Precalculus

Third Edition (3.0)

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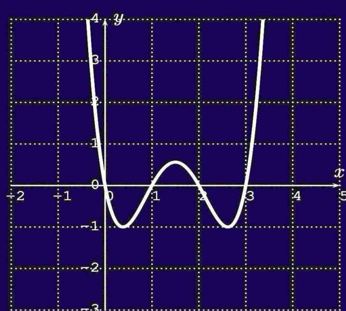

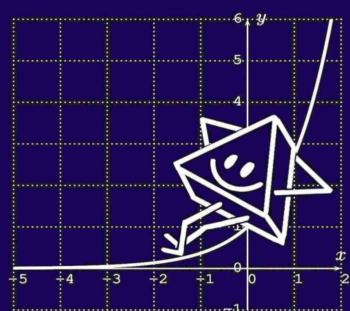

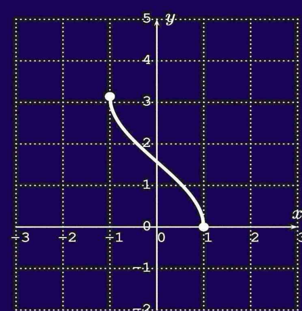

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Chapter 18

Graphing trigonometric functions

We now turn to function theoretic aspects of the trigonometric functions defined in the last chapter. In particular, we will study the graphs of trigonometric functions.

18.1 Graphs of $y = \sin(x)$, $y = \cos(x)$, and $y = \tan(x)$

To graph the functions $y = \sin(x)$, $y = \cos(x)$, and $y = \tan(x)$, we review a few trigonometric function values in the table below. Here, the angles x, which are the inputs of the trigonometric functions, are most conveniently taken in *radian* measure.

Note 18.1

We showed in the previous chapter how these function values were defined and how they can be computed, in particular, with the help of a calculator.

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\dots$
$\sin(x)$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$\dots$
$\cos(x)$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$\frac{-1}{2}$	$\frac{-\sqrt{2}}{2}$	$\frac{-\sqrt{3}}{2}$	-1	$\dots$
$\tan(x)$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undef.	$-\sqrt{3}$	-1	$\frac{-\sqrt{3}}{3}$	0	$\dots$

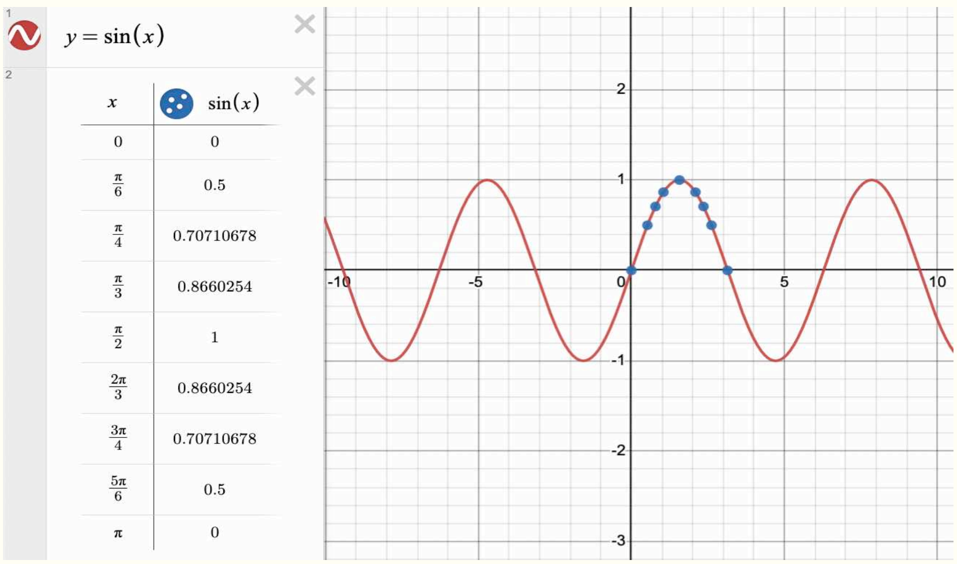
We start with the graph of the sine function.

Observation 18.2: Graph of $y = \sin(x)$

Graph the function $y = \sin(x)$.

Solution.

We graph the function $y = \sin(x)$ by plotting the corresponding (x, y) values in the plane. This can be done by hand, for example, by using the values from the above table and connecting the dots. While this is a great exercise, we will instead use the graphing calculator to do the main work for us.



From this, we make the following observations. The domain of $y = \sin(x)$ is all real numbers, $D = \mathbb{R}$, since $\sin(x)$ is defined for any angle x (see Definition 17.5).
Next, the graph is bounded (in the y-direction) between -1 and $+1$,

$$-1 \leq \sin(x) \leq 1 \quad \text{for all } x.$$

This follows from Definition 17.5, where we defined $\sin(x) = \frac{b}{r}$ with $-r \leq b \leq r$. Therefore, the range of $y = \sin(x)$ is $R = [-1, 1]$.
Moreover, the graph shows that $y = \sin(x)$ is a *periodic* function, that is, a function that repeats its output values after adding a fixed constant

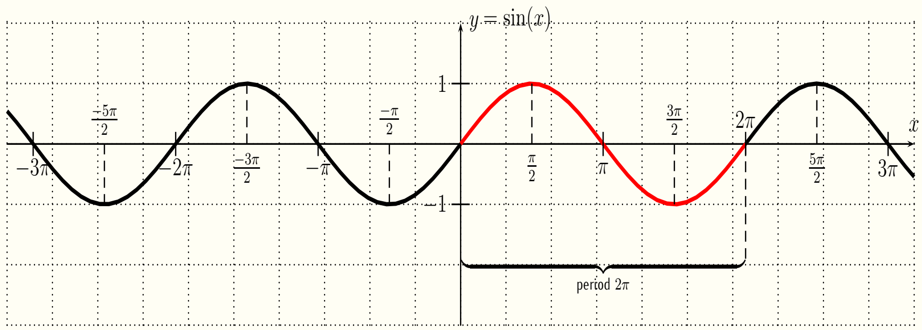
to the input. More precisely, $y = f(x)$ is *periodic* if there is a number $P \neq 0$ called a *period* so that

$$f(x + P) = f(x) \quad \text{for all } x.$$

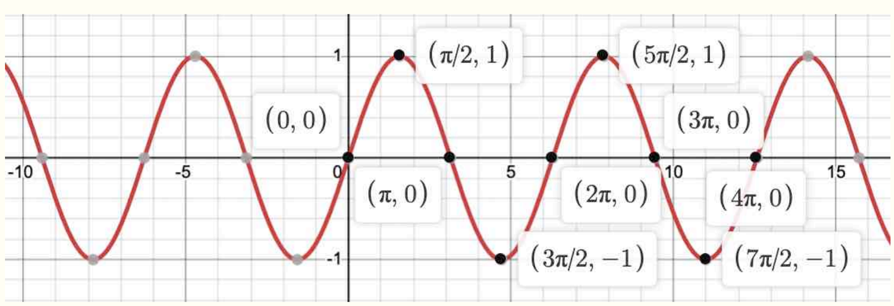
Note that $y = \sin(x)$ has a period of $P = 2\pi$, since the function does not change its value when adding $360° = 2\pi$ to its argument (and this is, in fact, the smallest positive number with that property):

$$\sin(x + 2\pi) = \sin(x)$$

The graph of $y = \sin(x)$ has the following specific values:



This can also be seen with the graphing calculator by clicking on the points of interest.



