Precalculus

Third Edition (3.0)

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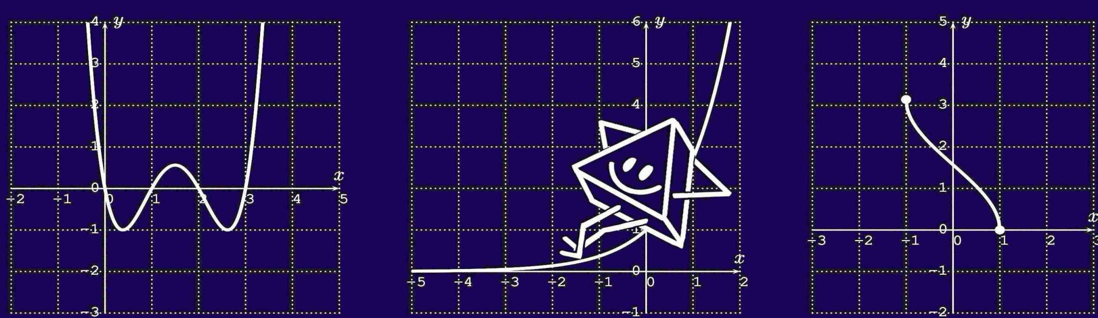

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Chapter 24

Sequences and series

In the next chapters, we will define and study sequences and series, which are concepts that are fundamental for calculus. Two specific types of sequences (arithmetic sequences and geometric sequences) will be studied in more detail.

24.1 Introduction to sequences and series

In this section, we define sequences and series.

Definition 24.1: Sequence

A **sequence** is an enumerated list of numbers. In other words, a sequence is a list of numbers

$$a_1, a_2, a_3, a_4, \ldots$$

where a_1 is the first number, a_2 is the second number, a_3 is the third number, etc.
We also denote the sequence by a or $\{a_n\}$ or $\{a_n\}_{n\geq 1}$.

Note that a sequence a is just an assignment, which assigns to each $n = 1, 2, 3, \ldots$ a number $a(n) = a_n$. In this sense, a sequence is just a function $a : \mathbb{N} \to R$ from the natural numbers $\mathbb{N}$ to a range R, which is a set of numbers such as, for example, the set of real or complex numbers.

Note 24.2

Here are some examples of sequences.

a) $4, 6, 8, 10, 12, 14, 16, 18, \ldots$

b) $1, 3, 9, 27, 81, 243, \ldots$

c) $+5, -5, +5, -5, +5, -5, \ldots$

d) $1, 1, 2, 3, 5, 8, 13, 21, 34, 55, \ldots$

e) $5, 8, -12, 4, 5.3, 7, -3, \sqrt{2}, 18, \frac{2}{3}, 9, \ldots$

For many of these sequences we can find explicit rules that describe how to obtain the individual terms.

a) Note that in the sequence in (a), we always add the fixed number 2 to the previous number to obtain the next, starting from the first term 4. Assuming that the following numbers in this sequence continue in this pattern, this would be an example of an *arithmetic sequence*, and we will study those in more detail in Section 24.2 below. We note, however, that knowing only a few numbers in a sequence is not enough to conclude that all of the following terms will, indeed, follow a similar pattern. To understand the complete sequence, we must specify all terms of the sequence (as is done, for example, in Examples 24.3 and 24.4 below).

b) In (b), we start with the first element 1 and multiply by the fixed number 3 to obtain the next term. Assuming this rule persists for the whole sequence, this would be an example of a *geometric sequence*, and we will study those in more detail in Chapter 25 below.

c) The sequence in (c) alternates between $+5$ and -5, starting from $+5$. Note that we can get from one term to the next by multiplying (-1) to the term. Assuming this as the rule for the whole sequence, this is another example of a geometric sequence.

d) In (d) we wrote the first few terms of a sequence called the **Fibonacci sequence**. In the Fibonacci sequence, each term is calculated by adding the previous two terms, starting with the first two terms 1 and 1:

$$1+1=2, \quad 1+2=3, \quad 2+3=5, \quad 3+5=8, \quad 5+8=13, \quad \ldots$$

e) Finally, the sequence in (e) does not seem to have any obvious rule by which the terms are generated.

