

# Trigonometric identities

## Lesson #21

### MAT 1375 Precalculus

New York City College of Technology CUNY



# Reciprocal identities and quotient identities

## Reciprocal and quotient identities

$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$

$$\csc(x) = \frac{1}{\sin(x)}$$

$$\sec(x) = \frac{1}{\cos(x)}$$

$$\cot(x) = \frac{1}{\tan(x)} = \frac{\cos(x)}{\sin(x)}$$

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Solution:

$$\sin(x) \cdot \cot(x) = \sin(x) \cdot \frac{\cos(x)}{\sin(x)} = \cos(x)$$

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Solution:

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- 2 Simplify  $\csc(x) \cdot \tan(x)$ .

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Solution:

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- 2 Simplify  $\csc(x) \cdot \tan(x)$ .

Solution:

$$\begin{aligned}\csc(x) \cdot \tan(x) &= \frac{1}{\sin(x)} \cdot \frac{\sin(x)}{\cos(x)} \\ &= \frac{1}{\cos(x)} = \sec(x)\end{aligned}$$

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Solution:

$$\sin(x) \cdot \cot(x) = \sin(x) \cdot \frac{\cos(x)}{\sin(x)} = \cos(x)$$

- 3 True or false?  $\frac{\sec(x)}{\cos(x)} = \cot(x)$

- 2 Simplify  $\csc(x) \cdot \tan(x)$ .

Solution:

$$\begin{aligned}\csc(x) \cdot \tan(x) &= \frac{1}{\sin(x)} \cdot \frac{\sin(x)}{\cos(x)} \\ &= \frac{1}{\cos(x)} = \sec(x)\end{aligned}$$

- 4 True or false?  $\frac{\sec(x)}{\csc(x)} = \tan(x)$



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- 1 Simplify  $\sin(x) \cdot \cot(x)$ .

Solution:

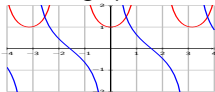
$$\sin(x) \cdot \cot(x) = \sin(x) \cdot \frac{\cos(x)}{\sin(x)} = \cos(x)$$

- 2 Simplify  $\csc(x) \cdot \tan(x)$ .

Solution:

$$\begin{aligned}\csc(x) \cdot \tan(x) &= \frac{1}{\sin(x)} \cdot \frac{\sin(x)}{\cos(x)} \\ &= \frac{1}{\cos(x)} = \sec(x)\end{aligned}$$

- 3 True or false?  $\frac{\sec(x)}{\cos(x)} = \cot(x)$   
False, their graphs are different:



$\frac{\sec(x)}{\cos(x)}$   
 $\cot(x)$

- 4 True or false?  $\frac{\sec(x)}{\csc(x)} = \tan(x)$

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Solution:

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Solution:

$$\begin{aligned}\csc(x) \cdot \tan(x) &= \frac{1}{\sin(x)} \cdot \frac{\sin(x)}{\cos(x)} \\ &= \frac{1}{\cos(x)} = \sec(x)\end{aligned}$$

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False, their graphs are different:



- 4 True or false?  $\frac{\sec(x)}{\csc(x)} = \tan(x)$   
True (same graphs!) Proof of identity:

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- 1 Simplify  $\sin(x) \cdot \cot(x)$ .

Solution:

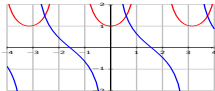
$$\sin(x) \cdot \cot(x) = \sin(x) \cdot \frac{\cos(x)}{\sin(x)} = \cos(x)$$

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Solution:

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- 3 True or false?  $\frac{\sec(x)}{\cos(x)} = \cot(x)$   
False, their graphs are different:



$\frac{\sec(x)}{\cos(x)}$   
 $\cot(x)$

- 4 True or false?  $\frac{\sec(x)}{\csc(x)} = \tan(x)$   
True (same graphs!) Proof of identity:

$$\begin{aligned}\frac{\sec(x)}{\csc(x)} &= \frac{\frac{1}{\cos(x)}}{\frac{1}{\sin(x)}} = \frac{1}{\cos(x)} \cdot \frac{\sin(x)}{1} \\ &= \frac{\sin(x)}{\cos(x)} = \tan(x)\end{aligned}$$