We say that $y = \sin(x)$ has a period of 2π, and an amplitude of 1. $\square$

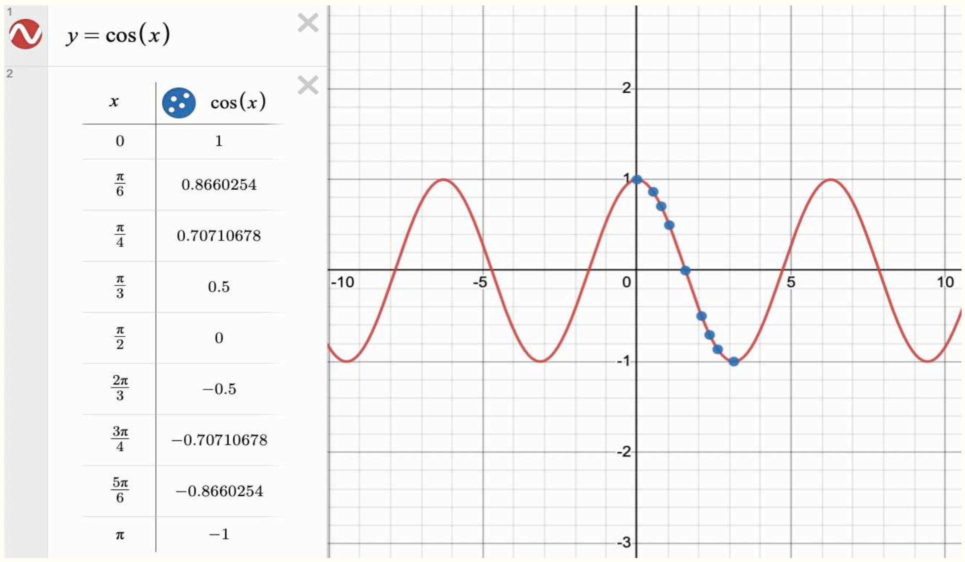
Next, we graph the cosine.

Observation 18.3: Graph of $y = \cos(x)$

Graph the function $y = \cos(x)$.

Solution.

Using the graphing calculator, we obtain the graph below.



x	$\cos(x)$
0	1
$\frac{\pi}{6}$	0.8660254
$\frac{\pi}{4}$	0.70710678
$\frac{\pi}{3}$	0.5
$\frac{\pi}{2}$	0
$\frac{2\pi}{3}$	−0.5
$\frac{3\pi}{4}$	−0.70710678
$\frac{5\pi}{6}$	−0.8660254
π	−1

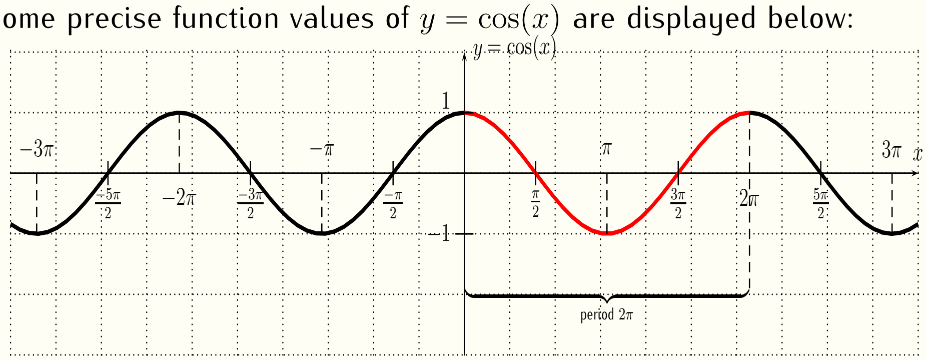
Just as we argued for the $\sin(x)$, we have the following similar observations for $y = \cos(x)$.

The domain of $y = \cos(x)$ is all real numbers, $D = \mathbb{R}$. The range is $R = [-1, 1]$, that is, $\cos(x)$ is bounded between -1 and 1.

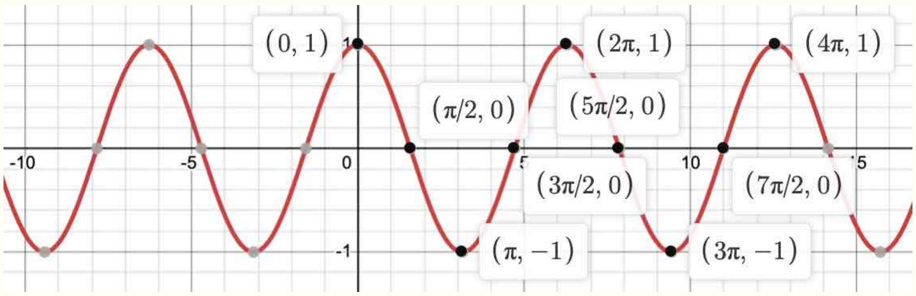
The function $y = \cos(x)$ is a periodic function with period $P = 2\pi$, that is,

$$\cos(x + 2\pi) = \cos(x) \quad \text{for all } x.$$

Some precise function values of $y = \cos(x)$ are displayed below:



These values can also be seen with the graphing calculator:



We say that $y = \cos(x)$ has a period of 2π, and an amplitude of 1. $\square$

Note 18.4

Many properties of $\sin$ and $\cos$ can be observed from the above graphs (as well as from the unit circle definition). For example, the graph of $y = \cos(x)$ appears to be that of $y = \sin(x)$ shifted to the left by $\frac{\pi}{2}$. Algebraically, this can be expressed with the following identity:

$$\boxed{\cos(x) = \sin\left(x + \frac{\pi}{2}\right)} \tag{18.1}$$

Moreover, the graph of $y = \sin(x)$ appears to be symmetric with respect to the origin, the graph of $y = \cos(x)$ appears to be symmetric with respect to the y-axis. Algebraically, this means that the sine and cosine functions satisfy the following relations:

$$\boxed{\sin(-x) = -\sin(x)} \quad \text{and} \quad \boxed{\cos(-x) = \cos(x)} \tag{18.2}$$

We will show in Observations 21.8 and 21.9, that these identities are indeed true.
Note, in particular, that this means that $y = \sin(x)$ is an odd function, while $y = \cos(x)$ is an even function (see Observation 4.24).