To fully describe a sequence, we must specify every term of the sequence. This can be done, for example, by giving a formula for the nth term a_n of the sequence.

Note 24.3

Consider the sequence $\{a_n\}$ with $a_n = 4n+3$. We can calculate the individual terms of this sequence:

$$\begin{array}{lrl} \text{first term:} & a_1 = & 4\cdot 1+3=7, \\ \text{second term:} & a_2 = & 4\cdot 2+3=11, \\ \text{third term:} & a_3 = & 4\cdot 3+3=15, \\ \text{fourth term:} & a_4 = & 4\cdot 4+3=19, \\ \text{fifth term:} & a_5 = & 4\cdot 5+3=23 \\ & \vdots & \end{array}$$

Thus, the sequence is: $7, 11, 15, 19, 23, 27, 31, 35, \ldots$
Furthermore, from the formula, we can directly calculate any higher term, for example, the 200th term is given by:

$$\text{200th term:} \quad a_{200} = 4\cdot 200+3=803$$

Example 24.4

Find the first 6 terms of each sequence.

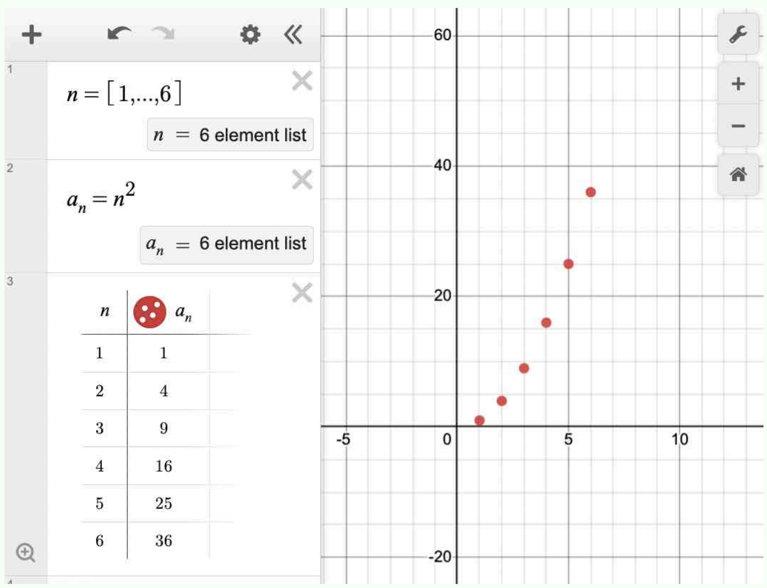
a) $a_n = n^2$ b) $a_n = \frac{n}{n+1}$ c) $a_n = (-1)^n$ d) $a_n = (-1)^{n+1}\cdot 2^n$

Solution.

a) We can easily calculate the first 6 terms of $a_n = n^2$ directly:

$$1, 4, 9, 16, 25, 36, \ldots$$

We can also use the calculator to produce the terms of a sequence. To this end, we first need to specify a finite list of indices n that we want to consider. To define the list $1, 2, 3, 4, 5, 6$ for our indices n, we write $n = [1, \ldots, 6]$. With this, we can generate the induced list for a_n by writing $a_n = n^2$. To see the values in this list, we generate a table using the + button (on the top left). We remind the reader that generating a table was described in Example 4.7 on page 57. We replace the input and output values (x_1 and y_1) of the table with n and a_n, respectively, which then shows the first six numbers of our sequence.



