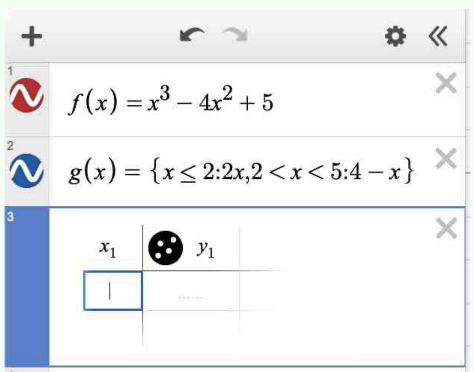
A direct way of computing function values is given by simply entering the wanted expression. Note that undefined function values (such as $g(6)$) are indeed stated as undefined.

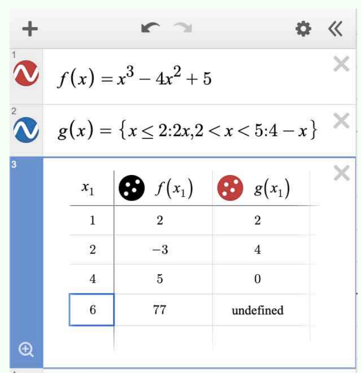


Another way to calculate function values comes from generating a table. Press the "Add Item" button + on top, and click on "table".



Modify the table by replacing y_1 with $f(x_1)$. We can also compute multiple function values, such as $f(x_1)$ and $g(x_1)$, by putting $g(x_1)$ next to $f(x_1)$. Below x_1, we enter the desired inputs.





Thus, $f(2) = -3$, $f(4) = 5$, $g(1) = 2$, $g(2) = 4$, and $g(6)$ is undefined. $\square$

4.2 *Optional section:* Exploring Desmos further

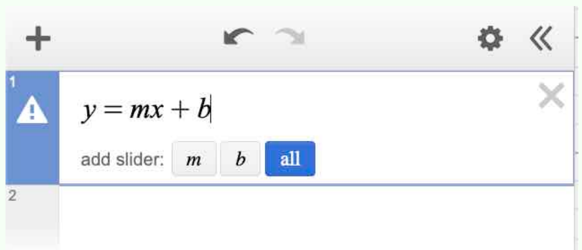
We now explore sliders in Desmos and we revisit the domain and range of a function, as well as the vertical line test. We also give an example of finding intersection points of two graphs.

Example 4.8

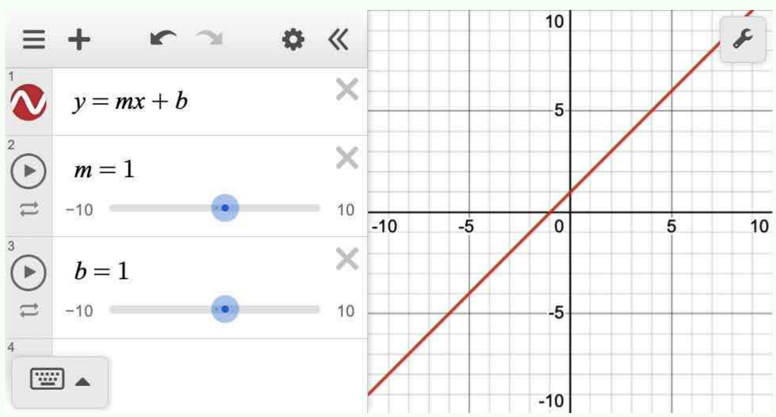
Explore the equation of a line $y = m \cdot x + b$ for various values of m and b using sliders.

Solution.

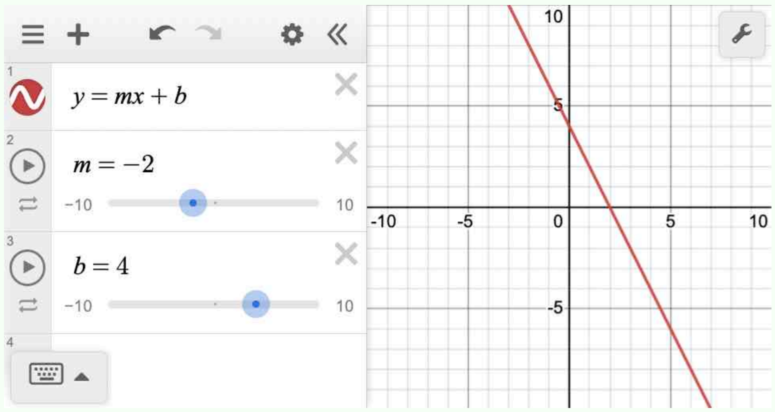
We enter $y = mx + b$ into Desmos.



We want m and b to be interpreted as constants, but these constants can be adjusted. This is precisely what sliders provide in Desmos. We therefore add the sliders m and b.



Changing the values for m and b, we instantly see the effect on its graph.



□

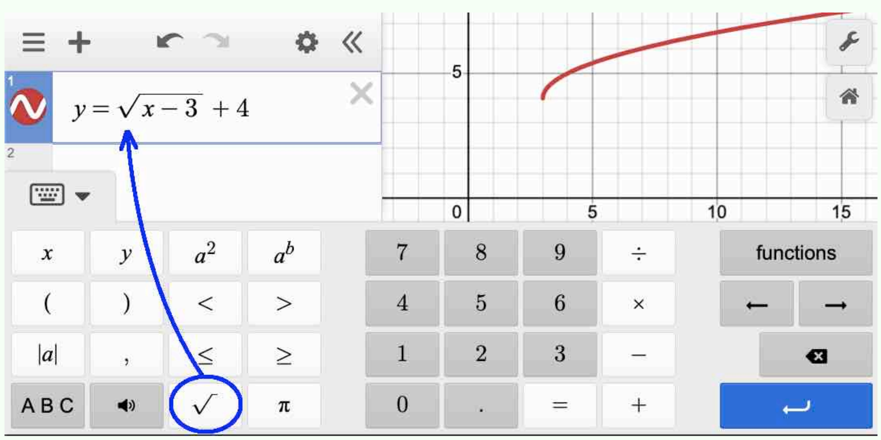
Example 4.9

Find the (approximate) domain and range of the following functions.