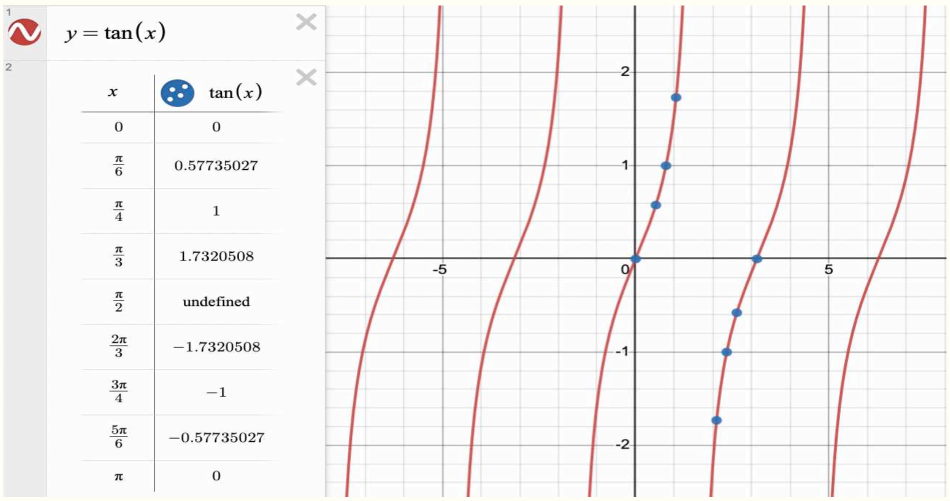
Finally, we also graph the tangent.

Observation 18.5: Graph of $y = \tan(x)$

Graph of $y = \tan(x)$.

Solution.

We graph the function $y = \tan(x)$ with the graphing calculator.



First, we observe that the tangent is periodic with a period of π:

$$\tan(x + \pi) = \tan(x) \tag{18.3}$$

Zooming into this graph, we see that $y = \tan(x)$ has vertical asymptotes $x = \frac{\pi}{2} \approx 1.6$ and $x = \frac{-\pi}{2} \approx -1.6$. This is also supported by the fact that $\tan(\frac{\pi}{2})$ and $\tan(-\frac{\pi}{2})$ are undefined. Since $y = \tan(x)$ is periodic, there are, in fact, infinitely many vertical asymptotes: $x = \frac{\pi}{2}, \frac{-\pi}{2}, \frac{3\pi}{2}, \frac{-3\pi}{2}, \frac{5\pi}{2}, \frac{-5\pi}{2}, \dots$, or, in short

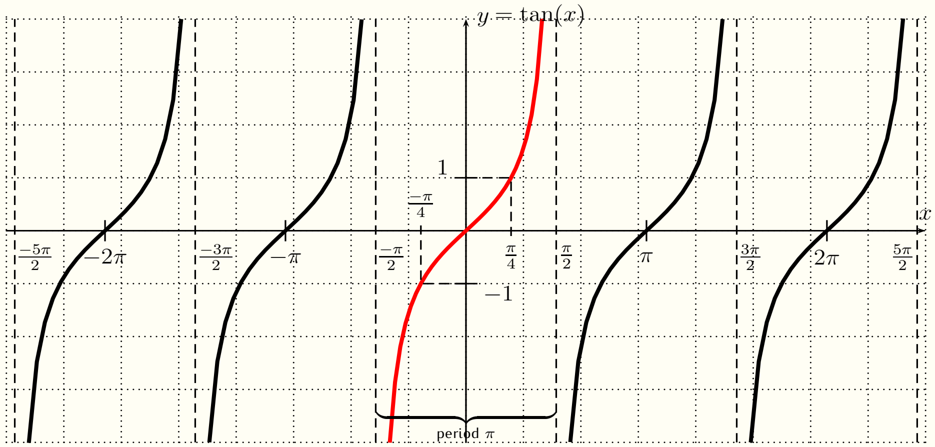
$$\text{asymptotes of } y = \tan(x): \quad x = n \cdot \frac{\pi}{2}, \text{ where } n = \pm 1, \pm 3, \pm 5, \dots$$

In particular, the domain of $y = \tan(x)$ is

$$D = \mathbb{R} - \left\{ x \ : \ x = n \cdot \frac{\pi}{2}, \text{ where } n = \pm 1, \pm 3, \pm 5, \dots \right\}$$

The range of $y = \tan(x)$ is all real numbers $R = \mathbb{R}$.

The graph of $y = \tan(x)$ with some more specific function values is shown below.



Furthermore, the tangent is an odd function, since it is symmetric with respect to the origin (see Observation 4.24):

$$\boxed{\tan(-x) = -\tan(x)} \tag{18.4}$$

□

18.2 Amplitude, period, and phase shift

Recall from Section 4.3 how adding or multiplying constants affects the graph of the function, such as:

- graph of $f(x) + c$ is the graph of $f(x)$ shifted up by c (or down when $c < 0$)
- graph of $f(x + c)$ is the graph of $f(x)$ shifted to the left by c (or to the right when $c < 0$)
- the graph of $c \cdot f(x)$ (for $c > 0$) is the graph of $f(x)$ stretched away from the x-axis by a factor c (or compressed when $0 < c < 1$)
- the graph of $f(c \cdot x)$ (for $c > 0$) is the graph of $f(x)$ compressed toward the y-axis by a factor c (or stretched away the y-axis when $0 < c < 1$)

With this, we can graph some variations of the basic trigonometric functions.

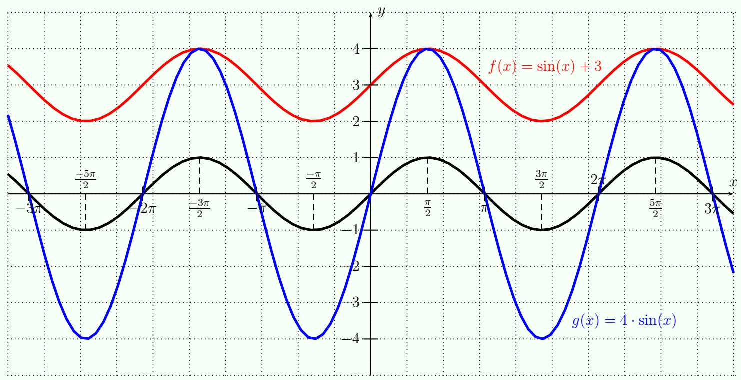
Example 18.6

Graph the functions:

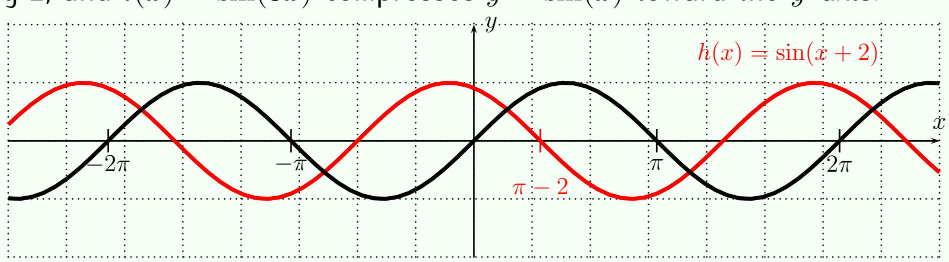
$$f(x) = \sin(x) + 3,\ \ g(x) = 4 \cdot \sin(x),\ \ h(x) = \sin(x+2),\ \ i(x) = \sin(3x)$$
$$j(x) = 2 \cdot \cos(x) + 3,\ \ k(x) = \cos(2x - \pi),\ \ l(x) = \tan(x+2) + 3$$

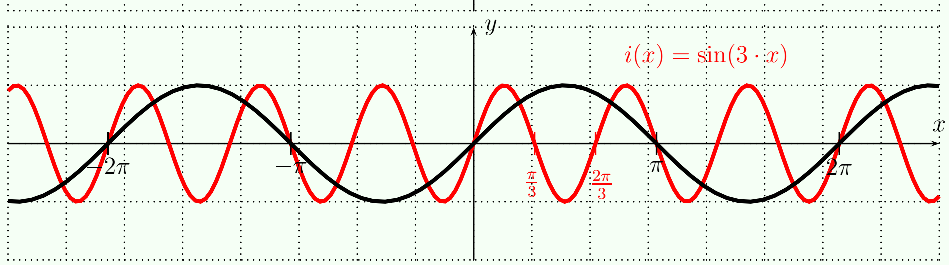
Solution.

