# Precalculus

Third Edition (3.0)

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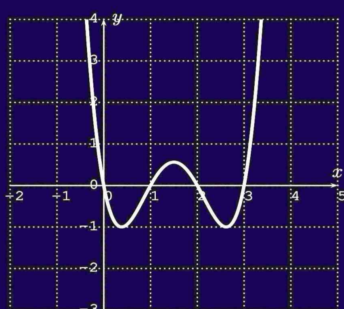

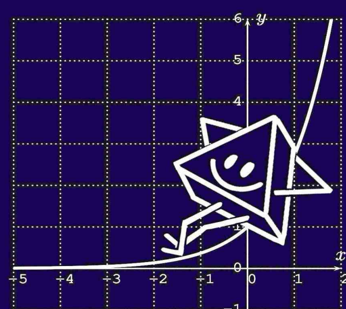

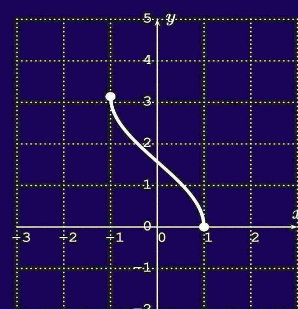



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# Chapter 23

# Complex numbers

We have already encountered complex numbers in the previous chapters. We now go a bit further by representing them in the plane and use the trigonometric functions to rewrite complex numbers in polar form. We will see that this can simplify the multiplication and division of complex numbers.

## 23.1 Polar form of complex numbers

We now recall the definition of complex numbers and show how to represent them in the complex plane.

**Definition 23.1: Imaginary unit**

We define the **imaginary unit** or **complex unit** to be

$$\text{imaginary unit:} \qquad \boxed{i = \sqrt{-1}} \tag{23.1}$$

In other words, $i$ is a solution of the equation:

$$\boxed{i^2 = -1} \tag{23.2}$$

Using the imaginary unit, a complex number is defined as a number with a real part and an imaginary part.

**Definition 23.2: Complex number**

A **complex number** is a number of the form

$$a + bi$$

where $a$ and $b$ are any real numbers, and $i$ is the complex unit. The number $a$ is called the **real part** of $a+bi$, and $bi$ is called the **imaginary part** of $a + bi$.
The set of all complex numbers is denoted by $\mathbb{C}$.

**Example 23.3**

Here are some examples of complex numbers:

$$3 + 2i, \quad 1 - 1 \cdot i, \quad \sqrt{2} + \pi \cdot i, \quad 5 + 0 \cdot i, \quad 0 + 3 \cdot i$$

Note that we can write $5 + 0 \cdot i = 5$, which has an imaginary part of zero, and so we see that the real number $5$ is also a complex number. Indeed, any real number $a = a + 0 \cdot i$ is also a complex number.
Note also that $0 + 3 \cdot i = 3i$ is a complex number, and similarly any multiple of $i$ is a complex number; these numbers are called *purely imaginary*.

We briefly recall the usual operations on complex numbers.

**Example 23.4**

Perform the operation.

a) $(2 - 3i) + (-6 + 4i)$ b) $(3 + 5i) \cdot (-7 + i)$ c) $\dfrac{5 + 4i}{3 + 2i}$

## Solution.

a) Adding real and imaginary parts, respectively, gives,

$$(2 - 3i) + (-6 + 4i) = 2 - 3i - 6 + 4i = -4 + i.$$

b) We multiply (using FOIL), and use that $i^2 = -1$.

$$\begin{aligned}(3 + 5i) \cdot (-7 + i) &= -21 + 3i - 35i + 5i^2 = -21 - 32i + 5 \cdot (-1) \\ &= -21 - 32i - 5 = -26 - 32i\end{aligned}$$

c) Recall that we may simplify a quotient of complex numbers by multiplying the complex conjugate of the denominator to both numerator and denominator.

$$\begin{aligned}\frac{5+4i}{3+2i} &= \frac{(5+4i)\cdot(3-2i)}{(3+2i)\cdot(3-2i)} = \frac{15-10i+12i-8i^2}{9-6i+6i-4i^2} \\ &= \frac{15+2i+8}{9+4} = \frac{23+2i}{13} = \frac{23}{13}+\frac{2}{13}i\end{aligned}$$

