Precalculus

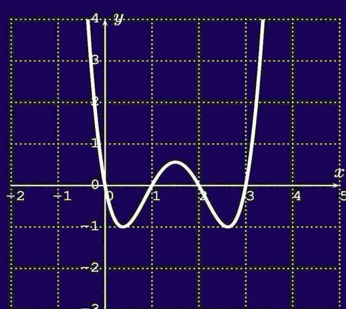
Third Edition (3.0)

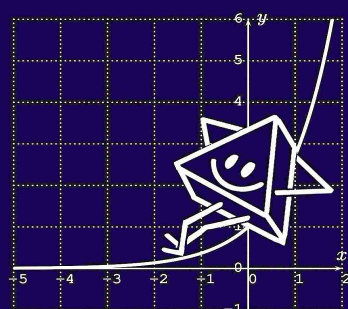
Thomas Tradler

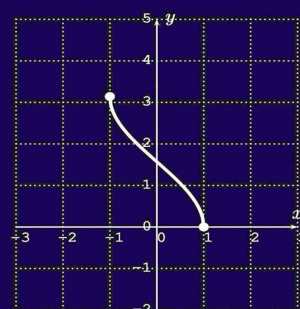
Holly Carley

The pages below contain a single chapter from the Precalculus textbook. The full textbook can be downloaded here:

Precalculus Textbook Download Page







This document was created with LaTeX.

The images of the Desmos graphing calculator were generated from the Desmos website at **https://www.desmos.com/calculator**

Chapter 15

Exponential equations and applications

We now turn to exponential equations, and discuss the application of population growth in Section 15.2. In the next chapter, we will study two more common applications of exponential functions.

15.1 Exponential equations

Recall from Observation 14.5 that both the exponential and the logarithmic functions are one-to-one:

$$b^x = b^y \quad \Leftrightarrow \quad x = y$$
$$\log_b(x) = \log_b(y) \quad \Leftrightarrow \quad x = y$$

In Section 14.2 we used the second equivalence to solve logarithmic equations. Now we use the first equivalence to solve exponential equations. Note that we can immediately apply this to exponential equations with a common base.

Example 15.1

Solve for x.

a) $2^{x+7} = 32$ b) $10^{2x-8} = 0.01$
c) $7^{2x-3} = 7^{5x+4}$ d) $5^{3x+1} = 25^{4x-7}$

Solution.
In these examples, we can always write both sides of the equation as

an exponential expression with the same base.

a) $2^{x+7} = 32 \implies 2^{x+7} = 2^5 \implies x + 7 = 5 \implies x = -2$

b) $10^{2x-8} = 0.01 \implies 10^{2x-8} = 10^{-2} \implies 2x - 8 = -2$
$\implies 2x = 6 \implies x = 3$

Here it is useful to recall the powers of 10, which were also used to solve the equation above.

$$\begin{array}{rcl} 10^4 & = & 10,000 \\ 10^3 & = & 1000 \\ 10^2 & = & 100 \\ 10^1 & = & 10 \\ 10^0 & = & 1 \\ 10^{-1} & = & 0.1 \\ 10^{-2} & = & 0.01 \\ 10^{-3} & = & 0.001 \\ 10^{-4} & = & 0.0001 \end{array}$$

In general $(n \geq 1)$: $\begin{cases} 10^n = 1\underbrace{00\cdots00}_{n \text{ zeros}} \\ 10^{-n} = \underbrace{0.0\cdots00}_{n \text{ zeros}}1 \end{cases}$

c) $7^{2x-3} = 7^{5x+4} \implies 2x - 3 = 5x + 4 \overset{(-5x+3)}{\implies} -3x = 7$
$\implies x = -\dfrac{7}{3}$

d) $5^{3x+1} = 25^{4x-7} \implies 5^{3x+1} = 5^{2\cdot(4x-7)}$
$\implies 3x + 1 = 2 \cdot (4x - 7)$
$\implies 3x + 1 = 8x - 14$
$\overset{(-8x-1)}{\implies} -5x = -15$
$\implies x = 3$

By a similar reasoning, we can solve equations involving logarithms whenever the bases coincide. □

To solve exponential equations that do not have a common base on both sides, we need to apply the logarithm, as stated in the following note.