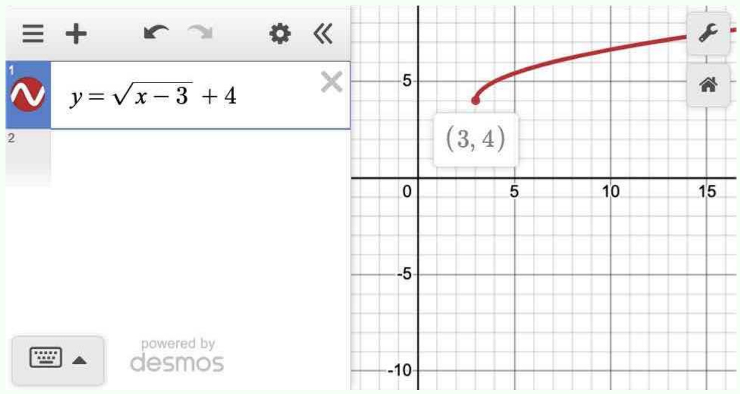
a) $f(x) = \sqrt{x-3} + 4$ b) $f(x) = x^2 + 8x - 7$

Solution.

a) Enter $y = \sqrt{x-3}+4$ into Desmos. Note that the square-root symbol can be entered by typing the letters $sqrt$, or alternatively, first show the keyboard by clicking  and then clicking the square-root symbol.

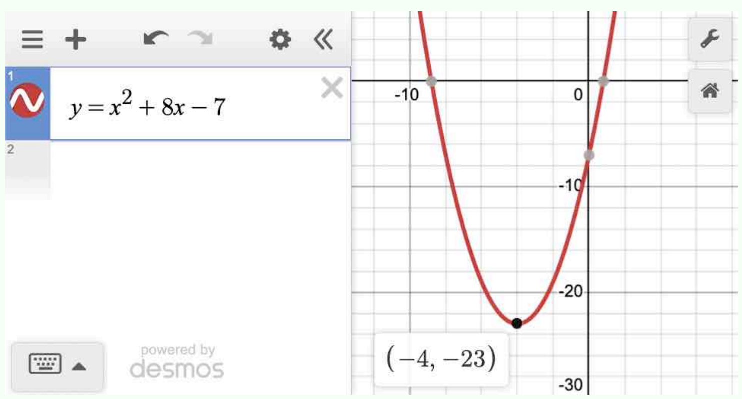


We see that the function starts from some vertex, and then increases as x increases. To find the vertex, click on it.



Thus, the domain and range appear to be $D=[3,\infty)$ and $R=[4,\infty)$.

b) Similarly, we can graph the function $y=x^2+8x-7$ and read off its domain and range. Note that there is no restriction on the inputs x, so that the domain is all real numbers.



From the graph, the domain and range are therefore $D=\mathbb{R}$ and $R=[-23,\infty)$.

$\square$

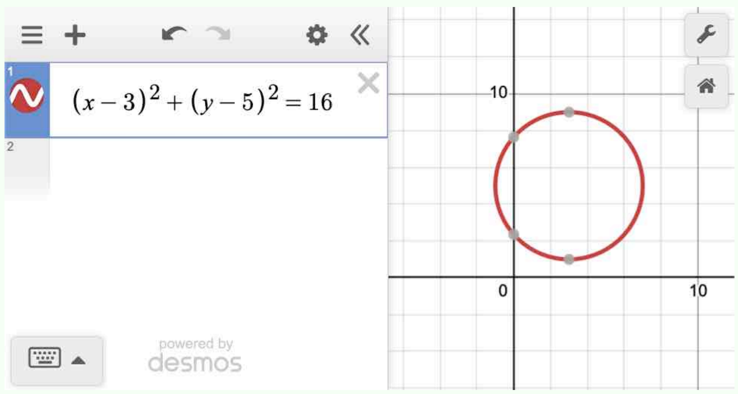
Example 4.10

Graph each of the following equations. Determine whether the graph is the graph of a function or not.

a) $(x-3)^2+(y-5)^2=16$ b) $3x^2+y^3+5xy=7$

Solution.

a) Graphing $(x-3)^2+(y-5)^2=16$ shows that we obtain a circle.

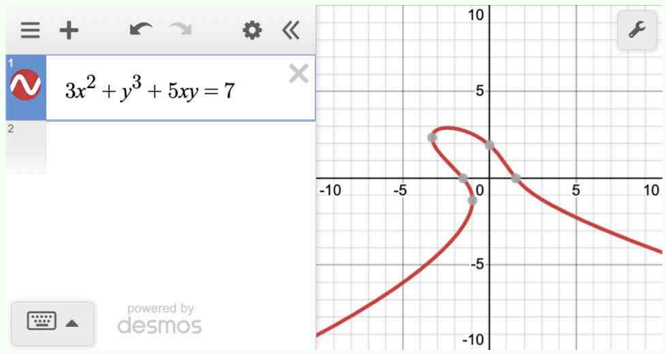


To see if this is the graph of a function, we can use the vertical line test (from Observation 3.15). The y-axis (which is the vertical line at $x=0$) intersects the circle at two points. This shows that the circle is not the graph of a function. Indeed, if we solve the equation for y, we get:

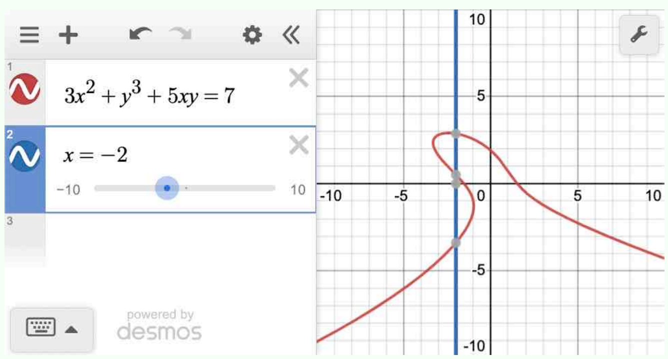
$$\begin{aligned}(x-3)^2+(y-5)^2=16 &\implies (y-5)^2=16-(x-3)^2\\ &\implies y-5=\pm\sqrt{16-(x-3)^2}\\ &\implies y=5\pm\sqrt{16-(x-3)^2}\end{aligned}$$

This shows that the circle is made up of two parts, the upper half-circle $y=5+\sqrt{16-(x-3)^2}$ and the lower half-circle $y=5-\sqrt{16-(x-3)^2}$, each of which *is* the graph of a function.

b) We can easily enter more complicated equations into Desmos.