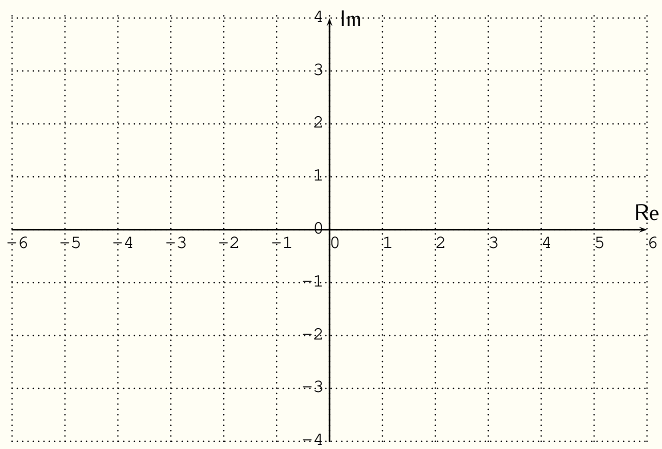
The real part of the solution is $\frac{23}{13}$; the imaginary part is $\frac{2}{13}i$.

□

Complex numbers can be pictured as points in the plane.

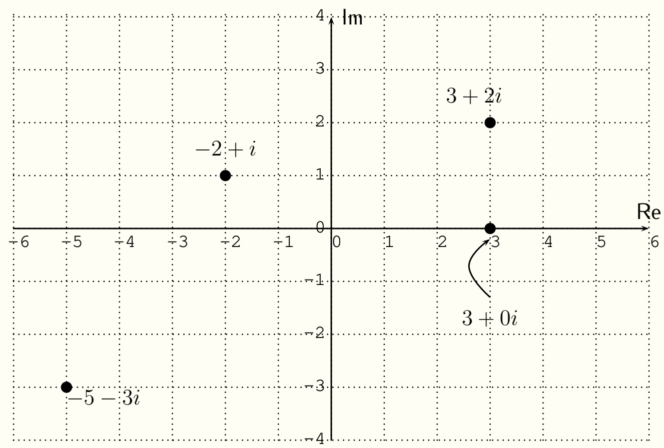
**Observation 23.5: Complex plane**

In analogy to Section 1.1, where we represented the real numbers on the number line, we can represent complex numbers in the *complex plane*:




The complex number $a+bi$ is represented as the point with coordinates

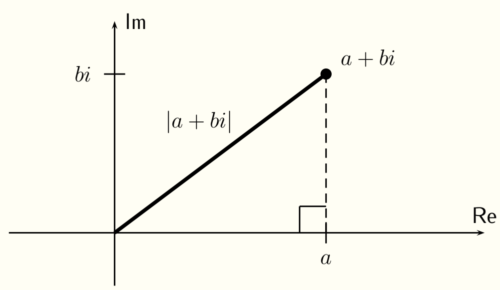
$(a, b)$ in the complex plane.




Just as with the magnitude and the direction angle of vectors, we can use the planar representation of a complex number to study its distance from the origin, as well as its direction angle. We start with the distance of a complex number $a + bi$ to the origin $0$, which is called the ***absolute value*** $|a + bi|$ of $a + bi$.

### Observation 23.6: Absolute value or modulus

Let $a + bi$ be a complex number. The **absolute value** or **modulus** of $a+bi$, denoted by $|a+bi|$, is the length between the point $a+bi$ in the complex plane and the origin $(0, 0)$.




Just as in Observation 22.5, we can use the Pythagorean theorem to calculate $|a + bi|$ as $a^2 + b^2 = |a + bi|^2$, and so

$$\boxed{|a + bi| = \sqrt{a^2 + b^2}} \tag{23.3}$$

**Example 23.7**

Find the absolute value of the complex numbers below.

a) $5-3i$ b) $-8-6i$ c) $-3+3i$ d) $4\sqrt{3}+4i$ e) $7i$

**Solution.**
The absolute values are calculated as follows.

a) $|5-3i| = \sqrt{5^2+(-3)^2} = \sqrt{25+9} = \sqrt{34}$

b) $|-8-6i| = \sqrt{(-8)^2+(-6)^2} = \sqrt{64+36} = \sqrt{100} = 10$

c) $|-3+3i| = \sqrt{(-3)^2+3^2} = \sqrt{9+9} = \sqrt{18} = \sqrt{9\cdot 2} = 3\cdot\sqrt{2}$

d) $\left|4\sqrt{3}+4i\right| = \sqrt{(4\sqrt{3})^2+4^2} = \sqrt{16\cdot 3+16} = \sqrt{64} = 8$