Note 15.2

An equation between two exponential expressions with the same base can be simplified using the fact that the exponential is one-to-one.

$$b^{f(x)} = b^{g(x)} \qquad \Longrightarrow \qquad f(x) = g(x)$$

To solve an equation between two exponential expressions with different bases, we first apply a logarithm and then solve for x. Indeed, using the identity $\log_b(x^n) = n \cdot \log_b(x)$ from (14.2), we can rewrite an exponent as a coefficient and solve from there:

$$\begin{aligned} a^{f(x)} = b^{g(x)} \qquad &\Longrightarrow \qquad \log(a^{f(x)}) = \log(b^{f(x)}) \\ &\Longrightarrow \qquad f(x) \cdot \log(a) = g(x) \cdot \log(b) \end{aligned}$$

Example 15.3

Solve for x.

a) $3^{x+5} = 8$ b) $13^{2x-4} = 6$ c) $5^{x-7} = 2^x$
d) $5.1^x = 2.7^{2x+6}$ e) $17^{x-2} = 3^{x+4}$ f) $7^{2x+3} = 11^{3x-6}$

Solution.

We solve these equations by applying a logarithm (both $\log$ or $\ln$ will work for solving the equation), and then we use the identity $\log_b(x^n) = n \cdot \log_b(x)$ from (14.2).

a) $3^{x+5} = 8 \implies \ln 3^{x+5} = \ln 8 \implies \quad (x+5) \cdot \ln 3 = \ln 8$

$$\implies x + 5 = \frac{\ln 8}{\ln 3} \implies \quad x = \frac{\ln 8}{\ln 3} - 5 \approx -3.11$$

b) $13^{2x-4} = 6 \implies \ln 13^{2x-4} = \ln 6 \implies \quad (2x-4) \cdot \ln 13 = \ln 6$

$$\begin{aligned} &\implies \quad 2x - 4 = \frac{\ln 6}{\ln 13} \implies \quad 2x = \frac{\ln 6}{\ln 13} + 4 \\ &\implies \quad x = \frac{\frac{\ln 6}{\ln 13} + 4}{2} = \frac{\ln 6}{2 \cdot \ln 13} + 2 \approx 2.35 \end{aligned}$$

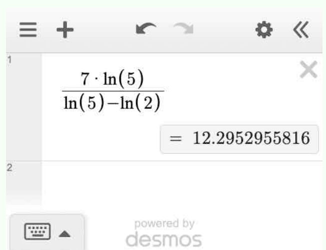
c) $5^{x-7} = 2^x \implies \ln 5^{x-7} = \ln 2^x \implies \quad (x-7) \cdot \ln 5 = x \cdot \ln 2$

At this point, the calculation will proceed differently than the calculations in parts (a) and (b). Since x appears on both sides of $(x-7)\cdot\ln 5 = x\cdot\ln 2$, we need to separate terms involving x from terms without x. That is, we need to distribute $\ln 5$ on the left:

$$(x-7)\cdot\ln 5 = x\cdot\ln 2 \implies x\cdot\ln 5 - 7\cdot\ln 5 = x\cdot\ln 2$$

Next, we separate the terms with x from those without x by adding $7\cdot\ln 5$ and subtracting $x\cdot\ln 2$ to both sides:

$$\begin{aligned}
&\implies\quad x\cdot\ln 5 - x\cdot\ln 2 = 7\cdot\ln 5\\
&\implies\quad x\cdot(\ln 5-\ln 2) = 7\cdot\ln 5\\
&\implies\quad x = \frac{7\cdot\ln 5}{\ln 5-\ln 2}\approx 12.30
\end{aligned}$$



We apply the same solution strategy that we used in (c) for the remaining parts (d)-(f).

d) $5.1^x = 2.7^{2x+6} \implies \ln 5.1^x = \ln 2.7^{2x+6}$

$$\begin{aligned}
&\implies\quad x\cdot\ln 5.1 = (2x+6)\cdot\ln 2.7\\
&\implies\quad x\cdot\ln 5.1 = 2x\cdot\ln 2.7 + 6\cdot\ln 2.7\\
&\implies\quad x\cdot\ln 5.1 - 2x\cdot\ln 2.7 = 6\cdot\ln 2.7\\
&\implies\quad x\cdot(\ln 5.1 - 2\cdot\ln 2.7) = 6\cdot\ln 2.7\\
&\implies\quad x = \frac{6\cdot\ln 2.7}{\ln 5.1-2\cdot\ln 2.7}\approx -16.68
\end{aligned}$$