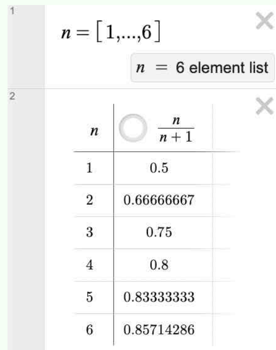
Note that we also get a graphical representation of the first six numbers in the sequence in the graph on the right.

b) We calculate the lowest terms of $a_n = \frac{n}{n+1}$:

$$a_1 = \frac{1}{1+1} = \frac{1}{2}, \quad a_2 = \frac{2}{2+1} = \frac{2}{3}, \quad a_3 = \frac{3}{3+1} = \frac{3}{4}, \quad \ldots$$

Identifying the pattern, we can simply write $a_1, \ldots, a_6$ as follows:

$$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}$$



n	$\frac{n}{n+1}$
1	0.5
2	0.66666667
3	0.75
4	0.8
5	0.83333333
6	0.85714286

Note that in the table on the right, we directly specified the output values $\frac{n}{n+1}$ without first defining the list a_n.

c) Since $(-1)^n$ is $+1$ for even n, but is -1 for odd n, the sequence $a_n = (-1)^n$ is:
$$-1, +1, -1, +1, -1, +1$$

d) Similar to part (c), $(-1)^{n+1}$ is -1 for even n, and is $+1$ for odd n. This, together with the calculation $2^1 = 2, 2^2 = 4, 2^3 = 8, 2^4 = 16$, etc., we get the first six terms of the sequence:

$$+2, -4, +8, -16, +32, -64$$

□

Another way to describe a sequence is by giving a *recursive* formula for the nth term a_n in terms of the lower terms. Here are some examples.

Example 24.5

Find the first 6 terms in the sequence described below.

a) $a_1 = 4$, and $a_n = a_{n-1} + 5$ for $n > 1$
b) $a_1 = 3$, and $a_n = 2 \cdot a_{n-1}$ for $n > 1$
c) $a_1 = 1$, $a_2 = 1$, and $a_n = a_{n-1} + a_{n-2}$ for $n > 2$

Solution.

a) The first term is explicitly given as $a_1 = 4$. Then we can calculate the following terms via $a_n = a_{n-1} + 5$:

$$\begin{aligned} a_2 &= a_1 + 5 = 4 + 5 = 9 \\ a_3 &= a_2 + 5 = 9 + 5 = 14 \\ a_4 &= a_3 + 5 = 14 + 5 = 19 \\ a_5 &= a_4 + 5 = 19 + 5 = 24 \\ &\vdots \end{aligned}$$

b) We start with $a_1 = 3$, and calculate $a_2 = 2 \cdot a_1 = 2 \cdot 3 = 6$, $a_3 = 2 \cdot a_2 = 2 \cdot 6 = 12$, $a_4 = 2 \cdot a_3 = 2 \cdot 12 = 24$, etc. We see that the effect of the recursive relation $a_n = 2 \cdot a_{n-1}$ is to double the previous number. The sequence is:

$$3, 6, 12, 24, 48, 96, 192, \ldots$$

c) Starting from $a_1 = 1$, and $a_2 = 1$, we can calculate the higher terms:

$$\begin{aligned} a_3 &= a_1 + a_2 = 1 + 1 = 2 \\ a_4 &= a_2 + a_3 = 1 + 2 = 3 \\ a_5 &= a_3 + a_4 = 2 + 3 = 5 \\ a_6 &= a_4 + a_5 = 3 + 5 = 8 \\ &\vdots \end{aligned}$$

Studying the sequence for a short while, we see that this is precisely the Fibonacci sequence from Example 24.2(d).

□

Note 24.6

There is no specific reason for using the indexing variable n in the sequence $\{a_n\}$. Indeed, we may as well use any other variable. For example, if the sequence $\{a_n\}_{n\geq 1}$ is given by the formula $a_n = 4n + 3$, then we can also write this as $a_k = 4k + 3$ or $a_i = 4i + 3$. In particular, the sequences $\{a_n\}_{n\geq 1} = \{a_k\}_{k\geq 1} = \{a_i\}_{i\geq 1} = \{a_j\}_{j\geq 1}$ are all identical as sequences.

We will be concerned with the task of adding terms in a sequence, such as $a_1 + a_2 + a_3 + \cdots + a_k$, for which we will use a standard summation notation.