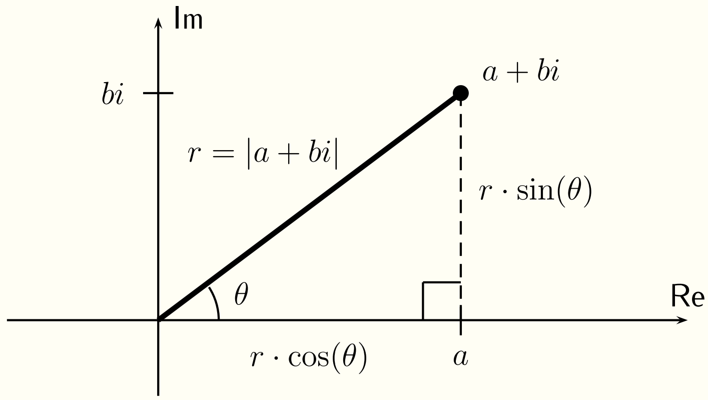
e) $|7i| = |0+7i| = \sqrt{0^2+(7)^2} = \sqrt{0+49} = 7$

□

Next, we apply the concept of the direction angle to a complex number.

**Observation 23.8: Angle or argument**

Let $a+bi$ be a complex number. Just as in Observation 22.7, we define the **angle** or **argument** of $a+bi$ to be the angle $\theta$ (read as "theta") determined by the line segment connecting the origin to $a+bi$.




Repeating the calculation from Observation 22.7, we write $r=|a+bi|$ for the absolute value, so that using (21.2), the coordinates $a$ and $b$ in the plane are given by $\sin(\theta) = \frac{b}{r}$ and $\cos(\theta) = \frac{a}{r}$. The angle is given by $\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} = \frac{\frac{b}{r}}{\frac{a}{r}} = \frac{b}{r} \cdot \frac{r}{a} = \frac{b}{a}$, which evaluates as in Note 22.9 to

$$\boxed{\theta = \begin{cases} \tan^{-1}(\frac{b}{a}) & \text{if } a+bi \text{ is in quadrant I or IV} \\ \tan^{-1}(\frac{b}{a}) + 180° & \text{if } a+bi \text{ is in quadrant II or III} \end{cases}} \tag{23.4}$$

We can rewrite a complex number via its absolute value and angle.

**Definition 23.9: Polar form**

For a complex number $a+bi$, we write $r = |a+bi|$ for the absolute value, and $\theta$ for the angle (given by (23.4)). Then, using that $a = r \cdot \cos(\theta)$, and $b = r \cdot \sin(\theta)$), the real and imaginary parts of the complex number $a + bi$ can be rewritten as follows:

$$a + bi = r \cdot \cos(\theta) + r \cdot \sin(\theta) \cdot i$$

After factoring $r$, we get:

$$\boxed{a + bi = r \cdot \big(\cos(\theta) + i \cdot \sin(\theta)\big)} \tag{23.5}$$

We say a complex number is in **polar form** if it is written in the form $r \cdot (\cos(\theta) + i \cdot \sin(\theta))$.
We say a complex number is in **standard form** or **rectangular form** if it is written as $a + bi$.

We can convert a complex number from standard form to polar form and vice versa, which we do in the next two examples.

**Example 23.10**

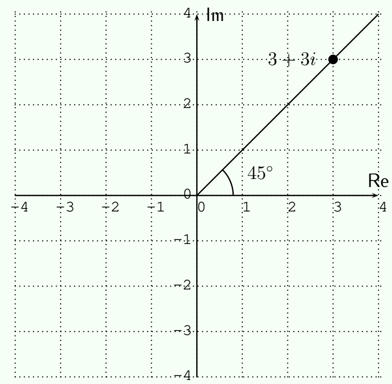
Convert the complex number to polar form.

a) $3 + 3i$ b) $-2 - 2\sqrt{3}i$ c) $-6\sqrt{3} + 6i$ d) $4 - 3i$ e) $-4i$

**Solution.**

a) First, the absolute value is $r = |3+3i| = \sqrt{3^2 + 3^2} = \sqrt{18} = \sqrt{9 \cdot 2} =$

$3\sqrt{2}$. Furthermore, since $a = 3$ and $b = 3$, we have $\tan(\theta) = \frac{b}{a} = \frac{3}{3} = 1$. To obtain $\theta$, we calculate $\tan^{-1}(1) = 45°$. Note that $45°$ is in the first quadrant, and so is the complex number $2 + 3i$