The functions f, g, h, and i have graphs that are variations of the basic $y = \sin(x)$ graph. The graph of $f(x) = \sin(x) + 3$ shifts the graph of $y = \sin(x)$ up by 3, whereas the graph of $g(x) = 4 \cdot \sin(x)$ stretches $y = \sin(x)$ away from the x-axis.

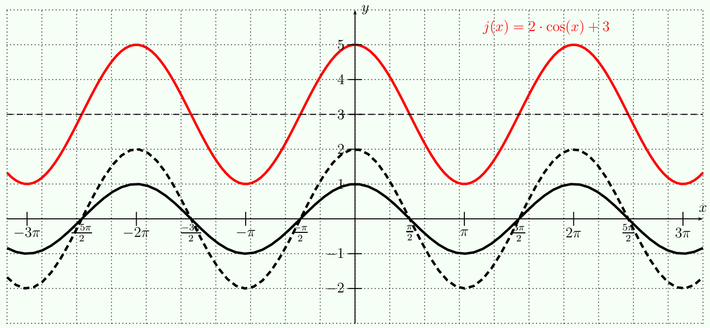


The graph of $h(x) = \sin(x+2)$ shifts the graph of $y = \sin(x)$ to the left by 2, and $i(x) = \sin(3x)$ compresses $y = \sin(x)$ toward the y-axis.

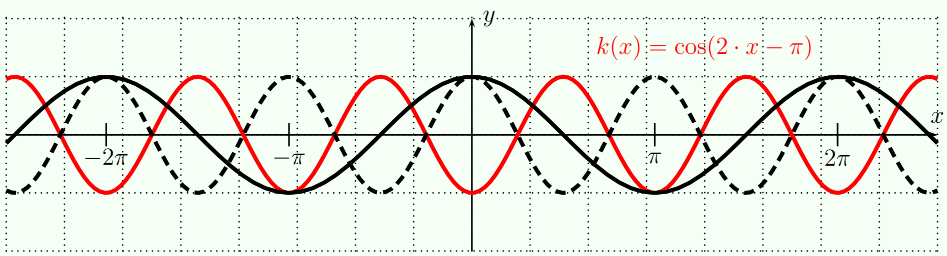




Next, $j(x) = 2 \cdot \cos(x) + 3$ has a graph of $y = \cos(x)$ stretched by a factor 2 away from the x-axis, and shifted up by 3.

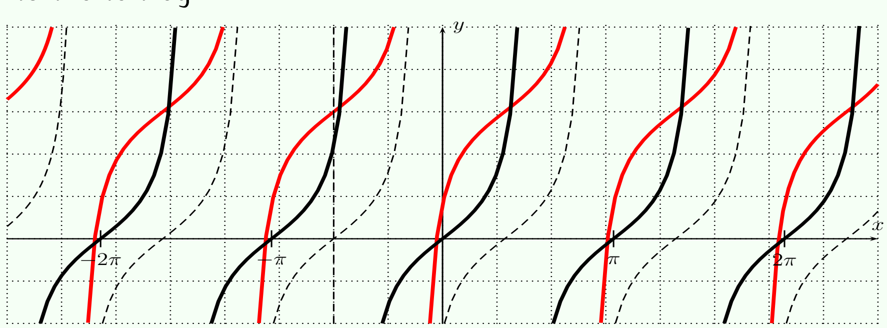


For the graph of $k(x) = \cos(2x - \pi) = \cos(2 \cdot (x - \frac{\pi}{2}))$, we need to compress the graph of $y = \cos(x)$ by a factor 2 (we obtain the graph of the function $y = \cos(2x)$), and then shift this by $\frac{\pi}{2}$ to the right.



We will explore this case below in more generality. In fact, whenever $y = \cos(b \cdot x + c) = \cos(b \cdot (x - \frac{-c}{b}))$, the graph of $y = \cos(x)$ is shifted to the right by $\frac{-c}{b}$, and compressed by a factor b, so that it has a period of $\frac{2\pi}{b}$.

Finally, $l(x) = \tan(x + 2) + 3$ shifts the graph of $y = \tan(x)$ up by 3 and to the left by 2.



$\square$

We collect some of the above observations in the following definition.

Definition 18.7: Amplitude, period, phase shift

Let f be one of the functions:

$$f(x) = a \cdot \sin(b \cdot x + c) \quad \text{or} \quad f(x) = a \cdot \cos(b \cdot x + c)$$

We define the **amplitude** A, the **period** P, and the **phase shift** S to be

$$\boxed{A = |a|} \quad \text{amplitude} \tag{18.5}$$

$$\boxed{P = \left|\frac{2\pi}{b}\right|} \quad \text{period} \tag{18.6}$$

$$\boxed{S = \frac{-c}{b}} \quad \text{phase shift} \tag{18.7}$$

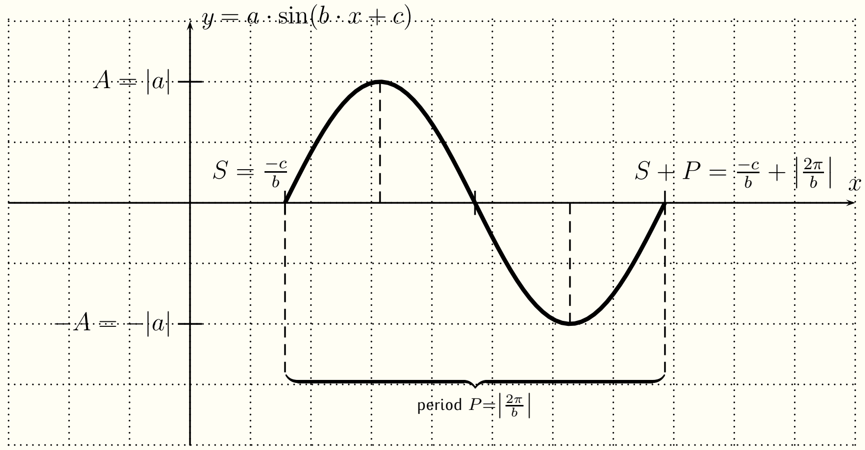
In physical applications, the period is sometimes denoted by $T = P = \left|\frac{2\pi}{b}\right|$ and the **frequency** f is defined as the reciprocal, $f = \frac{1}{P}$.