The vertical line at $x = -2$ intersects the graph at three points, which shows that it is not the graph of a function.



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Note 4.11: Equation of the circle

We recall that the equation

$$(x-h)^2 + (y-k)^2 = r^2$$

always forms a circle in the plane. Indeed, this equation describes a circle with center $C(h,k)$ and radius r.
This can easily be explored in Desmos using sliders; see Exercise 4.6 below.

In the last example of this section we solve an equation by determining the intersection of two graphs.

Example 4.12

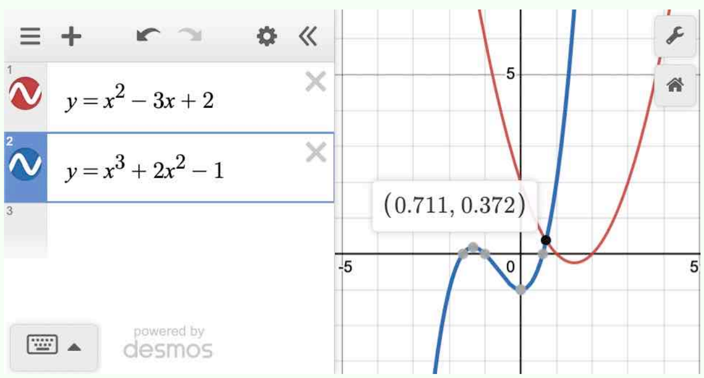
Solve the equation

$$x^2 - 3x + 2 = x^3 + 2x^2 - 1$$

Approximate your answer to the nearest thousandth.

Solution.

We can solve the equation by graphing the left-hand side $y = x^2 - 3x + 2$ and the right-hand side $y = x^3 + 2x^2 - 1$, and by determining those values of x where both sides are equal. This occurs precisely at the intersection of the two graphs. Graphing both functions and clicking on the intersection, we obtain:



The intersection is at $(x, y) \approx (0.711, 0.372)$. Therefore, the two sides of the equation are equal for $x \approx 0.711$ (in which case both the left-hand side and right-hand side are approximately 0.372). Therefore, $x \approx 0.711$ is the approximate solution. □

4.3 Graphs of basic functions and transformations

It will be useful to study the shape of graphs of some basic functions, which may then be taken as building blocks for more advanced and complicated functions.

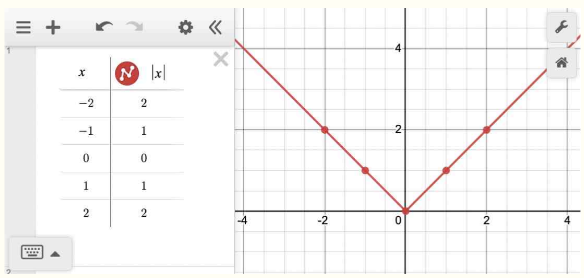
Observation 4.13: Basic function

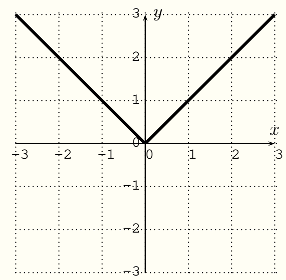
We start by examining the following functions, which we will sometimes refer to as *basic functions*:

$$y = |x|, \quad y = x^2, \quad y = x^3, \quad y = \sqrt{x}, \quad y = \frac{1}{x}$$

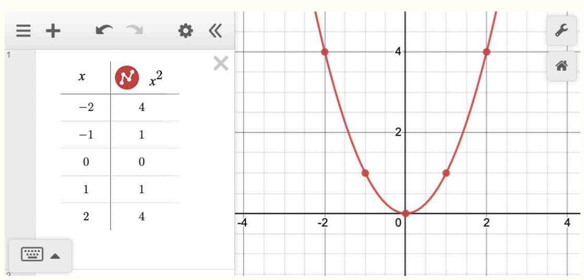
We can either graph these functions by hand by calculating a table, or by using the graphing calculator.

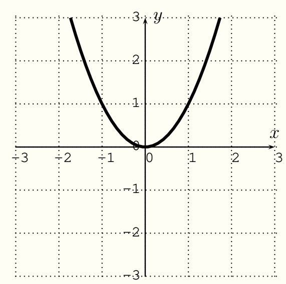
- We begin with the absolute value function $y = |x|$. The domain of $y = |x|$ is all real numbers, $D = \mathbb{R}$.



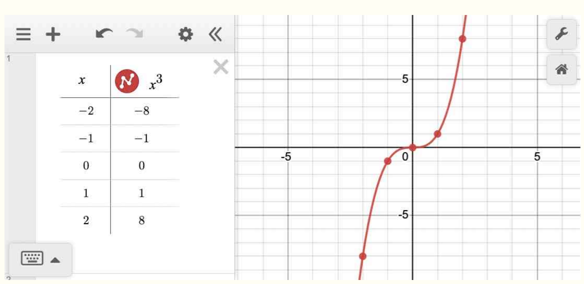


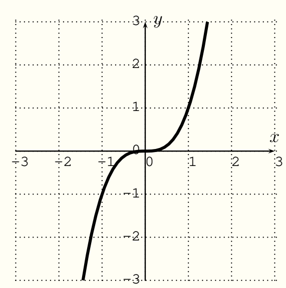
- Similarly, we obtain the graph for $y = x^2$, which is a parabola. The domain of the function $y = x^2$ is $D = \mathbb{R}$.



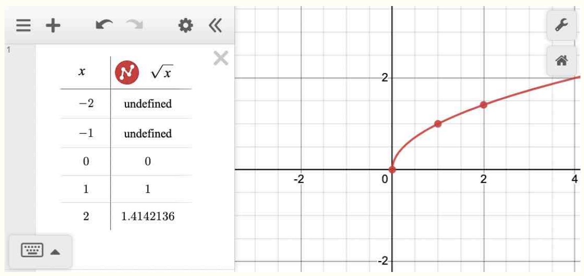


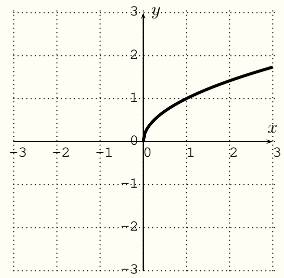
- Here is the graph for $y = x^3$. The domain is $D = \mathbb{R}$.