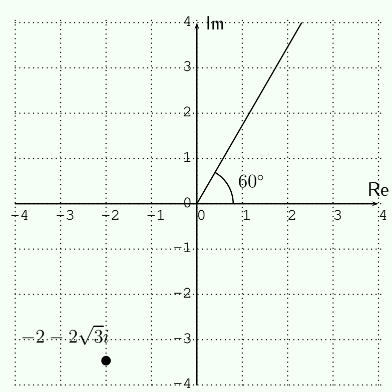

Therefore, $\theta = 45°$, and we obtain our answer:

$$2 + 3i = 3\sqrt{2} \cdot \big(\cos(45°) + i\sin(45°)\big).$$

b) For $-2 - 2\sqrt{3}i$, we first calculate the absolute value:

$$r = \sqrt{(-2)^2 + (-2\sqrt{3})^2} = \sqrt{4 + 4 \cdot 3} = \sqrt{4 + 12} = \sqrt{16} = 4.$$

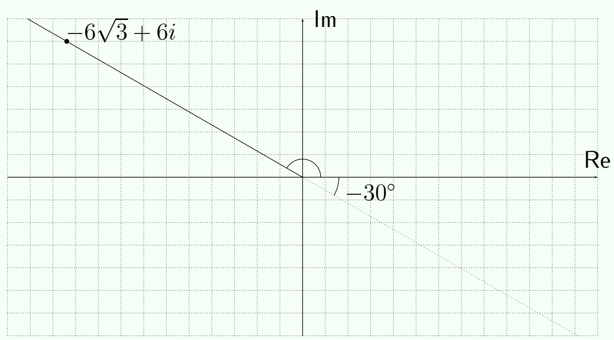
Furthermore, $\tan(\theta) = \frac{b}{a} = \frac{-2\sqrt{3}}{-2} = \sqrt{3}$. We have that $\tan^{-1}(\sqrt{3}) = 60°$. However, graphing the angle $60°$ and the number $-2 - 2\sqrt{3}i$, we see that $60°$ is in the first quadrant, whereas $-2 - 2\sqrt{3}i$ is in the third quadrant.



Therefore, we have to add $180°$ to $60°$ to get the correct angle for $-2-2\sqrt{3}i$, that is, $\theta = 60° + 180° = 240°$. Our complex number in polar form is

$$-2-2\sqrt{3}i = 4 \cdot \big(\cos\big(240°\big) + i \sin\big(240°\big)\big).$$

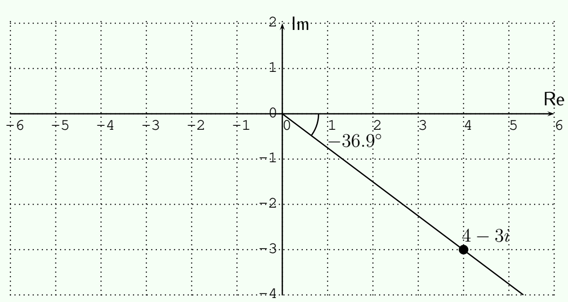
c) The absolute value of $-6\sqrt{3}+6i$ is $r = |-6\sqrt{3}+6i| = \sqrt{(-6\sqrt{3})^2+6^2} = \sqrt{36\cdot 3+36} = \sqrt{144} = 12$. The angle satisfies $\tan(\theta) = \frac{b}{a} = \frac{6}{-6\sqrt{3}} = \frac{1}{-\sqrt{3}}$, and $\tan^{-1}(\frac{1}{-\sqrt{3}}) = -30°$, which is in quadrant IV. Graphing $-6\sqrt{3}+6i$ in the complex plane shows it is in quadrant II.




Therefore, the angle is $\theta = -30° + 180° = 150°$, and so

$$-6\sqrt{3}+6i = 12 \cdot \big(\cos\big(150°\big) + i \sin\big(150°\big)\big).$$

d) For $4-3i$ we calculate $r = \sqrt{4^2+(-3)^2} = \sqrt{16+9} = \sqrt{25} = 5$. The angle $\tan^{-1}(\frac{-3}{4}) \approx -36.9°$ is in the fourth quadrant, and the complex number $4-3i$ is in the fourth quadrant as well.