Using the amplitude, period, and phase shift, we know precisely how the shape of a sine or cosine function has been shifted or stretched.

Observation 18.8: Graphing $\sin$ or $\cos$ over one full period

With the above definition, we analyze the graph of, for example, $f(x) = a \cdot \sin(b \cdot x + c)$ with positive $a > 0$ and $b > 0$ as follows.

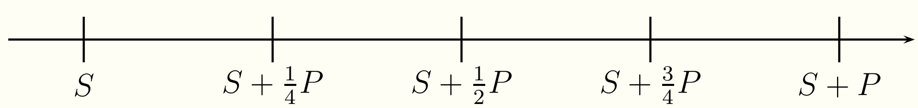
- First, consider $g(x) = a \cdot \sin(b \cdot x)$, that is, the function where we put $c = 0$. The graph of g is that of $y = \sin(x)$ stretched by a factor A away from the x-axis, and stretched (away or toward the y-axis) in such a way that it has a period of $P = \frac{2\pi}{b}$.
- Then, the graph of $f(x) = a \cdot \sin(b \cdot x + c) = a \cdot \sin\left(b \cdot \left(x + \frac{c}{b}\right)\right)$ is that of $g(x) = a \cdot \sin(b \cdot x)$ shifted by the phase shift $S = \frac{-c}{b}$. In other words, one full period of the graph starts at $(S, 0)$ and ends at $(S + P, 0)$.



We thus have the following strategy for graphing $f(x) = a \cdot \sin(b \cdot x + c)$ over one full period, starting at the phase shift S:

1. Mark the starting point of the period at $(S, 0) = (\frac{-c}{b}, 0)$.
2. Mark the endpoint of the period at $(S + P, 0) = (\frac{-c}{b} + \left|\frac{2\pi}{b}\right|, 0)$.
3. Draw the graph of $f(x) = a \cdot \sin(b \cdot x + c)$ from S to $S + P$ as the graph of $y = \sin(x)$ from 0 to 2π stretched and shifted so that it starts at S and ends at $S + P$, and so that it has an amplitude of A.

Note that the root(s), the maxima, and the minima of the graph within the drawn period are in equal distance from each other. The values of these can be found, for example, by calculating the midpoint between $(S, 0)$ and $(S + P, 0)$ and midpoints between these and the resulting midpoint:



A similar graph can be obtained for $f(x) = a \cdot \cos(b \cdot x + c)$ by replacing the graph of $y = \sin(x)$ with the graph of $y = \cos(x)$ over the period from 0 to 2π.
Moreover, note that when a and b are not positive, appropriate reflections may have to be applied as well.

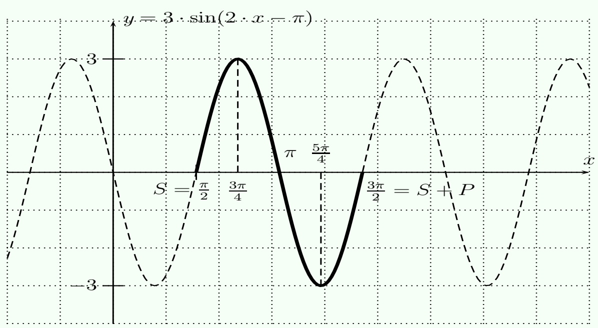
Example 18.9

Find the amplitude, period, and phase shift, and sketch the graph over one full period starting at the phase shift. Label all roots, maxima, and minima.

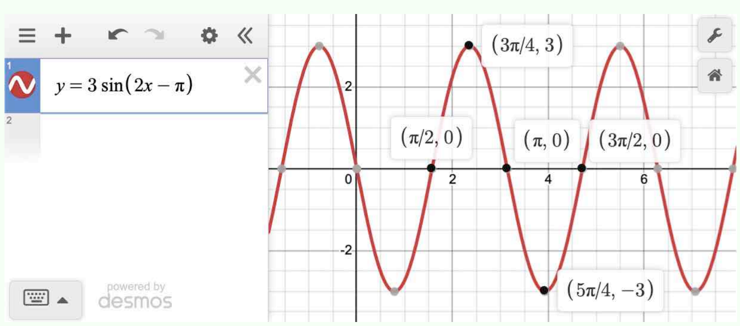
a) $f(x) = 3 \cdot \sin(2x - \pi)$ b) $f(x) = 4 \cdot \cos(5x - \frac{\pi}{2})$
c) $f(x) = 6 \cdot \sin(3x + \pi)$ d) $f(x) = -2 \cdot \cos(\frac{x}{2} - \frac{\pi}{4})$
e) $f(x) = 5 \cdot \sin\left(-2x + \frac{\pi}{3}\right)$

Solution.

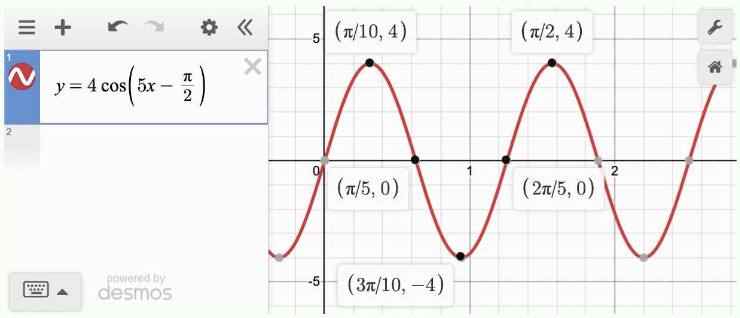
a) The amplitude is $A = |a| = 3$, the period is $P = \left|\frac{2\pi}{b}\right| = \left|\frac{2\pi}{2}\right| = \pi$, and the phase shift is $S = \frac{-c}{b} = \frac{-(-\pi)}{2} = \frac{\pi}{2}$. We graph one full period from $S = \frac{\pi}{2}$ to $S + P = \frac{\pi}{2} + \pi = \frac{3\pi}{2}$.



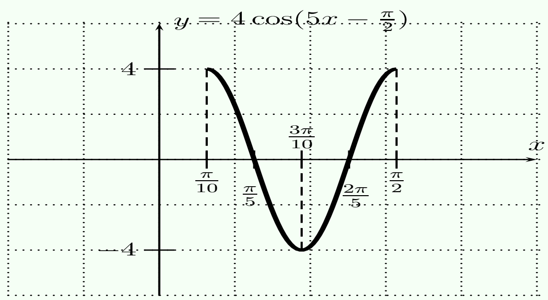
Note that the zero at the center of this period is given at $(\frac{\pi}{2} + \frac{3\pi}{2}) \div 2 = \frac{4\pi}{2} \cdot \frac{1}{2} = \pi$. The maximum is at $(\frac{\pi}{2} + \pi) \div 2 = \frac{3\pi}{2} \cdot \frac{1}{2} = \frac{3\pi}{4}$. The minimum is at $(\pi + \frac{3\pi}{2}) \div 2 = \frac{5\pi}{2} \cdot \frac{1}{2} = \frac{5\pi}{4}$. This is also confirmed with the graphing calculator.