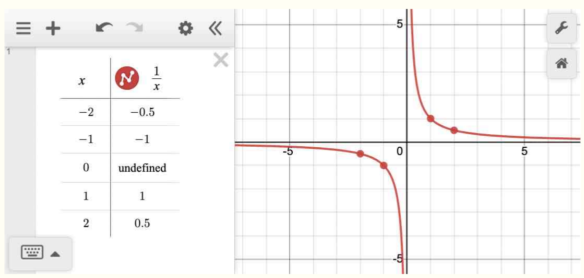


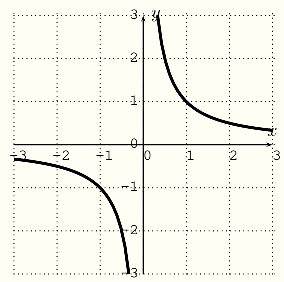
- Next we graph $y = \sqrt{x}$. The domain is $D = [0, \infty)$.



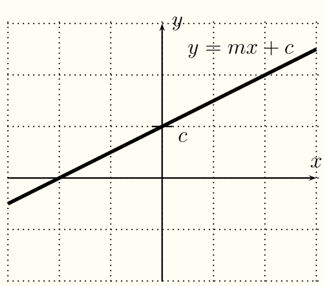


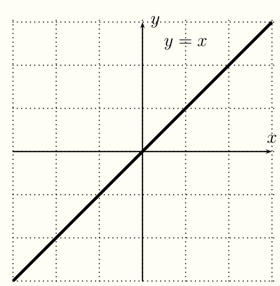
- Finally, here is the graph for $y = \frac{1}{x}$. The domain is $D = \mathbb{R} - \{0\}$.





These graphs together with the line $y = mx + b$ studied in Section 3.1 are our basic building blocks for more complicated graphs in the next sections. Note in particular, that the graph of $y = x$ is the diagonal line.

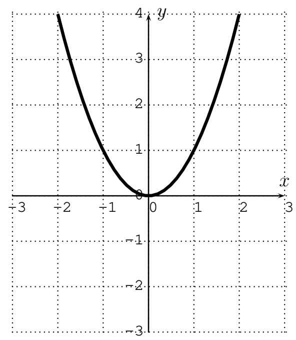
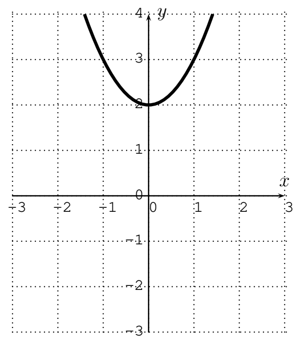
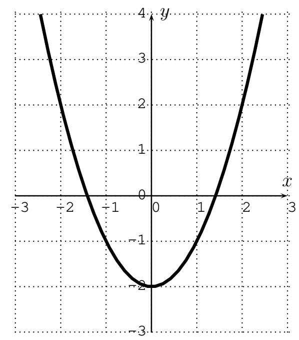




For a given function (such as one of the basic functions above), we now study how the graph of the function changes when performing elementary operations, such as adding, subtracting, or multiplying a constant number to the input or output. We will study the behavior in five specific transformations.

Adding or subtracting a constant to the output

Consider the following graphs:

$y = x^2$	$y = x^2 + 2$	$y = x^2 - 2$
		

We see that the function $y = x^2$ is shifted up by 2 units, respectively down by 2 units. In general, we have:

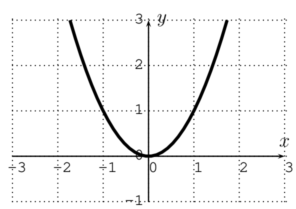
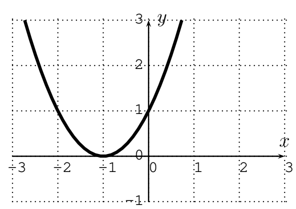
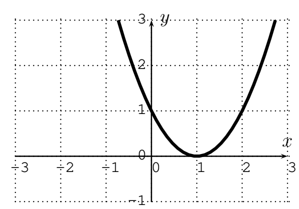
Observation 4.14: Adding or subtracting a constant to the output

Consider the graph of a function $y = f(x)$. Then, the graph of $y = f(x) + c$ is that of $y = f(x)$ shifted up or down by c.

- If c is positive, the graph is shifted up; if c is negative, the graph is shifted down.

Adding or subtracting a constant to the input

Next, we consider the transformation of $y = x^2$ given by adding or subtracting a constant to the input x.

$y = x^2$	$y = (x + 1)^2$	$y = (x - 1)^2$
		

Now we see that the function is shifted to the left or right. Note that $y = (x + 1)^2$ shifts the function to the left, which can be seen to be correct, since the input $x = -1$ gives the output $y = ((-1) + 1)^2 = 0^2 = 0$.

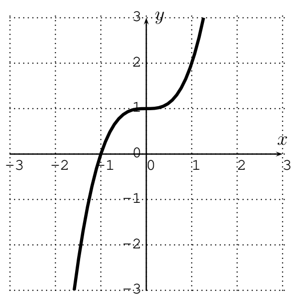
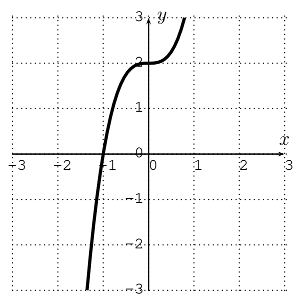
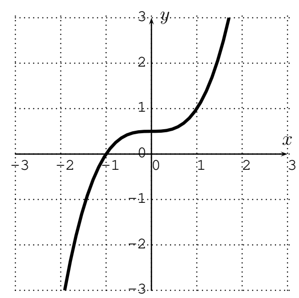
Observation 4.15: Adding or subtracting a constant to the input

Consider the graph of a function $y = f(x)$. Then, the graph of $y = f(x + c)$ is that of $y = f(x)$ shifted to the left or right by c.

- If c is positive, the graph is shifted to the left; if c is negative, the graph is shifted to the right.

Multiplying a positive constant to the output

Another transformation is given by multiplying the function by a fixed positive factor.

$y = x^3 + 1$	$y = 2 \cdot (x^3 + 1)$	$y = \frac{1}{2} \cdot (x^3 + 1)$
		

This time, the function is stretched away or compressed toward the x-axis.