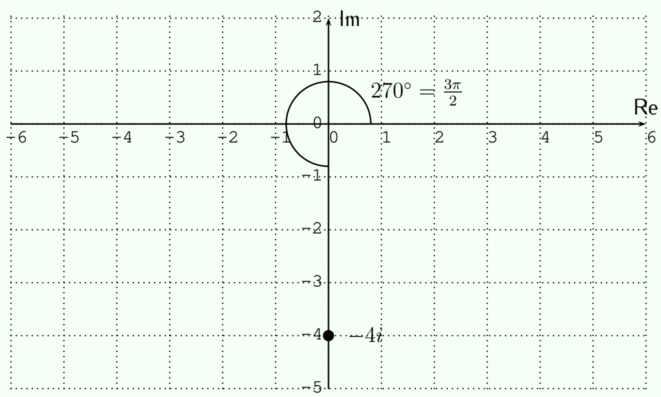
Therefore, $\theta \approx -36.9°$, and we write

$$4 - 3i \approx 5 \cdot \big(\cos(-36.9°) + i\sin(-36.9°)\big)$$

If we prefer an angle between $0°$ and $360°$, then we can also use the angle $-36.9° + 360° = 323.1°$, and write

$$4 - 3i \approx 5 \cdot \big(\cos(323.19°) + i\sin(323.1°)\big)$$

e) We calculate the absolute value of $0 - 4i$ as $r = \sqrt{0^2 + (-4)^2} = \sqrt{16} = 4$. However, when calculating the angle $\theta$ of $0 - 4i$, we are led to consider $\tan^{-1}(\frac{-4}{0})$, which is *undefined*! The reason for this can be seen by plotting the number $-4i$ in the complex plane.




The angle $\theta = 270°$ (or alternatively $\theta = -90°$), so that the complex number is

$$\begin{aligned} -4i &= 4 \cdot (\cos(270°) + i\sin(270°)) \\ &= 4 \cdot \Big(\cos\big(\frac{3\pi}{2}\big) + i\sin\big(\frac{3\pi}{2}\big)\Big) \end{aligned}$$

Note that we can always write our answer with an angle either in degree or radian mode, as we did in the last equality.

□

Conversely, we can convert a complex number from polar form to standard form $a + bi$ by evaluation the $\sin$ and the $\cos$.

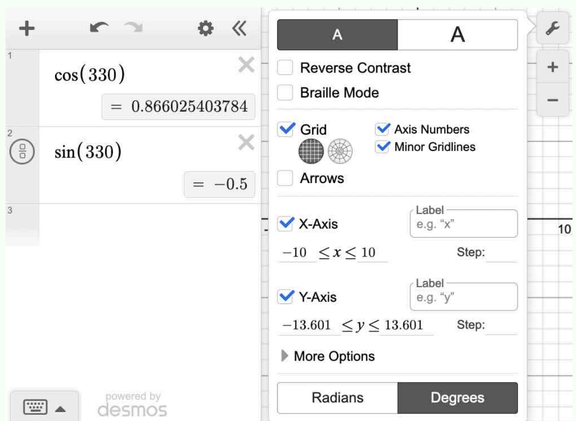
## Example 23.11

Convert the number from polar form to standard form $a + bi$.

a) $4 \cdot (\cos(330°) + i\sin(330°))$  b) $3 \cdot (\cos(117°) + i\sin(117°))$

### Solution.

a) We compute that $\cos(330°) = \frac{\sqrt{3}}{2}$ and $\sin(330°) = -\frac{1}{2}$, which is easily done with the calculator, since $\frac{\sqrt{3}}{2} \approx 0.866$ (review Example 17.10 if needed).




With this, we obtain the complex number in standard form.

$$
\begin{aligned}
4 \cdot (\cos(330°) + i\sin(330°)) &= 4 \cdot \left(\frac{\sqrt{3}}{2} + i\left(-\frac{1}{2}\right)\right) \\
&= \frac{4\sqrt{3}}{2} - i \cdot \frac{4}{2} = 2\sqrt{3} - 2 \cdot i
\end{aligned}
$$

b) Since we do not have an exact formula for $\cos(117°)$ or $\sin(117°)$, we use the calculator to obtain approximate values.

$$3 \cdot (\cos(117°) + i\sin(117°)) \approx 3 \cdot (-0.454 + i \cdot 0.891) = -1.362 + 2.673i$$

$\square$

## 23.2 Multiplication and division of complex numbers in polar form

It turns out that the angle of a product (or quotient) of complex numbers changes by adding (or subtracting) the angles. This is precisely stated in the following proposition.