e) $17^{x-2} = 3^{x+4} \implies \ln 17^{x-2} = \ln 3^{x+4}$

$$\implies\quad (x-2)\cdot\ln 17 = (x+4)\cdot\ln 3$$

$$\begin{aligned}
&\Longrightarrow \quad x \cdot \ln 17 - 2 \cdot \ln 17 = x \cdot \ln 3 + 4 \cdot \ln 3 \\
&\Longrightarrow \quad x \cdot \ln 17 - x \cdot \ln 3 = 2 \cdot \ln 17 + 4 \cdot \ln 3 \\
&\Longrightarrow \quad x \cdot (\ln 17 - \ln 3) = 2 \cdot \ln 17 + 4 \cdot \ln 3 \\
&\Longrightarrow \quad x = \frac{2 \cdot \ln 17 + 4 \cdot \ln 3}{\ln 17 - \ln 3} \approx 5.80
\end{aligned}$$

f) $7^{2x+3} = 11^{3x-6} \implies \ln 7^{2x+3} = \ln 11^{3x-6}$

$$\begin{aligned}
&\Longrightarrow \quad (2x+3) \cdot \ln 7 = (3x-6) \cdot \ln 11 \\
&\Longrightarrow \quad 2x \cdot \ln 7 + 3 \cdot \ln 7 = 3x \cdot \ln 11 - 6 \cdot \ln 11 \\
&\Longrightarrow \quad 2x \cdot \ln 7 - 3x \cdot \ln 11 = -3 \cdot \ln 7 - 6 \cdot \ln 11 \\
&\Longrightarrow \quad x \cdot (2 \cdot \ln 7 - 3 \cdot \ln 11) = -3 \cdot \ln 7 - 6 \cdot \ln 11 \\
&\Longrightarrow \quad x = \frac{-3 \cdot \ln 7 - 6 \cdot \ln 11}{2 \cdot \ln 7 - 3 \cdot \ln 11} \approx 6.13
\end{aligned}$$

□

Before we get to specific applications of exponential functions, we pause to explain how we can identify the base b and the coefficient c of an exponential function $f(x) = c \cdot b^x$.

Note 15.4

Let $f(x) = c \cdot b^x$ be an exponential function. Then, the parameters c and b in the function f are uniquely determined by knowing the function values $f(x_1)$ and $f(x_2)$ for any two distinct inputs x_1 and x_2.

Example 15.5

Let $f(x) = c \cdot b^x$. Determine the constant c and base b under the given conditions.

a) $f(0) = 5$, $f(1) = 20$ b) $f(0) = 3$, $f(4) = 48$
c) $f(2) = 160$, $f(7) = 5$ d) $f(-2) = 55$, $f(1) = 7$

Solution.

a) Applying $f(0) = 5$ to $f(x) = c \cdot b^x$, we get

$$5 = f(0) = c \cdot b^0 = c \cdot 1 = c$$

Indeed, in general, we always have $f(0) = c$ for any exponential function. The base b is then determined by substituting the second equation $f(1) = 20$.

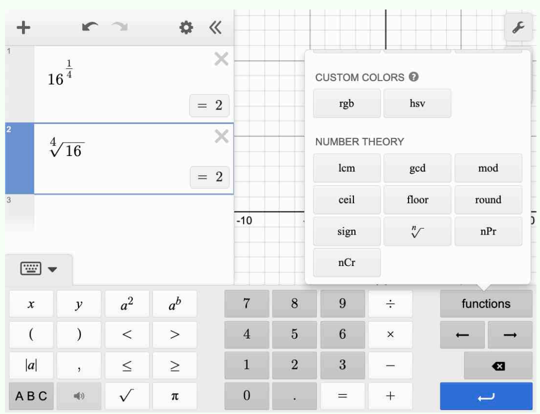
$$20 = f(1) = c \cdot b^1 = 5 \cdot b \quad \overset{(\div 5)}{\Longrightarrow} \quad b = 4$$

Therefore, $f(x) = 5 \cdot 4^x$. Note that in the last implication, we used that the base must be positive.

b) As before, we get $3 = f(0) = c \cdot b^0 = c$, and

$$\begin{aligned} 48 = f(4) = c \cdot b^4 = 3 \cdot b^4 \quad &\overset{(\div 3)}{\Longrightarrow} \quad 16 = b^4 \\ &\overset{(\text{exponentiate by } \frac{1}{4})}{\Longrightarrow} \quad b = 16^{\frac{1}{4}} = 2 \end{aligned}$$

Recall that $\sqrt[4]{a} = a^{\frac{1}{4}}$, and so the 4th root can be calculated with the graphing calculator either via the exponent $\frac{1}{4}$ or via the 4th root.