# Pythagorean identities

## Pythagorean identities

$$\sin^2(x) + \cos^2(x) = 1$$

$$\sec^2(x) = 1 + \tan^2(x)$$

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# Pythagorean identities

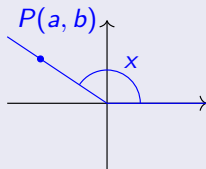
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Proof:



$$\sin(x) = b$$

$$\cos(x) = a$$

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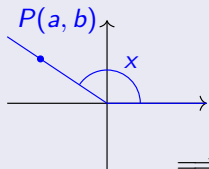
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$P$  on unit circle

$$\implies a^2 + b^2 = 1$$

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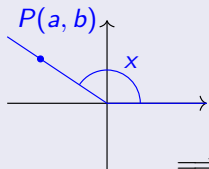
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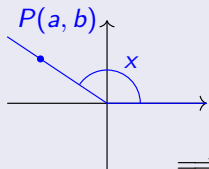
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Therefore:  $\sin^2(x) + \cos^2(x) = b^2 + a^2 = 1$

$$1 + \tan^2(x) = 1 + \left(\frac{b}{a}\right)^2 = \frac{a^2 + b^2}{a^2} = \frac{1}{a^2} = \sec^2(x)$$

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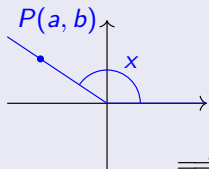
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1 Simplify:  $\frac{1 - \sin^2(x)}{\cos(x)}$

2 Simplify:  $\frac{1 - \sec^2(x)}{\tan(x)}$

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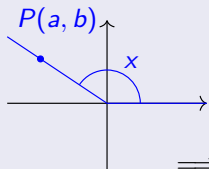
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1 Simplify:  $\frac{1 - \sin^2(x)}{\cos(x)}$

Solution:

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2 Simplify:  $\frac{1 - \sec^2(x)}{\tan(x)}$

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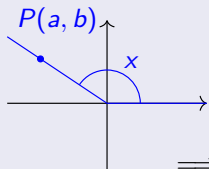
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1 Simplify:  $\frac{1 - \sin^2(x)}{\cos(x)}$

Solution:

$$\frac{1 - \sin^2(x)}{\cos(x)} = \frac{\cos^2(x)}{\cos(x)} = \cos(x)$$

2 Simplify:  $\frac{1 - \sec^2(x)}{\tan(x)}$

Solution:

$$\frac{1 - \sec^2(x)}{\tan(x)} = \frac{-\tan^2(x)}{\tan(x)} = -\tan(x)$$

3 Simplify:  $\frac{\cot^2(x) - \csc^2(x)}{\cos(x)}$

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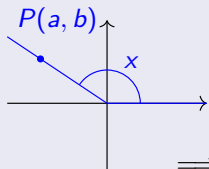
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$$1 + \cot^2(x) = 1 + \left(\frac{a}{b}\right)^2 = \frac{b^2 + a^2}{b^2} = \frac{1}{b^2} = \csc^2(x)$$

1 Simplify:  $\frac{1 - \sin^2(x)}{\cos(x)}$

Solution:

$$\frac{1 - \sin^2(x)}{\cos(x)} = \frac{\cos^2(x)}{\cos(x)} = \cos(x)$$

2 Simplify:  $\frac{1 - \sec^2(x)}{\tan(x)}$

Solution:

$$\frac{1 - \sec^2(x)}{\tan(x)} = \frac{-\tan^2(x)}{\tan(x)} = -\tan(x)$$

3 Simplify:  $\frac{\cot^2(x) - \csc^2(x)}{\cos(x)}$

Solution:

$$\begin{aligned} \frac{\cot^2(x) - \csc^2(x)}{\cos(x)} &= \frac{\cos(x)}{\cos(x)} \\ &= \frac{-1}{\cos(x)} = -\sec(x) \end{aligned}$$

## Pythagorean identities - exercises

4 Simplify:  $(\cos(x) + 1) \cdot (\cos(x) - 1)$

7 True or false?

$$\frac{\sec(x)}{\sin(x)} - \frac{\sin(x)}{\cos(x)} = \cot(x)$$

5 Simplify:  
 $(\tan(x) - \sec(x)) \cdot (\tan(x) + \sec(x))$

6 Simplify:  $(1 - \csc(x)) \cdot (1 + \csc(x))$

8 True or false?

$$\frac{\csc(x)}{\cos(x)} = \sin(x) + \frac{\sec(x)}{\cos(x)}$$

## Pythagorean identities - exercises

4 Simplify:  $(\cos(x) + 1) \cdot (\cos(x) - 1)$

Solution:

Recall:  $(a + b)(a - b) = a^2 - b^2$

7 True or false?

$$\frac{\sec(x)}{\sin(x)} - \frac{\sin(x)}{\cos(x)} = \cot(x)$$

5 Simplify:

$$(\tan(x) - \sec(x)) \cdot (\tan(x) + \sec(x))$$

6 Simplify:  $(1 - \csc(x)) \cdot (1 + \csc(x))$

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Solution:

Recall:  $(a + b)(a - b) = a^2 - b^2$

$$\begin{aligned} \implies & (\cos(x) + 1) \cdot (\cos(x) - 1) \\ & = \cos^2(x) - 1 = -\sin^2(x) \end{aligned}$$

5 Simplify:  
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$$\begin{aligned} \implies & (\cos(x) + 1) \cdot (\cos(x) - 1) \\ & = \cos^2(x) - 1 = -\sin^2(x) \end{aligned}$$

5 Simplify:

$$(\tan(x) - \sec(x)) \cdot (\tan(x) + \sec(x))$$

Solution:

$$\begin{aligned} \implies & (\tan(x) - \sec(x)) \cdot (\tan(x) + \sec(x)) \\ & = \tan^2(x) - \sec^2(x) = -1 \end{aligned}$$

6 Simplify:  $(1 - \csc(x)) \cdot (1 + \csc(x))$

7 True or false?

$$\frac{\sec(x)}{\sin(x)} - \frac{\sin(x)}{\cos(x)} = \cot(x)$$

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Solution:

Recall:  $(a + b)(a - b) = a^2 - b^2$

$$\begin{aligned}\implies & (\cos(x) + 1) \cdot (\cos(x) - 1) \\ & = \cos^2(x) - 1 = -\sin^2(x)\end{aligned}$$

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$$(\tan(x) - \sec(x)) \cdot (\tan(x) + \sec(x))$$

Solution:

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6 Simplify:  $(1 - \csc(x)) \cdot (1 + \csc(x))$

Solution:

$$\begin{aligned}\implies & (1 - \csc(x)) \cdot (1 + \csc(x)) \\ & = 1 - \csc^2(x) = -\cot^2(x)\end{aligned}$$

7 True or false?

$$\frac{\sec(x)}{\sin(x)} - \frac{\sin(x)}{\cos(x)} = \cot(x)$$

8 True or false?

$$\frac{\csc(x)}{\cos(x)} = \sin(x) + \frac{\sec(x)}{\cos(x)}$$

## Pythagorean identities - exercises

4 Simplify:  $(\cos(x) + 1) \cdot (\cos(x) - 1)$

Solution:

Recall:  $(a + b)(a - b) = a^2 - b^2$

$$\begin{aligned}\implies & (\cos(x) + 1) \cdot (\cos(x) - 1) \\ & = \cos^2(x) - 1 = -\sin^2(x)\end{aligned}$$

5 Simplify:

$$(\tan(x) - \sec(x)) \cdot (\tan(x) + \sec(x))$$

Solution:

$$\begin{aligned}\implies & (\tan(x) - \sec(x)) \cdot (\tan(x) + \sec(x)) \\ & = \tan^2(x) - \sec^2(x) = -1\end{aligned}$$

6 Simplify:  $(1 - \csc(x)) \cdot (1 + \csc(x))$

Solution:

$$\begin{aligned}\implies & (1 - \csc(x)) \cdot (1 + \csc(x)) \\ & = 1 - \csc^2(x) = -\cot^2(x)\end{aligned}$$

7 True or false?

$$\frac{\sec(x)}{\sin(x)} - \frac{\sin(x)}{\cos(x)} = \cot(x)$$

True. Both graphs coincide:



Proof of the identity:

8 True or false?

$$\frac{\csc(x)}{\cos(x)} = \sin(x) + \frac{\sec(x)}{\cos(x)}$$

# Pythagorean identities - exercises

4 Simplify:  $(\cos(x) + 1) \cdot (\cos(x) - 1)$

Solution:

Recall:  $(a + b)(a - b) = a^2 - b^2$

$$\begin{aligned} \implies & (\cos(x) + 1) \cdot (\cos(x) - 1) \\ & = \cos^2(x) - 1 = -\sin^2(x) \end{aligned}$$

5 Simplify:

$$(\tan(x) - \sec(x)) \cdot (\tan(x) + \sec(x))$$

Solution:

$$\begin{aligned} \implies & (\tan(x) - \sec(x)) \cdot (\tan(x) + \sec(x)) \\ & = \tan^2(x) - \sec^2(x) = -1 \end{aligned}$$

6 Simplify:  $(1 - \csc(x)) \cdot (1 + \csc(x))$

Solution:

$$\begin{aligned} \implies & (1 - \csc(x)) \cdot (1 + \csc(x)) \\ & = 1 - \csc^2(x) = -\cot^2(x) \end{aligned}$$

7 True or false?

$$\frac{\sec(x)}{\sin(x)} - \frac{\sin(x)}{\cos(x)} = \cot(x)$$

True. Both graphs coincide:



Proof of the identity:

$$\begin{aligned} \frac{\sec(x)}{\sin(x)} - \frac{\sin(x)}{\cos(x)} &= \frac{\sec(x)\cos(x) - \sin(x)\sin(x)}{\sin(x)\cos(x)} \\ &= \frac{1 - \sin^2(x)}{\sin(x)\cos(x)} = \frac{\cos^2(x)}{\sin(x)\cos(x)} \\ &= \frac{\cos(x)}{\sin(x)} = \cot(x) \end{aligned}$$

8 True or false?

$$\frac{\csc(x)}{\cos(x)} = \sin(x) + \frac{\sec(x)}{\cos(x)}$$

# Pythagorean identities - exercises

- 4 Simplify:  $(\cos(x) + 1) \cdot (\cos(x) - 1)$

Solution:

Recall:  $(a + b)(a - b) = a^2 - b^2$

$$\begin{aligned} \implies & (\cos(x) + 1) \cdot (\cos(x) - 1) \\ & = \cos^2(x) - 1 = -\sin^2(x) \end{aligned}$$

- 5 Simplify:

$$(\tan(x) - \sec(x)) \cdot (\tan(x) + \sec(x))$$

Solution:

$$\begin{aligned} \implies & (\tan(x) - \sec(x)) \cdot (\tan(x) + \sec(x)) \\ & = \tan^2(x) - \sec^2(x) = -1 \end{aligned}$$

- 6 Simplify:  $(1 - \csc(x)) \cdot (1 + \csc(x))$

Solution:

$$\begin{aligned} \implies & (1 - \csc(x)) \cdot (1 + \csc(x)) \\ & = 1 - \csc^2(x) = -\cot^2(x) \end{aligned}$$

- 7 True or false?

$$\frac{\sec(x)}{\sin(x)} - \frac{\sin(x)}{\cos(x)} = \cot(x)$$

True. Both graphs coincide:



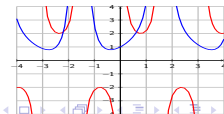
Proof of the identity:

$$\begin{aligned} \frac{\sec(x)}{\sin(x)} - \frac{\sin(x)}{\cos(x)} &= \frac{\sec(x)\cos(x) - \sin(x)\sin(x)}{\sin(x)\cos(x)} \\ &= \frac{1 - \sin^2(x)}{\sin(x)\cos(x)} = \frac{\cos^2(x)}{\sin(x)\cos(x)} \\ &= \frac{\cos(x)}{\sin(x)} = \cot(x) \end{aligned}$$

- 8 True or false?

$$\frac{\csc(x)}{\cos(x)} = \sin(x) + \frac{\sec(x)}{\cos(x)}$$

False, since graphs do not coincide:



## Even, odd trigonometric functions

$$\sin(-x) = -\sin(x) \quad \text{odd}$$

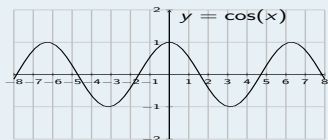
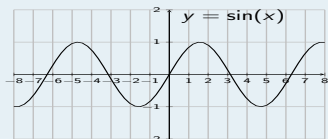
$$\cos(-x) = \cos(x) \quad \text{even}$$

$$\tan(-x) = -\tan(x) \quad \text{odd}$$

$$\csc(-x) = -\csc(x) \quad \text{odd}$$

$$\sec(-x) = \sec(x) \quad \text{even}$$

$$\cot(-x) = -\cot(x) \quad \text{odd}$$



# Even and odd trigonometric functions; beyond the unit circle

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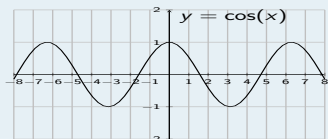
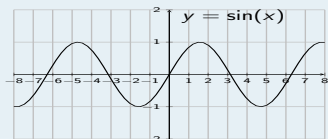
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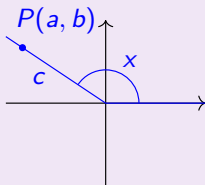
$$\cot(-x) = -\cot(x) \quad \text{odd}$$



## General point $P$ on the terminal side

$P(a, b)$  is a point on the terminal side of  $x$

$$a^2 + b^2 = c^2 \quad \implies \quad c = \sqrt{a^2 + b^2}$$



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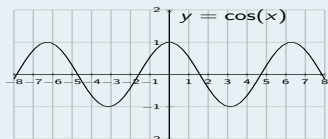
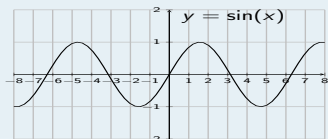
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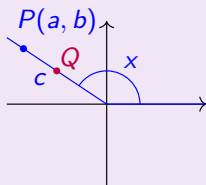
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$Q\left(\frac{a}{c}, \frac{b}{c}\right)$   
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# Even and odd trigonometric functions; beyond the unit circle

## Even, odd trigonometric functions

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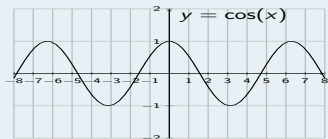
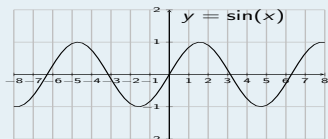
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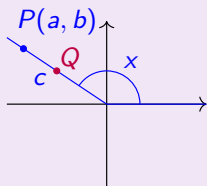
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$Q\left(\frac{a}{c}, \frac{b}{c}\right)$   
is on the  
unit circle

$$\sin(x) = \frac{b}{c} \quad \csc(x) = \frac{c}{b}$$

$$\cos(x) = \frac{a}{c} \quad \sec(x) = \frac{c}{a}$$

$$\tan(x) = \frac{b}{a} \quad \cot(x) = \frac{a}{b}$$



## Adding $\pm\frac{\pi}{2}$ or $\pm\pi$ ; addition, subtraction, half-, double-angle formulas

### Identities from adding $\pm\frac{\pi}{2}$ or $\pm\pi$

$$\sin(\pi - x) = \sin(x)$$

$$\cos(\pi - x) = -\cos(x)$$

$$\sin\left(x + \frac{\pi}{2}\right) = \cos x$$

$$\cos\left(x + \frac{\pi}{2}\right) = -\sin(x)$$

$$\sin\left(x - \frac{\pi}{2}\right) = -\cos x$$

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$$\cos\left(x - \frac{\pi}{2}\right) = \sin(x)$$

### Sum and difference formulas

$$\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$

$$\sin(\alpha - \beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$$

$$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

$$\cos(\alpha - \beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$$

$$\tan(\alpha + \beta) = \frac{\tan(\alpha) + \tan(\beta)}{1 - \tan(\alpha)\tan(\beta)}$$

$$\tan(\alpha - \beta) = \frac{\tan(\alpha) - \tan(\beta)}{1 + \tan(\alpha)\tan(\beta)}$$

### Half-angle formulas

$$\sin\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1 - \cos(\alpha)}{2}}$$

$$\cos\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1 + \cos(\alpha)}{2}}$$

$$\tan\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1 - \cos(\alpha)}{1 + \cos(\alpha)}}$$

### Double-angle formulas

$$\sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$$

$$\begin{aligned}\cos(2\alpha) &= \cos^2(\alpha) - \sin^2(\alpha) \\ &= 1 - 2\sin^2(\alpha) = 2\cos^2(\alpha) - 1\end{aligned}$$

$$\tan(2\alpha) = \frac{2\tan(\alpha)}{1 - \tan^2(\alpha)}$$