Definition 24.7: Series

A **series** is a sum of terms in a sequence. We denote the sum of the first k terms in a sequence with the following notation:

$$\sum_{n=1}^{k} a_n = a_1 + a_2 + \cdots + a_k \tag{24.1}$$

The summation symbol "$\sum$" comes from the Greek letter Σ, pronounced "sigma," which is the Greek symbol for the /s/ sound.
More generally, we denote the sum from the jth to the kth term (where $j \leq k$) in a sequence with the following notion:

$$\sum_{n=j}^{k} a_n = a_j + a_{j+1} + \cdots + a_k$$

Furthermore, for typesetting reasons, $\sum\limits_{n=j}^{k} a_n$ is sometimes also written as $\sum_{n=j}^{k} a_n$, where indices are placed next to the summation symbol "$\sum$" instead of above and below.

Example 24.8

Find the sum.

a) $\displaystyle\sum_{n=1}^{4} a_n,$ for $a_n = 7n + 3$

b) $\displaystyle\sum_{j=1}^{6} a_j,$ for $a_n = (-2)^n$

c) $\displaystyle\sum_{k=1}^{5} \left(4 + k^2\right)$

Solution.

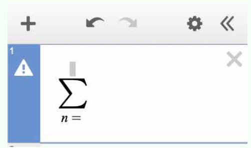
a) The first four terms a_1, a_2, a_3, a_4 of the sequence $\{a_n\}_{n\geq 1}$ are:

$$10, 17, 24, 31$$

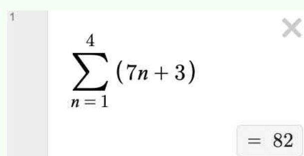
The sum is therefore:

$$\sum_{n=1}^{4} a_n = a_1 + a_2 + a_3 + a_4 = 10 + 17 + 24 + 31 = 82$$

We may also find the answer with the calculator. Typing the letters "sum" in the calculator will create the summation symbol:



Entering the specified values for the sequence, as well as where to begin and the end the sum, displays the answer.



Note that we need to place parentheses around the terms to be summed $7n + 3$. (Confirm this by observing that you get a different result without the parentheses!)

b) The first six terms of $\{a_n\}$ with $a_n = (-2)^n$ are:

$$\begin{array}{lll} (-2)^1 = -2, & (-2)^2 = 4, & (-2)^3 = -8, \\ (-2)^4 = 16, & (-2)^5 = -32, & (-2)^6 = 64 \end{array}$$

We calculate $\sum_{j=1}^{6} a_j$ by adding these first six terms. (Note that the sum is independent of the index j appearing in the sum $\sum_{j=1}^{6} a_j$, which we could also replace by any other index $\sum_{j=1}^{6} a_j = \sum_{n=1}^{6} a_n$, etc.) We get:

$$\begin{aligned} \sum_{j=1}^{6} a_j &= a_1 + a_2 + a_3 + a_4 + a_5 + a_6 \\ &= (-2) + 4 + (-8) + 16 + (-32) + 64 = 42 \end{aligned}$$

c) For the sum $\sum_{k=1}^{5}(4+k^2)$ we need to add the first five terms of the sequence $a_k = 4+k^2$. Calculating and adding the terms of this sequence, we obtain the sum:

$$\begin{aligned}\sum_{k=1}^{5}\left(4+k^2\right) &= (4+1^2)+(4+2^2)+(4+3^2)+(4+4^2)+(4+5^2)\\ &= (4+1)+(4+4)+(4+9)+(4+16)+(4+25)\\ &= 5+8+13+20+29\\ &= 75\end{aligned}$$

This answer can, of course, also be confirmed with the calculator.