Therefore, $f(x) = 3 \cdot 2^x$.

c) When $f(0)$ is not given, it is easiest to solve for b first. We can see this as follows. Since $160 = f(2) = c \cdot b^2$ and $5 = f(7) = c \cdot b^7$, the quotient of these equations eliminates c.

$$\begin{aligned} \frac{160}{5} = \frac{c \cdot b^2}{c \cdot b^7} = \frac{1}{b^5} \quad &\Longrightarrow \quad 32 = b^{-5} \\ &\overset{(\text{exponentiate by } (-\frac{1}{5}))}{\Longrightarrow} \quad b = 32^{-\frac{1}{5}} = \frac{1}{32^{\frac{1}{5}}} = \frac{1}{2} \end{aligned}$$

Then c is determined by any of the original equations.

$$160 = f(2) = c\cdot b^2 = c\cdot\left(\frac{1}{2}\right)^2 = c\cdot\frac{1}{4} \quad\Longrightarrow\quad c = 4\cdot 160 = 640$$

Therefore, $f(x) = 640\cdot\left(\frac{1}{2}\right)^x$.

d) This solution is similar to part (c).

$$\begin{aligned}\frac{55}{7} = \frac{f(-2)}{f(1)} = \frac{c\cdot b^{-2}}{c\cdot b^1} = \frac{1}{b^3} \quad &\Longrightarrow\quad b^3 = \frac{7}{55}\\ &\Longrightarrow\quad b = \left(\frac{7}{55}\right)^{\frac{1}{3}} \approx 0.503\end{aligned}$$

$$\begin{aligned}55 = f(-2) = c\cdot b^{-2} = c\cdot\left(\left(\frac{7}{55}\right)^{\frac{1}{3}}\right)^{-2} &= c\cdot\left(\frac{7}{55}\right)^{\frac{-2}{3}}\\ \Longrightarrow\quad c = \frac{55}{\left(\frac{7}{55}\right)^{\frac{-2}{3}}} &= 55\cdot\left(\frac{7^{\frac{2}{3}}}{55^{\frac{2}{3}}}\right)\\ &= 55^{\frac{1}{3}}\cdot 7^{\frac{2}{3}} = \sqrt[3]{55\cdot 7^2} = \sqrt[3]{2695} \approx 13.916\end{aligned}$$

Therefore, $f(x) = \sqrt[3]{2695}\cdot\left(\sqrt[3]{\frac{7}{55}}\right)^x$.

□

15.2 Applications of exponential functions

Exponential functions express situations where the growths of a quantity is proportional to the amount of the quantity at a given time. This makes exponential functions an important toy model for many applications. In this text we will use exponential functions to model the following:

- population growths or decline
- compound interest on an investment
- radioactive decay

In this section we will focus on population growth and decline, and we will study compound interest and radioacitve decay in the next chapter.

Example 15.6

The mass of a bacteria sample is $2 \cdot 1.02^t$ grams after t hours.

a) What is the mass of the bacteria sample after 4 hours?
b) When will the mass reach 10 grams?

Solution.

a) The formula for the mass y in grams after t hours is $y(t) = 2 \cdot 1.02^t$. Therefore, after 4 hours, the mass in grams is:

$$y(4) = 2 \cdot 1.02^4 \approx 2.16$$

b) We are seeking the number of hours t for which $y = 10$ grams. Therefore, we have to solve:

$$10 = 2 \cdot 1.02^t \quad \overset{(\div 2)}{\Longrightarrow} \quad 5 = 1.02^t$$

We need to solve for the variable in the exponent. In general, to solve for a variable in the exponent requires an application of a logarithm on both sides of the equation.

$$5 = 1.02^t \quad \overset{\text{(apply log)}}{\Longrightarrow} \quad \log(5) = \log(1.02^t)$$

Recall an important property that we can use to solve for t:

$$\log(x^t) = t \cdot \log(x) \tag{15.1}$$

Using (15.1), we can now solve for t as follows:

$$\begin{aligned} \log(5) = \log(1.02^t) \quad &\Longrightarrow \quad \log(5) = t \cdot \log(1.02) \\ &\overset{\text{(divide by } \log(1.02))}{\Longrightarrow} \quad t = \frac{\log(5)}{\log(1.02)} \approx 81.3 \end{aligned}$$

After approximately 81.3 hours, the mass will be 10 grams.