**Proposition 23.12**

Let $r_1(\cos(\theta_1) + i\sin(\theta_1))$ and $r_2(\cos(\theta_2) + i\sin(\theta_2))$ be two complex numbers in polar form. Then the product and quotient of these are given by:

$$\boxed{\begin{aligned} &r_1(\cos(\theta_1) + i\sin(\theta_1)) \cdot r_2(\cos(\theta_2) + i\sin(\theta_2)) \\ &\qquad = r_1 r_2 \cdot (\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)) \end{aligned}} \tag{23.6}$$

$$\boxed{\frac{r_1(\cos(\theta_1) + i\sin(\theta_1))}{r_2(\cos(\theta_2) + i\sin(\theta_2))} = \frac{r_1}{r_2} \cdot (\cos(\theta_1-\theta_2) + i\sin(\theta_1-\theta_2))} \tag{23.7}$$

*Proof.* The proof uses the addition formulas for trigonometric functions $\sin(\alpha+\beta)$ and $\cos(\alpha+\beta)$ from Proposition 17.11 on page 306.

$$\begin{aligned} &r_1(\cos(\theta_1) + i\sin(\theta_1)) \cdot r_2(\cos(\theta_2) + i\sin(\theta_2)) \\ &\quad = r_1 r_2 \cdot (\cos(\theta_1)\cos(\theta_2) + i\cos(\theta_1)\sin(\theta_2) + i\sin(\theta_1)\cos(\theta_2) + i^2\sin(\theta_1)\sin(\theta_2)) \\ &\quad = r_1 r_2 \cdot ((\cos(\theta_1)\cos(\theta_2) - \sin(\theta_1)\sin(\theta_2)) + i(\cos(\theta_1)\sin(\theta_2) + \sin(\theta_1)\cos(\theta_2))) \\ &\qquad\qquad = r_1 r_2 \cdot (\cos(\theta_1+\theta_2) + i\sin(\theta_1+\theta_2)) \end{aligned}$$

For the division formula, note that the multiplication formula (23.6) gives

$$\begin{aligned} r_2(\cos(\theta_2) + i\sin(\theta_2)) \cdot \frac{1}{r_2}(\cos(-\theta_2) + i\sin(-\theta_2)) &= r_2\frac{1}{r_2}(\cos(\theta_2-\theta_2) + i\sin(\theta_2-\theta_2)) \\ &= 1\cdot(\cos 0 + i\sin 0) = 1\cdot(1+i\cdot 0) = 1 \\ \Longrightarrow \qquad \frac{1}{r_2(\cos(\theta_2) + i\sin(\theta_2))} &= \frac{1}{r_2}(\cos(-\theta_2) + i\sin(-\theta_2)), \end{aligned}$$

so that

$$\begin{aligned} &\frac{r_1(\cos(\theta_1) + i\sin(\theta_1))}{r_2(\cos(\theta_2) + i\sin(\theta_2))} = r_1(\cos(\theta_1) + i\sin(\theta_1)) \cdot \frac{1}{r_2(\cos(\theta_2) + i\sin(\theta_2))} \\ &\quad = r_1(\cos(\theta_1) + i\sin(\theta_1)) \cdot \frac{1}{r_2}(\cos(-\theta_2) + i\sin(-\theta_2)) = \frac{r_1}{r_2}\cdot(\cos(\theta_1-\theta_2) + i\sin(\theta_1-\theta_2)). \end{aligned}$$

□

**Example 23.13**

Multiply or divide the complex numbers and write your answer in both polar form and standard form.

a) $5(\cos(48°) + i\sin(48°)) \cdot 8(\cos(87°) + i\sin(87°))$

b) $3(\cos(\frac{5\pi}{8}) + i\sin(\frac{5\pi}{8})) \cdot 12(\cos(\frac{7\pi}{8}) + i\sin(\frac{7\pi}{8}))$

c) $\dfrac{8(\cos(257°) + i\sin(257°))}{6(\cos(47°) + i\sin(47°))}$

d) $\dfrac{32(\cos(\frac{\pi}{4}) + i\sin(\frac{\pi}{4}))}{10(\cos(\frac{7\pi}{12}) + i\sin(\frac{7\pi}{12}))}$