□

24.2 The arithmetic sequence

We have already encountered examples of arithmetic sequences in the previous section. An arithmetic sequence is a sequence for which we add a constant number to get from one term to the next, for example:

$$8, \underbrace{}_{+3} 11, \underbrace{}_{+3} 14, \underbrace{}_{+3} 17, \underbrace{}_{+3} 20, \underbrace{}_{+3} 23, \ldots$$

Definition 24.9: Arithmetic sequence

A sequence $\{a_n\}$ is called an **arithmetic sequence** if any two consecutive terms have a *common difference* d. The arithmetic sequence is determined by d and the first value a_1. This can be written recursively as:

$$a_n = a_{n-1} + d \qquad \text{for } n \geq 2$$

Alternatively, we have the general formula for the nth term of the arithmetic sequence:

$$\boxed{a_n = a_1 + (n-1)\cdot d} \tag{24.2}$$

Example 24.10

Determine whether the terms below could be the terms of an arithmetic sequence. If so, then find the general formula for a_n in the form of Equation (24.2).

a)	$7, 13, 19, 25, 31, \ldots$	b)	$18, 13, 8, 3, -2, -7, \ldots$
c)	$10, 13, 16, 20, 23, \ldots$	d)	$a_n = 8 \cdot n + 3$

Solution.

a) Calculating the difference between two consecutive terms always gives the same answer: $13 - 7 = 6$, $19 - 13 = 6$, $25 - 19 = 6$, etc. Therefore, the common difference $d = 6$, which shows that these are the terms in an arithmetic sequence. Furthermore, the first term is $a_1 = 7$, so that the general formula for the nth term would be $a_n = 7 + (n - 1) \cdot 6$.

b) One checks that the common difference is $13-18 = -5$, $8-12 = -5$, etc., so that this could be an arithmetic sequence with $d = -5$. Since $a_1 = 18$, the general term would be $a_n = 18 + (n - 1) \cdot (-5)$ or $a_n = 18 - (n - 1) \cdot 5$.

c) We have $13 - 10 = 3$, but $20 - 16 = 4$, so that this *cannot* be an arithmetic sequence.

d) If we write out the first couple of terms of $a_n = 8n + 3$, we get the sequence:

$$11, 19, 27, 35, 43, 51, \ldots$$

From this, it seems reasonable to suspect that the above is an arithmetic sequence with common difference $d = 8$ and first term $a_1 = 11$. The general term of such an arithmetic sequence is

$$a_1 + (n - 1) \cdot d = 11 + (n - 1) \cdot 8 = 11 + 8n - 8 = 8n + 3 = a_n.$$

This shows that $a_n = 8n + 3 = 11 + (n - 1) \cdot 8$ is an arithmetic sequence.

□

Example 24.11

Find the formula of an arithmetic sequence $a_n = a_1 + (n-1) \cdot d$ with the properties given below.

a) $a_1 = -5$, and $a_9 = 27$
b) $d = 12$, and $a_6 = 68$
c) $a_5 = 38$, and $a_{16} = 115$

Solution.

a) We are given $a_1 = -5$, but we still need to find the common difference d. Plugging $a_9 = 27$ into $a_n = a_1 + (n-1) \cdot d$, we obtain

$$27 = a_9 = -5 + (9-1) \cdot d = -5 + 8d \overset{(+5)}{\Longrightarrow} 32 = 8d \overset{(\div 8)}{\Longrightarrow} 4 = d$$

The nth term is therefore, $a_n = -5 + (n-1) \cdot 4$.

b) We know that $d = 12$, so we only need to find a_1. Plugging $a_6 = 68$ into $a_n = a_1 + (n-1) \cdot d$, we can solve for a_1:

$$68 = a_6 = a_1 + (6-1) \cdot 12 = a_1 + 5 \cdot 12 = a_1 + 60 \overset{(-60)}{\Longrightarrow} a_1 = 68 - 60 = 8$$

The nth term is therefore, $a_n = 8 + (n-1) \cdot 12$.

c) In this case, neither a_1 nor d are given. However, the two conditions $a_5 = 38$ and $a_{16} = 115$ give two equations in the two unknowns a_1 and d.

$$\begin{cases} 38 = a_5 = a_1 + (5-1) \cdot d \\ 115 = a_{16} = a_1 + (16-1) \cdot d \end{cases} \Longrightarrow \begin{cases} 38 = a_1 + 4 \cdot d \\ 115 = a_1 + 15 \cdot d \end{cases}$$