□

Example 15.7

The population size of a country was 12.7 million in the year 2010, and 14.3 million in the year 2020.

a) Assuming an exponential growth for the population size, find the formula for the population depending on the year t.

b) What will the population size be in the year 2025, assuming the formula holds until then?

c) When will the population reach 18 million?

Solution.

a) The growth is assumed to be exponential, so that $y(t) = c \cdot b^t$ describes the population size depending on the year t, where we set $t = 0$ corresponding to the year 2010. Then the example describes $y(0) = c$ as $c = 12.7$, which we assume in units of millions of people. To find the base b, we substitute the data of $t = 10$ and $y(t) = 14.3$ into $y(t) = c \cdot b^t$.

$$\begin{aligned} 14.3 = 12.7 \cdot b^{10} \quad &\implies \quad \frac{14.3}{12.7} = b^{10} \quad \implies \left(\frac{14.3}{12.7}\right)^{\frac{1}{10}} = (b^{10})^{\frac{1}{10}} = b \\ &\implies \quad b = \left(\frac{14.3}{12.7}\right)^{\frac{1}{10}} \approx 1.012 \end{aligned}$$

The formula for the population size is $y(t) \approx 12.7 \cdot 1.012^t$.

b) We calculate the population size in the year 2025 by setting $t = 2025 - 2010 = 15$:

$$y(15) = 12.7 \cdot 1.012^{15} \approx 15.2$$

c) We seek t so that $y(t) = 18$. We solve for t using the logarithm.

$$\begin{aligned} 18 = 12.7 \cdot 1.012^t \quad &\implies \quad \frac{18}{12.7} = 1.012^t \\ &\implies \quad \log\left(\frac{18}{12.7}\right) = \log(1.012^t) \end{aligned}$$

$$\Longrightarrow \quad \log\left(\frac{18}{12.7}\right) = t \cdot \log(1.012)$$
$$\Longrightarrow \quad t = \frac{\log\left(\frac{18}{12.7}\right)}{\log(1.012)} \approx 29.2$$

Adding 29.2 years to the year 2010, we see that the population will reach 18 million in the year 2039.

□

In many instances the exponential function $f(x) = c \cdot b^x$ is given via a rate of growth r.

Definition 15.8: Rate of growth

An exponential function with a **rate of growth** r is a function $f(x) = c \cdot b^x$ with base
$$\boxed{b = e^r}$$

Note 15.9

Some textbooks use a different convention than the one given in Definition 15.8 for the rate of growth. Indeed, sometimes a function with rate of growth r is defined as an exponential function with base $b = 1 + r$, whereas we use a base $b = e^r$. Since e^r can be expanded as $e^r = 1 + r + \frac{r^2}{2} + \dots$, this shows that the two versions only vary by a difference of order 2 (that is they differ by $\frac{r^2}{2}$ plus higher powers of r), and so, for small r, the base $1 + r$ and the base e^r are approximately equal.

Example 15.10

The number of PCs that are sold in the US in the year 2021 is approximately 350 million. Assuming that the number grows exponentially at a constant rate of 3.6% per year, how many PCs will be sold in the year 2027?

Solution.

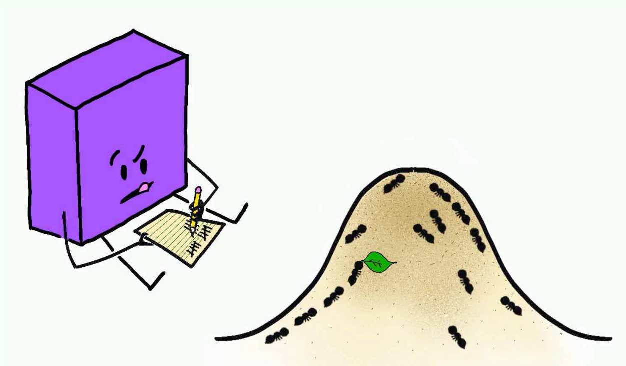
Since the rate of growth is $r = 3.6\% = 0.036$, we obtain a base of $b = e^r = e^{0.036}$. Therefore, we will model the number of PCs sold (in millions of PCs) by the function $y(t) = c \cdot (e^{0.036})^t = c \cdot e^{0.036 \cdot t}$. If we set $t = 0$ for the year 2021, we find that $c = 350$, so the number of sales is given by $y(t) = 350 \cdot e^{0.036 \cdot t}$. Since the year 2027 corresponds to $t = 2027 - 2021 = 6$, we can calculate the number of sales in the year 2027 as