**Solution.**

We will multiply and divide the complex numbers using Equations (23.6) and (23.7), respectively, and then convert them to standard form.

a) For the product of the two complex numbers, we multiply the absolute values and add the angles.

$$5(\cos(48°) + i\sin(48°)) \cdot 8(\cos(87°) + i\sin(87°))$$
$$= 5\cdot 8\cdot(\cos(48°+87°)+i\sin(48°+87°)) = 40(\cos(135°)+i\sin(135°))$$

To write this in standard form, we evaluate $\cos(135°) = -\frac{\sqrt{2}}{2}$ and $\sin(135°) = \frac{\sqrt{2}}{2}$. Thus, we get

$$40 \cdot \Big(-\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\Big) = -\frac{40\sqrt{2}}{2} + i\frac{40\sqrt{2}}{2} = -20\sqrt{2} + 20\sqrt{2}i$$

b) Similarly, we obtain the next product.

$$3\Big(\cos(\frac{5\pi}{8}) + i\sin(\frac{5\pi}{8})\Big) \cdot 12\Big(\cos(\frac{7\pi}{8}) + i\sin(\frac{7\pi}{8})\Big)$$
$$= 36\Big(\cos(\frac{5\pi}{8} + \frac{7\pi}{8}) + i\sin(\frac{5\pi}{8} + \frac{7\pi}{8})\Big)$$

Now, $\frac{5\pi}{8} + \frac{7\pi}{8} = \frac{5\pi + 7\pi}{8} = \frac{12\pi}{8} = \frac{3\pi}{2}$, for which $\cos(\frac{3\pi}{2}) = 0$ and $\sin(\frac{3\pi}{2}) = -1$. Therefore, we obtain that the product is

$$36\Big(\cos(\frac{3\pi}{2}) + i\sin(\frac{3\pi}{2})\Big) = 36(0 + i\cdot(-1)) = -36i$$

c) Next, we calculate

$$\frac{8(\cos(257°)+i\sin(257°))}{6(\cos(47°)+i\sin(47°))} = \frac{4}{3}\cdot\Big(\cos(210°)+i\sin(210°)\Big).$$

Computing $\cos(210°) = -\frac{\sqrt{3}}{2}$ and $\sin(210°) = -\frac{1}{2}$, we obtain

$$\begin{aligned}\frac{4}{3}\cdot\Big(\cos(210°)+i\sin(210°)\Big) &= \frac{4}{3}\cdot\Big(-\frac{\sqrt{3}}{2}-i\cdot\frac{1}{2}\Big)\\ &= -\frac{4\cdot\sqrt{3}}{3\cdot 2}-i\cdot\frac{4\cdot 1}{3\cdot 2} = -\frac{2\sqrt{3}}{3}-\frac{2}{3}\cdot i\end{aligned}$$

d) For the quotient, we use the subtraction formula (23.7).

$$\frac{32(\cos(\frac{\pi}{4})+i\sin(\frac{\pi}{4}))}{10(\cos(\frac{7\pi}{12})+i\sin(\frac{7\pi}{12}))} = \frac{32}{10}\Big(\cos(\frac{\pi}{4}-\frac{7\pi}{12})+i\sin(\frac{\pi}{4}-\frac{7\pi}{12})\Big)$$

The difference in the argument of $\cos$ and $\sin$ is given by

$$\frac{\pi}{4}-\frac{7\pi}{12} = \frac{3\pi-7\pi}{12} = \frac{-4\pi}{12} = -\frac{\pi}{3}$$

and $\cos(-\frac{\pi}{3}) = \frac{1}{2}$ and $\sin(-\frac{\pi}{3}) = -\frac{\sqrt{3}}{2}$. With this, we obtain

$$\begin{aligned}\frac{32(\cos(\frac{\pi}{4})+i\sin(\frac{\pi}{4}))}{10(\cos(\frac{7\pi}{12})+i\sin(\frac{7\pi}{12}))} &= \frac{32}{10}\Big(\cos(-\frac{\pi}{3})+i\sin(-\frac{\pi}{3})\Big)\\ &= \frac{16}{5}\cdot\Big(\frac{1}{2}-i\frac{\sqrt{3}}{2}\Big) = \frac{16}{10}-\frac{16\sqrt{3}}{10}\cdot i = \frac{8}{5}-\frac{8\sqrt{3}}{5}\cdot i\end{aligned}$$

□

## 23.3 Exercises

### Exercise 23.1

Plot the complex numbers in the complex plane.