To solve this system of equations, we need to recall the methods for doing so. One convenient method is the addition/subtraction method. For this, we subtract the top equation from the bottom equation:

$$\begin{array}{rrcll} & 115 & = & a_1 & +15 \cdot d \\ -(& 38 & = & a_1 & +4 \cdot d \quad) \\ \hline & 77 & = & & +11 \cdot d \end{array} \quad \overset{(\div 11)}{\Longrightarrow} \quad 7 = d$$

Plugging $d = 7$ into either of the two equations gives a_1. We plug it into the first equation $38 = a_1 + 4d$:

$$38 = a_1 + 4 \cdot 7 \quad \Longrightarrow \quad 38 = a_1 + 28 \quad \overset{(-28)}{\Longrightarrow} \quad 10 = a_1$$

Therefore, the nth term is given by $a_n = 10 + (n-1) \cdot 7$.

$\square$

We want to find a sum of terms in an arithmetic sequence. Since the arithmetic sequence is given by an easy and straightforward rule, it turns out that one can find a nice formula for the sum of the first k terms in the sequence, as well.

Note 24.12: Summing integers from 1 to 100

Find the sum of the first 100 integers, starting from 1. In other words, we want to find the sum of $1 + 2 + 3 + \cdots + 99 + 100$. Note that the sequence $1, 2, 3, \ldots$ is an arithmetic sequence. Instead of adding all these 100 numbers, we use a trick that will turn out to work for any arithmetic sequence:
In order to compute $S = 1 + 2 + 3 + \cdots + 98 + 99 + 100$, we write the sum for these numbers twice, once in ascending order, and once in descending order:

$$\begin{array}{rccccccccccccc} 2 \cdot S = & & 1 & + & 2 & + & 3 & \cdots & + & 98 & + & 99 & + & 100 \\ & + & 100 & + & 99 & + & 98 & \cdots & + & 3 & + & 2 & + & 1 \end{array}$$

Note that the second line is just S, but we are adding the terms in reverse order. Adding the terms vertically, that is, adding the two terms in each column, each sum is precisely 101:

$$2 \cdot S = 101 + 101 + 101 + \cdots + 101 + 101 + 101$$

Note that there are 100 terms of 101 on the right-hand side. So,

$$2S = 100 \cdot 101 \text{ and therefore } S = \frac{100 \cdot 101}{2} = 5050.$$

According to lore, this formula was discovered by the German mathematician Carl Friedrich Gauss as a child in primary school.

An appropriate generalization of the previous note yields a computation that applies to any arithmetic sequence.

Observation 24.13: Arithmetic series

Let $\{a_n\}$ be an arithmetic sequence whose nth term is given by the formula $a_n = a_1 + (n-1) \cdot d$. Then the sum $a_1 + a_2 + \cdots + a_{k-1} + a_k$ is given by adding $(a_1 + a_k)$ precisely $\frac{k}{2}$ times:

$$\boxed{\sum_{n=1}^{k} a_n = \frac{k}{2} \cdot (a_1 + a_k)} \qquad (24.3)$$

Proof. For the proof of Equation (24.3), we write $S = a_1 + a_2 + \cdots + a_{k-1} + a_k$. We then add it to itself, but in reverse order:

$$\begin{array}{rcccccccccccc} 2 \cdot S = & & a_1 & + & a_2 & + & a_3 & \cdots & + & a_{k-2} & + & a_{k-1} & + & a_k \\ & + & a_k & + & a_{k-1} & + & a_{k-2} & \cdots & + & a_3 & + & a_2 & + & a_1 \end{array}.$$

Now note that in general, $a_\ell + a_m = a_1 + (\ell - 1) \cdot d + a_1 + (m-1) \cdot d = 2a_1 + (\ell + m - 2) \cdot d$. Since the vertical terms are always terms a_ℓ and a_m with $\ell + m = k + 1$, these add to $a_\ell + a_m = 2a_1 + (k-1) \cdot d$. We see that adding vertically gives

$$\begin{aligned} 2 \cdot S &= (2a_1 + (k-1) \cdot d) + \cdots + (2a_1 + (k-1) \cdot d) \\ &= k \cdot (2a_1 + (k-1) \cdot d) = k \cdot (a_1 + (a_1 + (k-1) \cdot d)) = k \cdot (a_1 + a_k). \end{aligned}$$

Dividing by 2 gives the desired result. □

Here are some examples in which we apply formula (24.3).