$$y(4) = 350 \cdot e^{0.036} \approx 434.$$

Approximately 434 million PCs will be sold in the year 2027. $\square$

Example 15.11

The size of an ant colony is decreasing at a rate of 1% per month. How long will it take until the colony has reached 80% of its original size?



Solution.

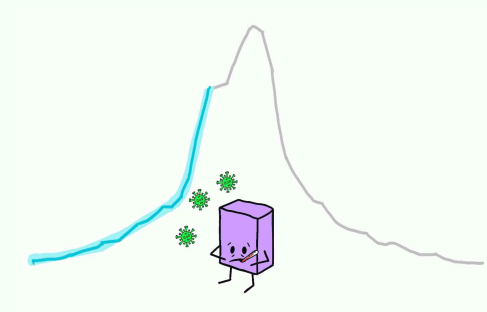
Since the population size is decreasing, the rate is negative, that is $r = -1\% = -0.01$. We therefore obtain the base $b = e^r = e^{-0.01}$. We have a colony size of $y(t) = c \cdot e^{-0.01 \cdot t}$ after t months, where c is the original size. We need to find t so that the size is at 80% of its original size c, that is, $y(t) = 80\% \cdot c = 0.8 \cdot c$.

$$\begin{aligned}
0.8 \cdot c = c \cdot e^{-0.01 \cdot t} \quad & \overset{(\div c)}{\Longrightarrow} \quad 0.8 = e^{-0.01 \cdot t} \\
& \Longrightarrow \quad \ln(0.8) = \ln(e^{-0.01 \cdot t}) \\
& \Longrightarrow \quad \ln(0.8) = -0.01 \cdot t \cdot \underbrace{\ln(e)}_{=1} \\
& \Longrightarrow \quad t = \frac{\ln(0.8)}{-0.01} \approx 22.3
\end{aligned}$$

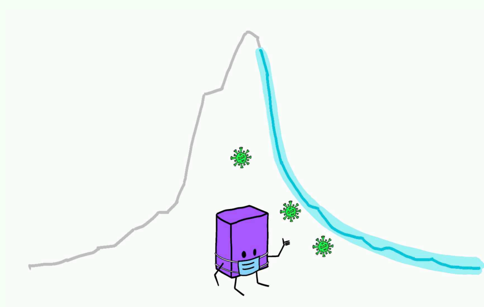
After approximately 22.3 months, the ant colony has decreased to 80% of its original size. $\square$

Example 15.12

a) The number of flu cases in the fall was increasing at a rate of 9.8% per week. How long did it take for the number of flu cases to double?



b) The number of flu cases in the spring was decreasing at a rate of 15% per week. How long did it take for the number of flu cases to decrease to a quarter of its size?



Solution.

a) The rate of change is $r = 9.8\% = 0.098$ per week, so that the number of flu cases is an exponential function with base $b = e^{0.098}$. Therefore, $f(x) = c \cdot e^{0.098 \cdot x}$ denotes the number of flu cases, with c being the initial number of cases at the time corresponding to $x = 0$. In order for the number of flu cases to double, $f(x)$ has to reach twice its initial size, that is:

$$\begin{aligned} f(x) = 2c \quad &\Longrightarrow \quad 2c = c \cdot e^{0.098 \cdot x} \\ &\overset{(\div c)}{\Longrightarrow} \quad 2 = e^{0.098 \cdot x} \end{aligned}$$

$$\Longrightarrow \quad \ln(2) = \ln(e^{0.098\cdot x})$$
$$\Longrightarrow \quad \ln(2) = 0.098 \cdot x \ln(e)$$
$$\Longrightarrow \quad x = \frac{\ln(2)}{0.098} \approx 7.07$$

Therefore, it took about 7.07 weeks until the number of flu cases doubled.