Observation 4.16: Multiplying a positive constant to the output

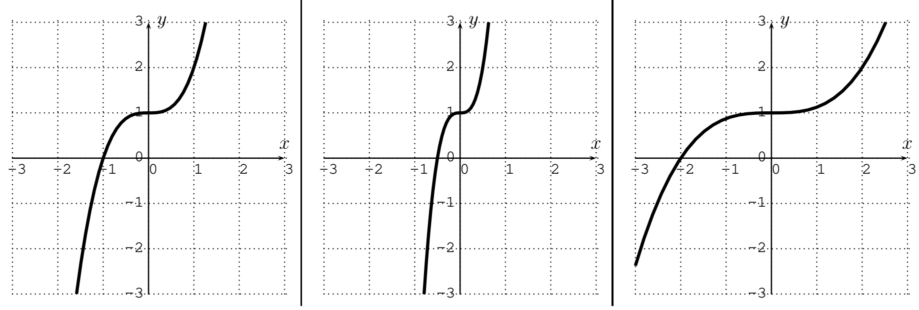
Consider the graph of a function $y = f(x)$ and let $c > 0$. Then, the graph of $y = c \cdot f(x)$ is that of $y = f(x)$ stretched away or compressed toward the x-axis by a factor c.

- If $c > 1$, the graph is stretched away from the x-axis; if $0 < c < 1$, the graph is compressed toward the x-axis.

Multiplying a positive constant to the input

Similarly, we can multiply the input by a positive factor.

$y = x^3 + 1$	$y = (2 \cdot x)^3 + 1$	$y = (\frac{1}{2} \cdot x)^3 + 1$



This time, the function is stretched away or compressed toward the y-axis.

Observation 4.17: Multiplying a positive constant to the input

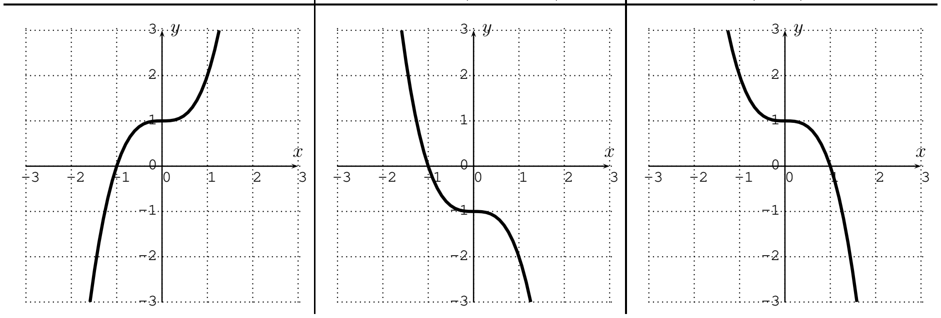
Consider the graph of a function $y = f(x)$ and let $c > 0$. Then, the graph of $y = f(c \cdot x)$ is that of $y = f(x)$ stretched away or compressed toward the y-axis by a factor c.

- If $c > 1$, the graph is compressed toward the y-axis; if $0 < c < 1$, the graph is stretched away from the y-axis.

Multiplying (-1) to the input or output

The last transformation is given by multiplying (-1) to the input or output, as displayed in the following chart.

$y = x^3 + 1$	$y = -(x^3 + 1)$	$y = (-x)^3 + 1$



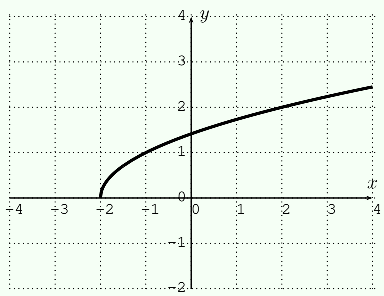
Here, the function is reflected either about the x-axis or about the y-axis.

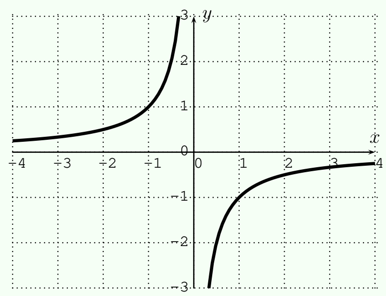
Observation 4.18: Multiplying (-1) to the input or output

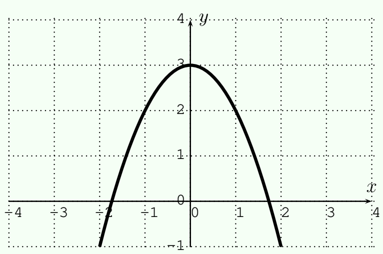
Consider the graph of a function $y = f(x)$. Then, the graph of $y = -f(x)$ is that of $y = f(x)$ reflected about the x-axis. Furthermore, the graph of $y = f(-x)$ is that of $y = f(x)$ reflected about the y-axis.

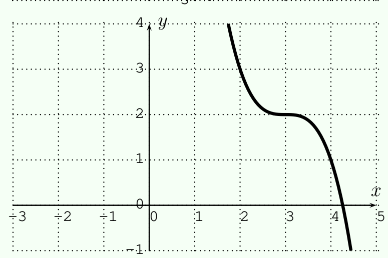
Example 4.19

Guess the formula for the function based on the basic graphs in Section 4.3 and the transformations described above.

a) 

b) 

c) 

d) 

Solution.

a) This is the square-root function shifted to the left by 2. Thus, by Observation 4.14, this is the function $f(x) = \sqrt{x+2}$.

b) This is the graph of $y = \frac{1}{x}$ reflected about the x-axis (or also $y = \frac{1}{x}$ reflected about the y-axis). In either case, we obtain the rule $y = -\frac{1}{x}$.

c) This is a parabola reflected about the x-axis and then shifted up by 3. Thus, we get:

	$y = x^2$
reflecting about the x-axis gives	$y = -x^2$
shifting the graph up by 3 gives	$y = -x^2 + 3$

d) Starting from the graph of the cubic equation $y = x^3$, we need to reflect about the x-axis (or also y-axis), then shift up by 2 and to the right by 3. These transformations affect the formula as follows:

	$y = x^3$
reflecting about the x-axis gives	$y = -x^3$
shifting up by 2 gives	$y = -x^3 + 2$
shifting the the right by 3 gives	$y = -(x-3)^3 + 2$

All of the above answers can be checked by graphing the function with the graphing calculator. □

Example 4.20

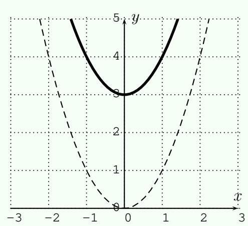
Sketch the graph of the function based on the basic graphs in Section 4.3 and the transformations described above.