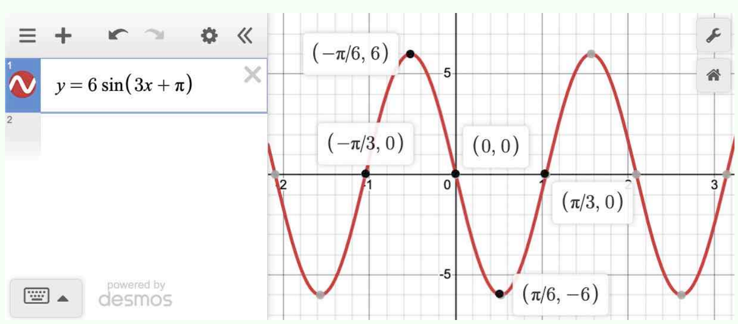
b) For $f(x) = 4 \cdot \cos(5x - \frac{\pi}{2})$, we have the amplitude $A = 4$, period $P = \frac{2\pi}{5}$, and phase shift $S = \frac{-(-\pi/2)}{5} = \frac{\pi}{2} \cdot \frac{1}{5} = \frac{\pi}{10}$. Therefore, the endpoint of the period is at $S + P = \frac{\pi}{10} + \frac{2\pi}{5} = \frac{5\pi}{10} = \frac{\pi}{2}$.



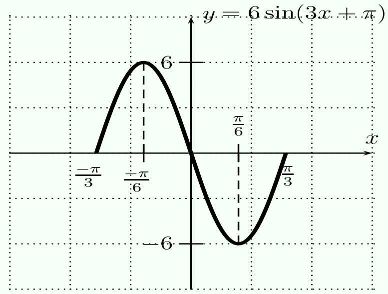
The minimum is at $\left(\frac{\pi}{10} + \frac{\pi}{2}\right) \div 2 = \frac{6\pi}{10} \cdot \frac{1}{2} = \frac{3\pi}{10}$, the roots are at $\left(\frac{\pi}{10} + \frac{3\pi}{10}\right) \div 2 = \frac{4\pi}{10} \cdot \frac{1}{2} = \frac{\pi}{5}$ and $\left(\frac{3\pi}{10} + \frac{\pi}{2}\right) \div 2 = \frac{8\pi}{10} \cdot \frac{1}{2} = \frac{2\pi}{5}$.



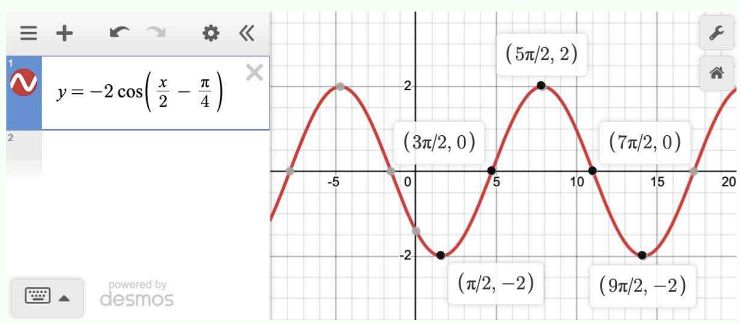
c) For $f(x) = 6 \cdot \sin(3x + \pi)$, we have the amplitude $A = 6$, period $P = \frac{2\pi}{3}$, and phase shift $S = \frac{-\pi}{3}$. Therefore, the endpoint of the period is at $S + P = \frac{-\pi}{3} + \frac{2\pi}{3} = \frac{\pi}{3}$.



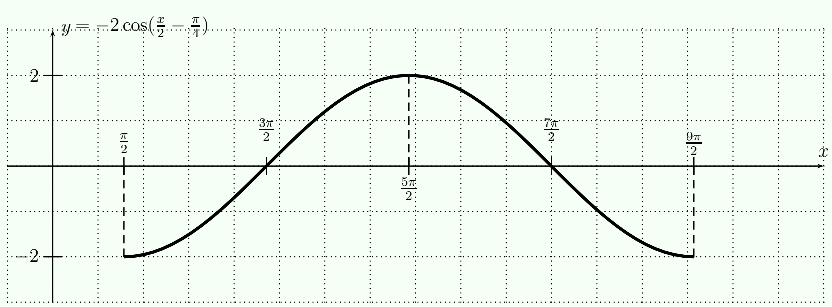
The root is at $\left(\frac{-\pi}{3} + \frac{\pi}{3}\right) \div 2 = 0$, the maximum is at $\left(\frac{-\pi}{3} + 0\right) \div 2 = \frac{-\pi}{3} \cdot \frac{1}{2} = \frac{-\pi}{6}$, the minimum is at $\left(0 + \frac{\pi}{3}\right) \div 2 = \frac{\pi}{3} \cdot \frac{1}{2} = \frac{\pi}{6}$.



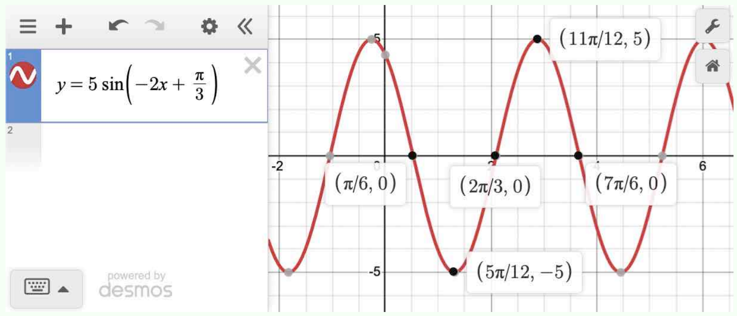
d) For $f(x) = -2 \cdot \cos(\frac{x}{2} - \frac{\pi}{4})$, we have the amplitude $A = |-2| = 2$, period $P = \frac{2\pi}{\frac{1}{2}} = 2\pi \cdot \frac{2}{1} = 4\pi$, and phase shift $S = \frac{-(-\pi/4)}{\frac{1}{2}} = \frac{\pi}{4} \cdot \frac{2}{1} = \frac{\pi}{2}$. Therefore, the endpoint of the period is at $S + P = \frac{\pi}{2} + 4\pi = \frac{9\pi}{2}$.