b) Since the number of flu cases was decreasing, the rate of growth is negative, $r = -15\% = -0.15$ per week, so that we have an exponential function with base $b = e^r = e^{-0.15}$. To reach a quarter of its initial number of flu cases, we set $f(x) = c \cdot e^{-0.15\cdot x}$ equal to $\frac{1}{4}c$.

$$\frac{1}{4}c = c \cdot e^{-0.15\cdot x} \quad \overset{(\div c)}{\Longrightarrow} \quad \frac{1}{4} = e^{-0.15\cdot x}$$
$$\Longrightarrow \quad \ln(\frac{1}{4}) = -0.15 \cdot x \cdot \ln(e)$$
$$\Longrightarrow \quad x = \frac{\ln(\frac{1}{4})}{-0.15} \approx 9.24$$

It therefore took about 9.24 weeks until the number of flu cases decreased to a quarter.

□

15.3 Exercises

Exercise 15.1

Solve for x without using a calculator.

a)	$6^{x-2} = 36$	b)	$2^{3x-8} = 16$
c)	$10^{5-x} = 0.0001$	d)	$5^{5x+7} = \frac{1}{125}$
e)	$2^x = 64^{x+1}$	f)	$4^{x+3} = 32^x$
g)	$13^{4+2x} = 1$	h)	$3^{x+2} = 27^{x-3}$
i)	$25^{7x-4} = 5^{2-3x}$	j)	$9^{5+3x} = 27^{8-2x}$

Exercise 15.2

Solve for x. First find the exact answer as an expression involving logarithms. Then approximate the answer to the nearest hundredth using a calculator.

a) $4^x = 57$	b) $9^{x-2} = 7$	c) $2^{x+1} = 31$
d) $3.8^{2x+7} = 63$	e) $5^{x+5} = 8^x$	f) $3^{x+2} = 0.4^x$
g) $1.02^{2x-9} = 4.35^x$	h) $4^{x+1} = 5^{x+2}$	i) $9^{3-x} = 4^{x-6}$
j) $2.4^{7-2x} = 3.8^{3x+4}$	k) $4^{9x-2} = 9^{2x-4}$	l) $1.95^{-3x-4} = 1.2^{4-7x}$

Exercise 15.3

Assuming that $f(x) = c \cdot b^x$ is an exponential function, find the constants c and b from the given conditions.

a) $f(0) = 4$, $f(1) = 12$	b) $f(0) = 5$, $f(3) = 40$
c) $f(0) = 3200$, $f(6) = 0.0032$	d) $f(3) = 12$, $f(5) = 48$
e) $f(-1) = 4$, $f(2) = 500$	f) $f(2) = 3$, $f(4) = 15$

Exercise 15.4

The number of downloads of a certain software application was 8.4 million in the year 2017 and 13.6 million in the year 2022.

a) Assuming an exponential growth for the number of downloads, find the formula for the downloads depending on the year t.

b) Assuming the number of downloads will follow the formula from part (a), what will the number of downloads be in the year 2026?

c) In what year will the number of downloaded applications reach the 25 million barrier?

Exercise 15.5

The population size of a city was $79,000$ in the year 1998 and $136,000$ in the year 2013. Assume that the population size follows an exponential function.

a) Find the formula for the population size.

b) What is the population size in the year 2030?

c) What is the population size in the year 2035?

d) When will the city reach its expected maximum capacity of one million residents?

Exercise 15.6

The population of a city decreases at a rate of 2.3% per year. After how many years will the population be at 90% of its current size? Round your answer to the nearest tenth.

Exercise 15.7

A big company plans to expand its franchise and, with this, its number of employees. For tax reasons, it is most beneficial to expand the number of employees at a rate of 5% per year. If the company currently has 4730 employees, how many years will it take until the company has 6000 employees? Round your answer to the nearest hundredth.

Exercise 15.8

An ant colony has a population size of 4000 ants and is increasing at a rate of 3% per week. How long will it take until the ant population has doubled? Round your answer to the nearest tenth.

Exercise 15.9

The size of a beehive is decreasing at a rate of 15% per month. How long will it take for the beehive to be at half of its current size? Round your answer to the nearest hundredth.

Exercise 15.10

If the population size of the world is increasing at a rate of 0.5% per year, how long does it take until the world population doubles in size? Round your answer to the nearest tenth.