a) $4+2i$ b) $-3-5i$ c) $6-2i$ d) $-5+i$ e) $-2i$
f) $\sqrt{2}-\sqrt{2}i$ g) $7$ h) $i$ i) $0$ j) $2i-\sqrt{3}$

### Exercise 23.2

Add, subtract, multiply, and divide as indicated.

a) $(5-2i)+(-2+6i)$ b) $(-9-i)-(5-3i)$
c) $(3+2i)\cdot(4+3i)$ d) $(-2-i)\cdot(-1+4i)$
e) $\frac{2+3i}{2+i}$ f) $(5+5i)\div(2-4i)$

### Exercise 23.3

Find the absolute value $|a+bi|$ of the given complex number, and simplify your answer as much as possible.

a) $|4+3i|$ b) $|6-6i|$ c) $|-3i|$ d) $|-2-6i|$
e) $|\sqrt{8}-i|$ f) $|-2\sqrt{3}-2i|$ g) $|-5|$ h) $|-\sqrt{17}+4\sqrt{2}i|$

### Exercise 23.4

Convert the complex number into polar form $r(\cos(\theta)+i\sin(\theta))$.

a) $2+2i$ b) $4\sqrt{3}-4i$ c) $-7+7\sqrt{3}i$ d) $-5-5i$
e) $8-8i$ f) $-8+8i$ g) $-\sqrt{5}-\sqrt{15}i$ h) $\sqrt{7}-\sqrt{21}i$
i) $-5-12i$ j) $6i$ k) $-10$ l) $-\sqrt{3}+3i$

### Exercise 23.5

Convert the complex number into the standard form $a+bi$.

a) $6(\cos(150°)+i\sin(150°))$ b) $10(\cos(315°)+i\sin(315°))$
c) $2(\cos(90°)+i\sin(90°))$ d) $\cos(\frac{\pi}{6})+i\sin(\frac{\pi}{6})$
e) $\frac{1}{2}(\cos(\frac{7\pi}{6})+i\sin(\frac{7\pi}{6}))$ f) $6(\cos(-\frac{5\pi}{12})+i\sin(-\frac{5\pi}{12}))$

## Exercise 23.6

Multiply the complex numbers and write the answer in standard form $a + bi$.

a) $4(\cos(27°) + i\sin(27°)) \cdot 10(\cos(123°) + i\sin(123°))$

b) $7(\cos(182°) + i\sin(182°)) \cdot 6(\cos(43°) + i\sin(43°))$

c) $(\cos(\frac{13\pi}{12}) + i\sin(\frac{13\pi}{12})) \cdot (\cos(\frac{7\pi}{12}) + i\sin(\frac{7\pi}{12}))$

d) $8(\cos(\frac{3\pi}{7}) + i\sin(\frac{3\pi}{7})) \cdot 1.5(\cos(\frac{4\pi}{7}) + i\sin(\frac{4\pi}{7}))$

e) $0.2(\cos(196°) + i\sin(196°)) \cdot 0.5(\cos(88°) + i\sin(88°))$

f) $4(\cos(\frac{7\pi}{8}) + i\sin(\frac{7\pi}{8})) \cdot 0.25(\cos(\frac{-5\pi}{24}) + i\sin(\frac{-5\pi}{24}))$

## Exercise 23.7

Divide the complex numbers and write the answer in standard form $a+bi$.

a) $\dfrac{18(\cos(320°) + i\sin(320°))}{3(\cos(110°) + i\sin(110°))}$

b) $\dfrac{10(\cos(207°) + i\sin(207°))}{15(\cos(72°) + i\sin(72°))}$

c) $\dfrac{7(\cos(\frac{11\pi}{15}) + i\sin(\frac{11\pi}{15}))}{3(\cos(\frac{\pi}{15}) + i\sin(\frac{\pi}{15}))}$

d) $\dfrac{\cos(\frac{8\pi}{5}) + i\sin(\frac{8\pi}{5})}{2(\cos(\frac{\pi}{10}) + i\sin(\frac{\pi}{10}))}$

e) $\dfrac{42(\cos(\frac{7\pi}{4}) + i\sin(\frac{7\pi}{4}))}{7(\cos(\frac{5\pi}{12}) + i\sin(\frac{5\pi}{12}))}$

f) $\dfrac{30(\cos(-175°) + i\sin(-175°))}{18(\cos(144°) + i\sin(144°))}$