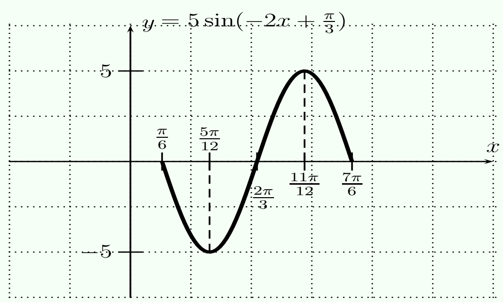
Since the coefficient $a = -2$ is negative, we need to graph the cosine reflected over the x-axis. We have the maximum at $\left(\frac{\pi}{2} + \frac{9\pi}{2}\right) \div 2 = \frac{10\pi}{2} \cdot \frac{1}{2} = \frac{5\pi}{2}$, the roots are at $\left(\frac{\pi}{2} + \frac{5\pi}{2}\right) \div 2 = \frac{6\pi}{2} \cdot \frac{1}{2} = \frac{3\pi}{2}$ and $\left(\frac{5\pi}{2} + \frac{9\pi}{2}\right) \div 2 = \frac{14\pi}{2} \cdot \frac{1}{2} = \frac{7\pi}{2}$.



e) For $f(x) = 5 \cdot \sin\left(-2x + \frac{\pi}{3}\right)$, we have the amplitude $A = 5$, period $P = \left|\frac{2\pi}{-2}\right| = \pi$, and phase shift $S = \frac{-(\pi/3)}{-2} = \frac{\pi}{3} \cdot \frac{1}{2} = \frac{\pi}{6}$. Therefore, the endpoint of the period is at $S + P = \frac{\pi}{6} + \pi = \frac{7\pi}{6}$.

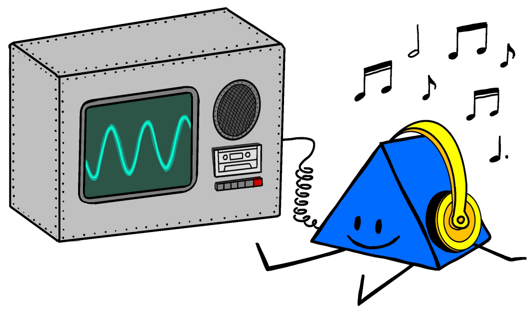


To graph $f(x)$, we recall from (18.2) that $\sin(-x) = -\sin(x)$, so that $f(x)$ can be rewritten as $f(x) = 5 \cdot \sin\left(-(2x - \frac{\pi}{3})\right) = -5 \cdot \sin\left(2x - \frac{\pi}{3}\right)$. Writing $f(x)$ in this way gives the same period and phase shift as before, since now $P = \left|\frac{2\pi}{2}\right| = \pi$ and $S = \frac{-(-\pi/3)}{2} = \frac{\pi}{3} \cdot \frac{1}{2} = \frac{\pi}{6}$. Since $f(x) = -5 \cdot \sin\left(2x - \frac{\pi}{3}\right)$ has a negative leading coefficient, we need to reflect the sine graph over the x-axis. Thus, we get the root at $\left(\frac{\pi}{6} + \frac{7\pi}{6}\right) \div 2 = \frac{8\pi}{6} \cdot \frac{1}{2} = \frac{8\pi}{12} = \frac{2\pi}{3}$; the minimum is at $\left(\frac{\pi}{6} + \frac{2\pi}{3}\right) \div 2 = \frac{5\pi}{6} \cdot \frac{1}{2} = \frac{5\pi}{12}$, the maximum is at $\left(\frac{2\pi}{3} + \frac{7\pi}{6}\right) \div 2 = \frac{11\pi}{6} \cdot \frac{1}{2} = \frac{11\pi}{12}$.

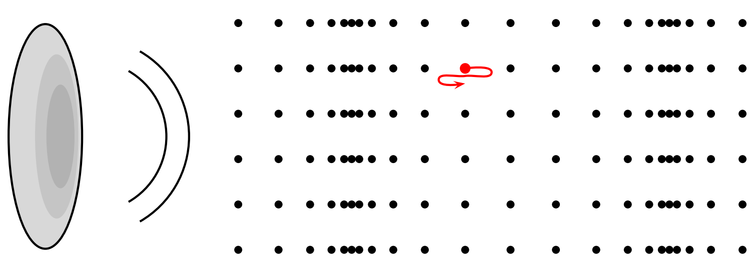


□

Application: Sound waves



Sound is created from a source, for example, from a loudspeaker. A vibrating membrane within the loudspeaker creates vibrations of the molecules in the surrounding air, creating regions of higher and lower pressures.[1] These differences in pressures transmit via sound waves and are perceived by the brain as sound.



The above picture shows a vibrating molecule (in red) in regions of variable pressure. Note that it is not any overall motion of the molecules that create the sound wave, but molecules vibrating at their equilibrium positions. The displacement $s(t)$ of a molecule from its equilibrium position is given by a $\sin$ or $\cos$ function in time.

$$s(t) = a \cdot \cos(\omega t + c) \tag{18.8}$$

In the formula above, ω is called the angular frequency, and so the period is given by $P = \frac{2\pi}{\omega}$. If we recall that the frequency $f = \frac{1}{P}$ is the reciprocal of the period, we see that $\omega = \frac{2\pi}{P} = 2\pi f$.

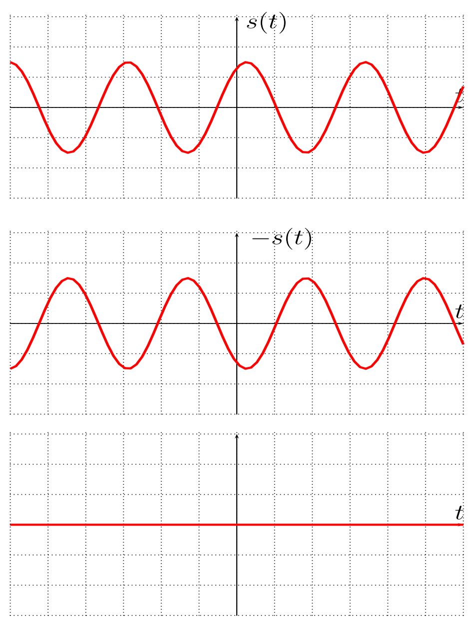
[1] For more information, see https://openstax.org/books/university-physics-volume-1/pages/17-1-sound-waves

We can interpret the amplitute, period, and phase shift from (18.8) in terms of the perceived sound as follows:

- Sound waves can be perceived by humans for frequencies ranging from 20 Hz to $20,000$ Hz. The higher the frequency (or the smaller the period), the higher the perceived pitch of the sound.

- The amplitude is perceived as the volume of the sound.

- While the phase shift cannot be perceived directly from the sound, we can see the effect of the phase shift from applications such as noise canceling headphones. The idea for this is that for a given displacement $s(t)$ of a molecule, if we can create a displacement in the opposite direction, then the overall effect is no displacement at all, which yields a cancellation of the sound.



18.3 Exercises

Exercise 18.1

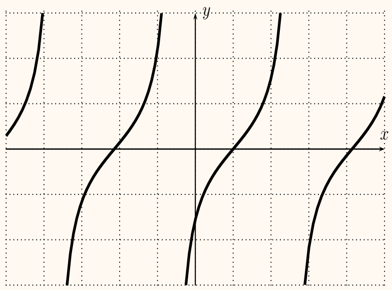
Graph the function and describe how the graph can be obtained from one of the basic graphs $y = \sin(x)$, $y = \cos(x)$, or $y = \tan(x)$.