Example 24.14

Find the value of the arithmetic series.

a) Find the sum $a_1 + \cdots + a_{60}$ for the arithmetic sequence

$$a_n = 2 + (n-1) \cdot 13.$$

b) Determine the value of the sum:

$$\sum_{n=1}^{1001} (5 - 6n)$$

c) Find the sum of the first 333 terms of the sequence

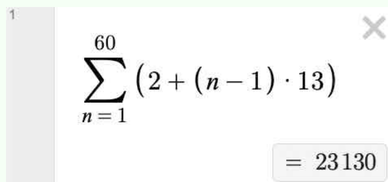
$$15, 11, 7, 3, -1, -5, -9, \ldots$$

Solution.

a) The sum is given by the formula (24.3): $\sum_{n=1}^{k} a_n = \frac{k}{2} \cdot (a_1 + a_k)$. Here, $k = 60$, and $a_1 = 2$ and $a_{60} = 2 + 13 \cdot (60 - 1) = 2 + 13 \cdot 59 = 2 + 767 = 769$. We obtain a sum of

$$a_1 + \cdots + a_{60} = \sum_{n=1}^{60} a_n = \frac{60}{2} \cdot (2 + 769) = 30 \cdot 771 = 23,130.$$

We may confirm this with the calculator as described in Example 24.8 (on page 419) of the previous section.



b) Again, we use the above formula $\sum_{n=1}^{k} a_n = \frac{k}{2} \cdot (a_1 + a_k)$, in which the arithmetic sequence is given by $a_n = 5 - 6n$ and $k = 1001$. Using the values $a_1 = 5 - 6 \cdot 1 = 5 - 6 = -1$ and $a_{1001} = 5 - 6 \cdot 1001 = 5 - 6006 = -6001$, we obtain:

$$\begin{aligned} \sum_{n=1}^{1001} (5 - 6n) &= \frac{1001}{2}(a_1 + a_{1001}) = \frac{1001}{2}((-1) + (-6001)) \\ &= \frac{1001}{2} \cdot (-6002) = 1001 \cdot (-3001) = -3,004,001 \end{aligned}$$

c) First note that the given numbers $15, 11, 7, 3, -1, -5, -9, \dots$ are the beginning of an arithmetic sequence. The sequence is determined by the first term $a_1 = 15$ and common difference $d = 11 - 15 = -4$. The nth term is given by $a_n = 15 - (n-1) \cdot 4$, and summing the first $k = 333$ terms gives:

$$\sum_{n=1}^{333} a_n = \frac{333}{2} \cdot (a_1 + a_{333})$$

We still need to find a_{333} in the above formula:

$$a_{333} = 15 - (333 - 1) \cdot 4 = 15 - 332 \cdot 4 = 15 - 1328 = -1313$$

This gives a total sum of

$$\sum_{n=1}^{333} a_n = \frac{333}{2} \cdot (15 + (-1313)) = \frac{35}{2} \cdot (-1298) = -216,117.$$

□

24.3 Exercises

Exercise 24.1

Find the first seven terms of the sequence.

a) $a_n = 3n$	b) $a_n = 5n + 3$	c) $a_n = n^2 + 2$
d) $a_n = n$	e) $a_n = (-1)^{n+1}$	f) $a_n = \frac{\sqrt{n+1}}{n}$
g) $a_k = 10^k$	h) $a_i = 5 + (-1)^i$	i) $a_n = \sin(\frac{\pi}{2} \cdot n)$

Exercise 24.2

Find the first six terms of the sequence.

a) $a_1 = 5$, $a_n = a_{n-1} + 3$ for $n \geq 2$
b) $a_1 = 7$, $a_n = 10 \cdot a_{n-1}$ for $n \geq 2$
c) $a_1 = 1$, $a_n = 2 \cdot a_{n-1} + 1$ for $n \geq 2$
d) $a_1 = 6$, $a_2 = 4$, $a_n = a_{n-1} - a_{n-2}$ for $n \geq 3$