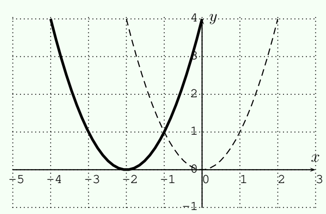
a) $y = x^2 + 3$ b) $y = (x+2)^2$ c) $y = |x-3| - 2$
d) $y = 2 \cdot \sqrt{x+1}$ e) $y = -\left(\frac{1}{x} + 2\right)$ f) $y = (-x+1)^3$

Solution.

a) This is the parabola $y = x^2$ shifted up by 3. The graph is shown below.

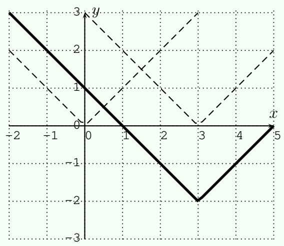
b) $y = (x+2)^2$ is the parabola $y = x^2$ shifted 2 units to the left.

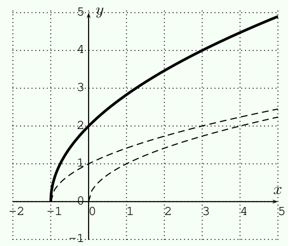
a) 

b) 

c) The graph of the function $f(x) = |x-3| - 2$ is the absolute value shifted to the right by 3 and down by 2. (Alternatively, we can first shift down by 2 and then to the right by 3.)

d) Similarly, to get from the graph of $y = \sqrt{x}$ to the graph of $y = \sqrt{x+1}$, we shift the graph to the left, and then for $y = 2 \cdot \sqrt{x+1}$, we need to stretch the graph by a factor 2 away from the x-axis. (Alternatively, we could first stretch the the graph away from the x-axis, then shift the graph by 1 to the left.)

c) 

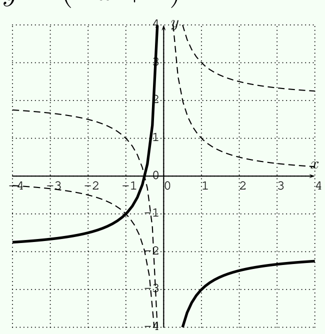
d) 

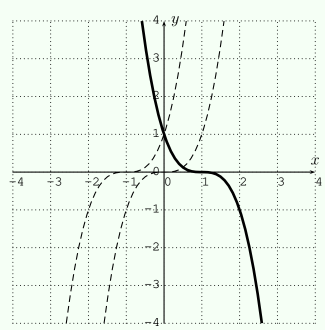
e) For $y = -(\frac{1}{x} + 2)$, we start with $y = \frac{1}{x}$ and add 2, giving $y = \frac{1}{x} + 2$, which shifts the graph up by 2. Then, taking the negative gives $y = -(\frac{1}{x} + 2)$, which corresponds to reflecting the graph about the x-axis.

Note that in this case, we cannot perform these transformations in the opposite order, since the negative of $y = \frac{1}{x}$ gives $y = -\frac{1}{x}$, and adding 2 gives $y = -\frac{1}{x} + 2$, which is *not* equal to $-(\frac{1}{x} + 2)$.

f) We start with $y = x^3$. Adding 1 to the argument, $y = (x+1)^3$, shifts its graph to the left by 1. Then, applying a minus to the argument gives $y = (-x+1)^3$, which reflects the graph about the y-axis.

Here, the order in which we perform these transformations is again important. In fact, if we first take the negative of the argument, we obtain $y = (-x)^3$. Then, adding one to the argument would give $y = (-(x+1))^3 = (-x-1)^3$, which is different than our given function $y = (-x+1)^3$.

e) 

f) 

All these solutions may also be checked easily by using the graphing calculator. □

Example 4.21

a) The graph of $f(x) = |x^3 - 5|$ is stretched away from the y-axis by a factor of 3. What is the formula for the new function?

b) The graph of $f(x) = \sqrt{6x^2 + 3}$ is shifted up 5 units, and then reflected about the x-axis. What is the formula for the new function?

c) How are the graphs of $y = 2x^3 + 5x - 9$ and $y = 2(x-2)^3 + 5(x-2) - 9$ related?

d) How are the graphs of $y = (x-2)^2$ and $y = (-x+3)^2$ related?

Solution.

a) Based on Observation 4.17 on page 68, we have to multiply the argument by $\frac{1}{3}$. The new function is therefore:

$$f\left(\frac{1}{3} \cdot x\right) = \left|\left(\frac{1}{3} \cdot x\right)^3 - 5\right| = \left|\frac{1}{27} \cdot x^3 - 5\right|$$

b) After the shift, we have the graph of a new function $y = \sqrt{6x^2 + 3} + 5$. Then, a reflection about the x-axis gives the graph of the function $y = -(\sqrt{6x^2 + 3} + 5)$.

c) Based on Observation 4.15 on page 67, we see that we need to shift the graph of $y = 2x^3 + 5x - 9$ by 2 units to the right.

d) The formulas can be transformed into each other as follows:

We begin with	$y = (x-2)^2$.
Replacing x by $x+5$ gives	$y = ((x+5) - 2)^2 = (x+3)^2$.
Replacing x by $-x$ gives	$y = ((-x) + 3)^2 = (-x+3)^2$.

Therefore, we have performed a shift to the left by 5, and then a reflection about the y-axis.

We want to point out that there is a second solution for this problem:

We begin with $y = (x-2)^2$.
Replacing x by $-x$ gives
$y = ((-x) - 2)^2 = (-x-2)^2$.
Replacing x by $x-5$ gives
$$y = (-(x-5) - 2)^2 = (-x+5-2)^2 = (-x+3)^2.$$

Therefore, we could also first perform a reflection about the y-axis, and then shift the graph to the right by 5.

□

Some of the above functions have special symmetries, which we investigate now.

Definition 4.22: Even function, odd function

A function f is called **even** if $f(-x) = f(x)$ for all x.
A function f is called **odd** if $f(-x) = -f(x)$ for all x.