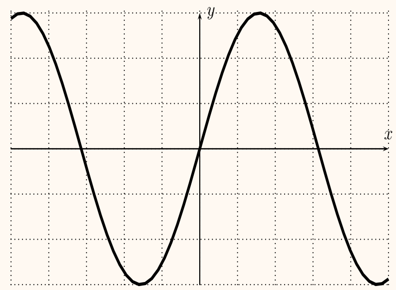
a) $f(x) = \sin(x) + 2$ b) $f(x) = \cos(x - \pi)$ c) $f(x) = \tan(x) - 4$
d) $f(x) = 5 \cdot \sin(x)$ e) $f(x) = \cos(2 \cdot x)$ f) $f(x) = \sin(x - 2) - 5$

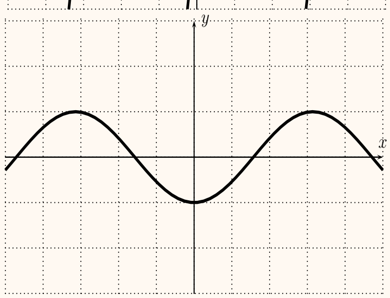
Exercise 18.2

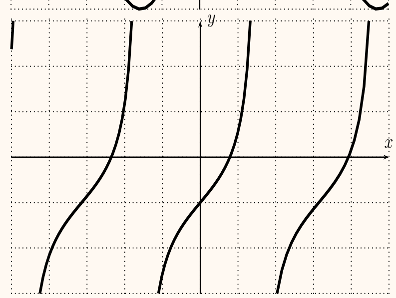
Identify the formulas with the graphs.

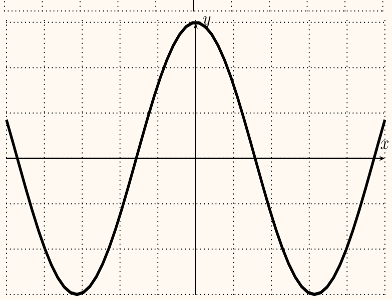
$f(x) = \sin(x) + 2$, $g(x) = \tan(x - 1)$, $h(x) = 3\sin(x)$
$i(x) = 3\cos(x)$, $j(x) = \cos(x - \pi)$, $k(x) = \tan(x) - 1$

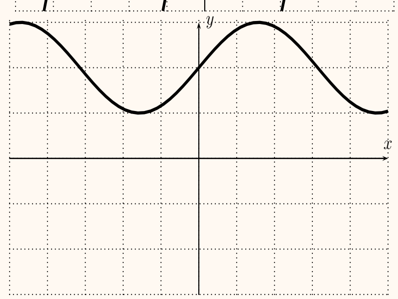
a) 

b) 

c) 

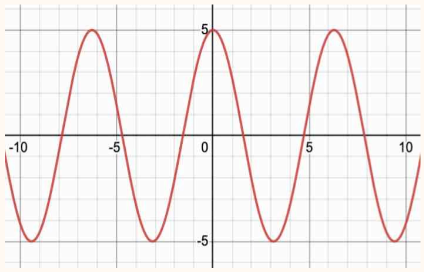
d) 

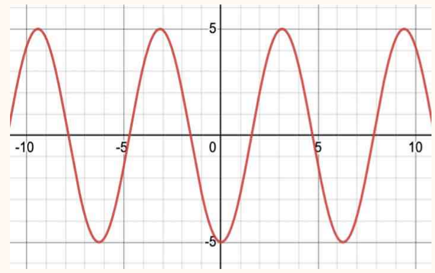
e) 

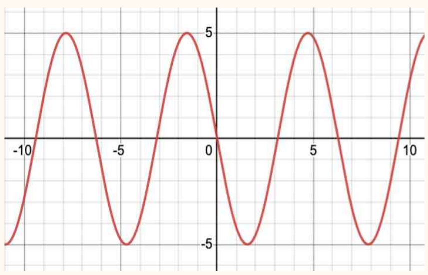
f) 

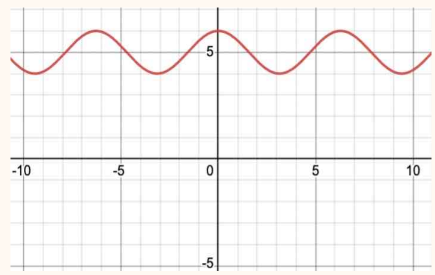
Exercise 18.3

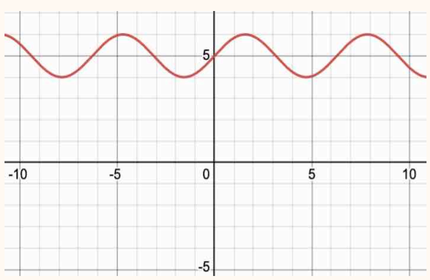
Find the formula of a function whose graph is the one displayed below.

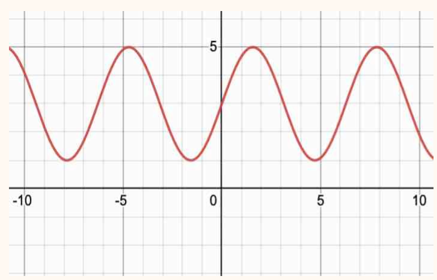
a) 

b) 

c) 

d) 

e) 

f) 

Exercise 18.4

Find the amplitude, period, and phase shift of the function.

a) $f(x) = 5\sin(2x+\pi)$
b) $f(x) = 3\sin(4x-\frac{\pi}{2})$
c) $f(x) = 4\sin(6x)$
d) $f(x) = 2\cos(7x+\frac{\pi}{4})$
e) $f(x) = 8\cos(2x-3\pi)$
f) $f(x) = 3\sin(\frac{x}{4})$
g) $f(x) = -4\cos(5x+\frac{\pi}{3})$
h) $f(x) = 7\sin(\frac{1}{2}x-\frac{6\pi}{5})$
i) $f(x) = \cos(-2x)$
j) $f(x) = 6\cos(\pi x-\pi)$

Exercise 18.5

Find the amplitude, period, and phase shift of the function. Use this information to graph the function over a full period. Label all roots, maxima, and minima of the function.

a) $y = 5\cos(2x)$
b) $y = -4\sin(\pi x)$
c) $y = 4\sin(5x - \pi)$
d) $y = 6\cos(2x - \pi)$
e) $y = 5\sin(2x - \frac{\pi}{2})$
f) $y = 7\cos(3x - \frac{\pi}{2})$
g) $y = 5\sin(3x - \frac{\pi}{4})$
h) $y = 3\sin(4x + \pi)$
i) $y = 2\cos(5x + \pi)$
j) $y = 4\sin(2x + \frac{\pi}{2})$
k) $y = 3\cos(6x + \frac{\pi}{2})$
l) $y = 3\cos\left(2x + \frac{\pi}{4}\right)$
m) $y = 7\sin\left(\frac{1}{4}x + \frac{\pi}{4}\right)$
n) $y = -2\sin\left(\frac{1}{5}x - \frac{\pi}{10}\right)$
o) $y = \frac{1}{3}\cos\left(\frac{14}{5}x - \frac{6\pi}{5}\right)$