Exercise 24.3

Find the value of the series.

a) $\sum_{n=1}^{4} a_n$, where $a_n = 5n$	b) $\sum_{k=1}^{5} a_k$, where $a_k = k$
c) $\sum_{i=1}^{4} a_i$, where $a_n = n^2$	d) $\sum_{n=1}^{6} (n-4)$
e) $\sum_{k=1}^{3} (k^2 + 4k - 4)$	f) $\sum_{j=1}^{4} \frac{1}{j+1}$

Exercise 24.4

Is the sequence below part of an arithmetic sequence? If it is part of an arithmetic sequence, find the formula for the nth term a_n in the form $a_n = a_1 + (n-1) \cdot d$.

a) $5, 8, 11, 14, 17, \dots$

b) $-10, -7, -4, -1, 2, \dots$

c) $-1, 1, -1, 1, -1, 1, \dots$

d) $18, 164, 310, 474, \dots$

e) $73.4, 51.7, 30, \dots$

f) $9, 3, -3, -8, -14, \dots$

g) $4, 4, 4, 4, 4, \dots$

h) $-2.72, -2.82, -2.92, -3.02, -3.12, \dots$

i) $\sqrt{2}, \sqrt{5}, \sqrt{8}, \sqrt{11}, \dots$

j) $\frac{-3}{5}, \frac{-1}{10}, \frac{2}{5}, \dots$

k) $a_n = 4 + 5 \cdot n$

l) $a_j = 2 \cdot j - 5$

m) $a_n = n^2 + 8n + 15$

n) $a_k = 9 \cdot (k+5) + 7k - 1$

Exercise 24.5

Determine the general nth term a_n of an arithmetic sequence $\{a_n\}$ with the data given below.

a) $d = 4$, and $a_8 = 57$

b) $d = -3$, and $a_{99} = -70$

c) $a_1 = 14$, and $a_7 = -16$

d) $a_1 = -80$, and $a_5 = 224$

e) $a_3 = 10$, and $a_{14} = -23$

f) $a_{20} = 2$, and $a_{60} = 32$

Exercise 24.6

Determine the value of the indicated term of the given arithmetic sequence.

a) if $a_1 = 8$, and $a_{15} = 92$, find a_{19}

b) if $d = -2$, and $a_3 = 31$, find a_{81}

c) if $a_1 = 0$, and $a_{17} = -102$, find a_{73}

d) if $a_7 = 128$, and $a_{37} = 38$, find a_{26}

Exercise 24.7

Determine the sum of the arithmetic sequence.

a) Find the sum $a_1 + \dots + a_{48}$ for the arithmetic sequence $a_n = 4n + 7$.

b) Find the sum $\sum_{n=1}^{21} a_n$ for the arithmetic sequence $a_n = 2 - 5n$.

c) Find the sum: $\sum_{n=1}^{99}(10 \cdot n + 1)$

d) Find the sum: $\sum_{n=1}^{200}(-9 - n)$

e) Find the sum of the first 100 terms of the arithmetic sequence:

$$2, 4, 6, 8, 10, 12, \dots$$

f) Find the sum of the first 83 terms of the arithmetic sequence:

$$25, 21, 17, 13, 9, 5, \dots$$

g) Find the sum of the first 75 terms of the arithmetic sequence:

$$2012, 2002, 1992, 1982, \dots$$

h) Find the sum of the first 16 terms of the arithmetic sequence:

$$-11, -6, -1, \dots$$

i) Find the sum of the first 99 terms of the arithmetic sequence:

$$-8, -8.2, -8.4, -8.6, -8.8, -9, -9.2, \dots$$

j) Find the sum

$$7 + 8 + 9 + 10 + \cdots + 776 + 777$$

k) Find the sum of the first 40 terms of the arithmetic sequence:

$$5, 5, 5, 5, 5, \dots$$