Example 4.23

Determine if the following functions are even, odd, or neither:

$$\begin{array}{llll} f(x) = x^2, & g(x) = x^3, & h(x) = x^4, & k(x) = x^5, \\ l(x) = 4x^5 + 7x^3 - 2x, & & m(x) = x^2 + 5x & \end{array}$$

Solution.
The function $f(x) = x^2$ is even, since $f(-x) = (-x)^2 = x^2$. Similarly, $g(x) = x^3$ is odd, $h(x) = x^4$ is even, and $k(x) = x^5$ is odd, since

$$\begin{aligned} g(-x) &= (-x)^3 = -x^3 = -g(x) \\ h(-x) &= (-x)^4 = x^4 = h(x) \\ k(-x) &= (-x)^5 = -x^5 = -k(x) \end{aligned}$$

Indeed, we see that a function $y = x^n$ is even precisely when n is even, and $y = x^n$ is odd precisely when n is odd. (These examples are in fact the motivation behind defining even and odd functions as in Definition 4.22 above.)
Next, in order to determine if the function l is even or odd, we calculate $l(-x)$ and compare it with $l(x)$.

$$\begin{aligned} l(-x) &= 4(-x)^5 + 7(-x)^3 - 2(-x) = -4x^5 - 7x^3 + 2x \\ &= -(4x^5 + 7x^3 - 2x) = -l(x) \end{aligned}$$

Therefore, l is an odd function.

Finally, for $m(x) = x^2 + 5x$, we calculate $m(-x)$ as follows:

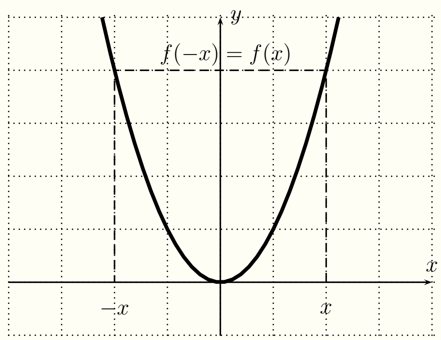
$$m(-x) = (-x)^2 + 5(-x) = x^2 - 5x$$

Note that m is not an even function, since $x^2 - 5x \neq x^2 + 5x$. Furthermore, m is also not an odd function, since $x^2 - 5x \neq -(x^2 + 5x)$. Therefore, m is a function that is *neither* even *nor* odd. □

Observation 4.24: Graph of even or odd function

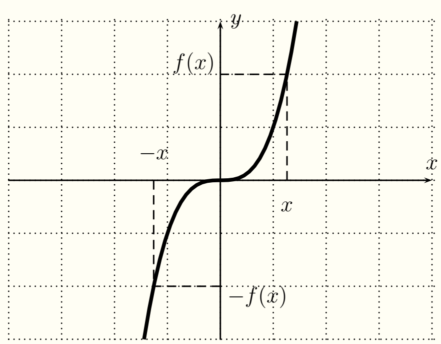
An even function f is symmetric with respect to the $y-$axis (if you reflect the graph of f about the $y-$axis, you get the same graph back), since even functions satisfy $f(-x) = f(x)$:

Example $y = x^2$:



An odd function f is symmetric with respect to the origin (if you reflect the graph of f about the $y-$axis and then about the $x-$axis, you get the same graph back), since odd functions satisfy $f(-x) = -f(x)$:

Example $y = x^3$:



4.4 Exercises

Exercise 4.1

Graph the function in Desmos.

a) $y = 3x - 5$ b) $y = x^2 - 3x - 2$ c) $y = x^4 - 3x^3 + 2x - 1$
d) $y = \sqrt{x^2 - 4}$ e) $y = \frac{4x+3}{2x+5}$ f) $y = |x + 3|$

Exercise 4.2

For each of the functions below, use Desmos to find all roots, all local maxima, all local minima, and the y-intercept.

a) $f(x) = x^3 + 4x^2 - 2x - 9$ b) $f(x) = x^3 - 6x^2 + 7x + 4$
c) $f(x) = -4x^3 + 3x^2 + 7x + 1$ d) $f(x) = 5x^3 + 2x^2$
e) $f(x) = x^4 - x^3 - 4x^2 + 1$ f) $f(x) = -x^4 + 5x^3 - 4x + 3$
g) $f(x) = x^5 + 2x^4 - x^3 - 3x^2 - x$ h) $f(x) = \sqrt{|2^x - 3|} - 2x + 3$

Exercise 4.3

Determine the domain and range using Desmos.

a) $y = |x - 2| + 5$ b) $y = -2x + 7$
c) $y = x^2 - 6x + 4$ d) $y = -x^2 - 8x + 3$
e) $y = 3 + \sqrt{x + 5}$ f) $y = 6 - x + \sqrt{4 - x}$
g) $y = x^4 - 8x^2 + 9$ h) $y = \frac{x-2}{x-3}$

Exercise 4.4

Use Desmos to determine whether the equation describes a function or not.

a) $x^2 + 2y - 3x = 7$ b) $x^2 + 2y^2 - 3x = 7$
c) $y^2 + 8y + 15 = x$ d) $y^3 + x^2 + y + x = 1$
e) $y = \frac{2x-5}{x-3}$ f) $x^2 + \left(y - \sqrt{|x|}\right)^2 = 1$

Exercise 4.5

Solve the equation for y and graph all branches in Desmos in the same window.

a) $x^2 + y^2 = 4$ b) $(x+5)^2 + y^2 = 15$
c) $(x-1)^2 + (y-2)^2 = 9$ d) $y^2 = x^2 + 3$

Exercise 4.6

Set up the general equation of a circle in Desmos, where the center and the radius can be changed using sliders. If a circle of radius 3 with center at the origin $(0,0)$ is shifted 4 units to the right and shifted 2 units down, then what is its equation?

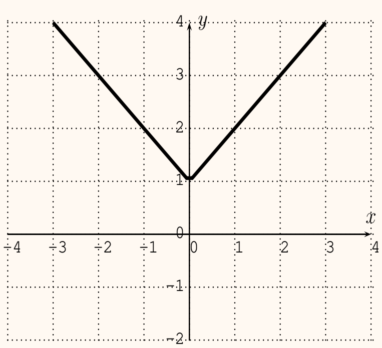
Exercise 4.7

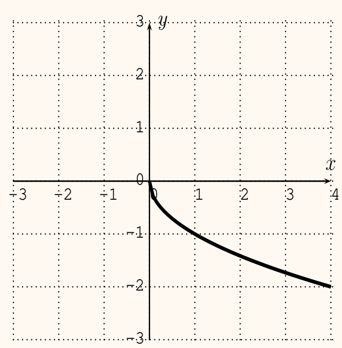
Use Desmos to find all solutions of the equation. Round your answer to the nearest thousandth.

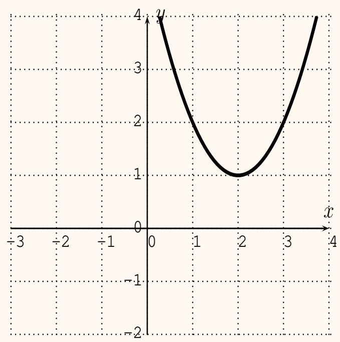
a) $x^3 + 3 = x^5 + 7$ b) $4x^3 + 6x^2 - 3x - 2 = 0$
c) $\frac{2x}{x-3} = \frac{x^2+2}{x+1}$ d) $5^{3x+1} = x^5 + 6$
e) $x^3 + x^2 = x^4 - x^2 + x$ f) $3x^2 = x^3 - x^2 + 3x$

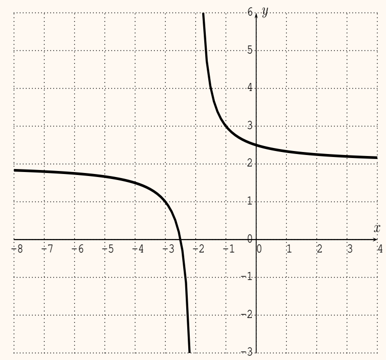
Exercise 4.8

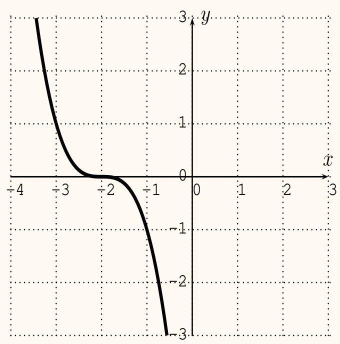
Find a possible formula of the graph displayed below.

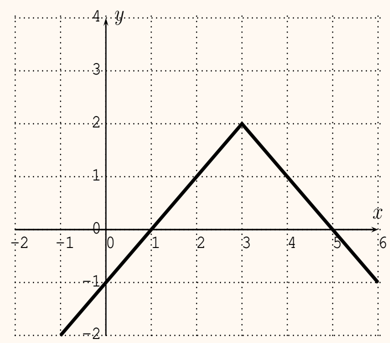
a) 

b) 

c) 

d) 

e) 

f) 

Exercise 4.9

Sketch the graph of the function based on the basic graphs and their transformation described in Section 4.3. Confirm your answer by graphing the function with the graphing calculator.

a) $f(x) = \lvert x \rvert - 3$	b) $f(x) = \frac{1}{x+2}$
c) $f(x) = -x^2$	d) $f(x) = (x-1)^3$
e) $f(x) = \sqrt{-x}$	f) $f(x) = 4 \cdot \lvert x - 3 \rvert$
g) $f(x) = -\sqrt{x} + 1$	h) $f(x) = (\frac{1}{2} \cdot x)^2 + 3$

Exercise 4.10

Consider the graph of $f(x) = x^2 - 7x + 1$. Find the formula of the function that is given by performing the following transformations on the graph.

a) Shift the graph of f down by 4.
b) Shift the graph of f to the left by 3 units.
c) Reflect the graph of f about the x-axis.
d) Reflect the graph of f about the y-axis.
e) Stretch the graph of f away from the y-axis by a factor 3.
f) Compress the graph of f toward the y-axis by a factor 2.

Exercise 4.11

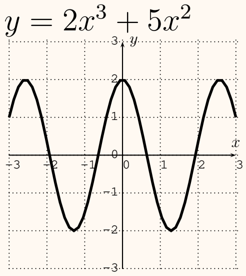
How are the graphs of f and g related?

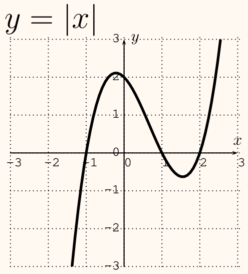
a)	$f(x) = \sqrt{x}$,	$g(x) = \sqrt{x-5}$
b)	$f(x) = \lvert x \rvert$,	$g(x) = 2 \cdot \lvert x \rvert$
c)	$f(x) = (x+1)^3$,	$g(x) = (x-3)^3$
d)	$f(x) = x^2 + 3x + 5$,	$g(x) = (2x)^2 + 3(2x)^2 + 5$
e)	$f(x) = \frac{1}{x+3}$,	$g(x) = -\frac{1}{x}$
f)	$f(x) = 2 \cdot \lvert x \rvert$,	$g(x) = \lvert x+1 \rvert + 1$

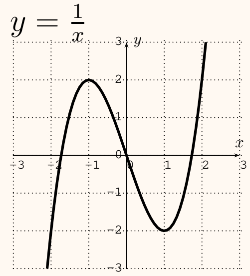
Exercise 4.12

Determine if the function is even, odd, or neither.

a) $y = 2x^3$ b) $y = 5x^2$ c) $y = 3x^4 - 4x^2 + 5$
d) $y = 2x^3 + 5x^2$ e) $y = \lvert x \rvert$ f) $y = \frac{1}{x}$

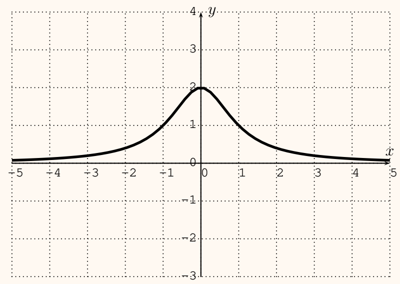
g) 

h) 

i) 

Exercise 4.13

The graph of the function $y = f(x)$ is displayed below.



Sketch the graph of the following functions.

a) $y = f(x) + 1$	b) $y = f(x-3)$	c) $y = -f(x)$
d) $y = 2f(x)$	e) $y = f(2x)$	f) $y = f(\frac{1}{2}x)$