Trigonometric equations

Lesson #20

MAT 1375 Precalculus

New York City College of Technology CUNY

Solving equations $\cos(x) = c$

Example

Solve for x: $\cos(x) = \frac{1}{2}$

Solving equations $\cos(x) = c$

Example

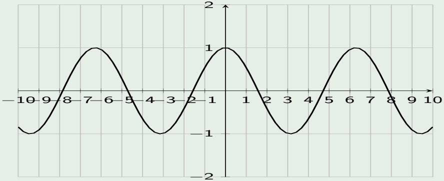
Solve for x: $\cos(x) = \frac{1}{2}$

One solution comes from

$$x = \arccos(\tfrac{1}{2}) = \frac{\pi}{3}$$

Example

Solve for x: $\cos(x) = \frac{1}{2}$

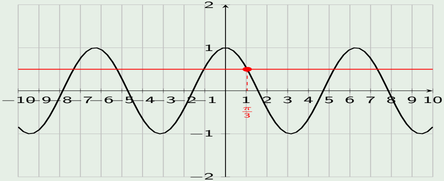


One solution comes from

$$x = \arccos\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

Example

Solve for x: $\cos(x) = \frac{1}{2}$

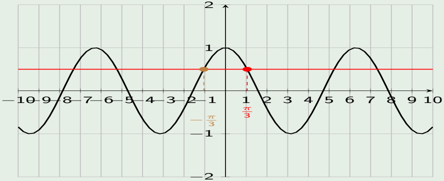


One solution comes from

$$x = \arccos(\tfrac{1}{2}) = \frac{\pi}{3}$$

Example

Solve for x: $\cos(x) = \frac{1}{2}$



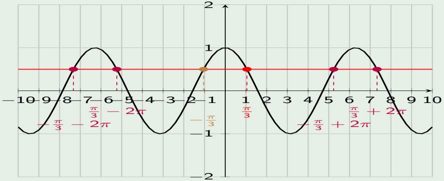
One solution comes from

$$x = \arccos(\tfrac{1}{2}) = \frac{\pi}{3}$$

A second solution is: $x = -\frac{\pi}{3}$.

Example

Solve for x: $\cos(x) = \frac{1}{2}$



One solution comes from

$$x = \arccos\left(\tfrac{1}{2}\right) = \frac{\pi}{3}$$

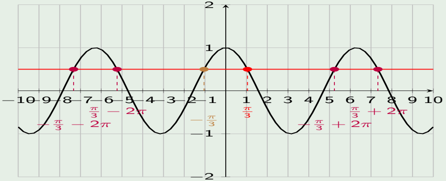
A second solution is: $x = -\frac{\pi}{3}$.

All other solutions come from adding or subtracting 2πs. The general solution is:

$$\begin{cases} x = \frac{\pi}{3} + n \cdot 2\pi, & \text{for } n \in \mathbb{Z} \\ x = -\frac{\pi}{3} + n \cdot 2\pi, & \text{for } n \in \mathbb{Z} \end{cases}$$

Example

Solve for x: $\cos(x) = \frac{1}{2}$



One solution comes from

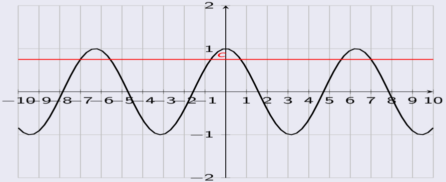
$$x = \arccos(\tfrac{1}{2}) = \frac{\pi}{3}$$

A second solution is: $x = -\frac{\pi}{3}$.

All other solutions come from adding or subtracting 2πs. The general solution is:

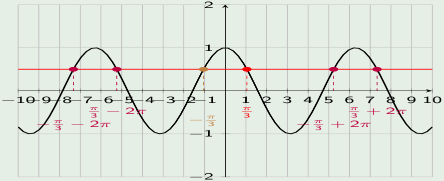
$$\begin{cases} x = \ \ \frac{\pi}{3} + n \cdot 2\pi, & \text{for } n \in \mathbb{Z} \\ x = -\frac{\pi}{3} + n \cdot 2\pi, & \text{for } n \in \mathbb{Z} \end{cases}$$

Solving $\cos(x) = c$



Example

Solve for x: $\cos(x) = \frac{1}{2}$



One solution comes from

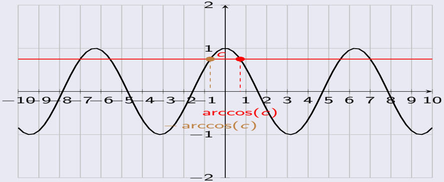
$$x = \arccos\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

A second solution is: $x = -\frac{\pi}{3}$.

All other solutions come from adding or subtracting 2πs. The general solution is:

$$\begin{cases} x = \frac{\pi}{3} + n \cdot 2\pi, & \text{for } n \in \mathbb{Z} \\ x = -\frac{\pi}{3} + n \cdot 2\pi, & \text{for } n \in \mathbb{Z} \end{cases}$$

Solving $\cos(x) = c$



One solution comes from

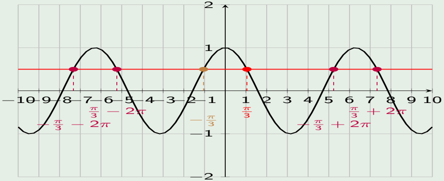
$$x = \arccos(c)$$

A second solution is: $x = -\arccos(c)$.

Solving equations $\cos(x) = c$

Example

Solve for x: $\cos(x) = \frac{1}{2}$



One solution comes from

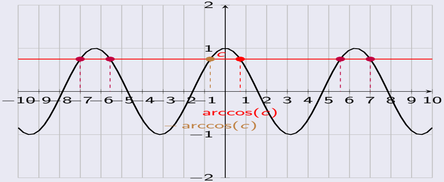
$$x = \arccos\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

A second solution is: $x = -\frac{\pi}{3}$.

All other solutions come from adding or subtracting 2πs. The general solution is:

$$\begin{cases} x = \frac{\pi}{3} + n \cdot 2\pi, & \text{for } n \in \mathbb{Z} \\ x = -\frac{\pi}{3} + n \cdot 2\pi, & \text{for } n \in \mathbb{Z} \end{cases}$$

Solving $\cos(x) = c$



One solution comes from

$$x = \arccos(c)$$

A second solution is: $x = -\arccos(c)$.

All other solutions come from adding or subtracting multiple of 2πs:

$$\boxed{\begin{array}{ll} x = \arccos(c) + n \cdot 2\pi, & \text{for } n \in \mathbb{Z} \\ x = -\arccos(c) + n \cdot 2\pi, & \text{for } n \in \mathbb{Z} \end{array}}$$

Solve $\cos(x) = c$

Solution:

$$\begin{aligned} x &= \arccos(c) + n \cdot 2\pi, && \text{for } n \in \mathbb{Z} \\ x &= -\arccos(c) + n \cdot 2\pi, && \text{for } n \in \mathbb{Z} \end{aligned}$$

1. Solve for x: $\cos(x) = \frac{\sqrt{2}}{2}$

Solve $\cos(x) = c$

Solution:

$$\begin{aligned} x &= \arccos(c) + n \cdot 2\pi, & \text{for } n \in \mathbb{Z} \\ x &= -\arccos(c) + n \cdot 2\pi, & \text{for } n \in \mathbb{Z} \end{aligned}$$

1. Solve for x: $\cos(x) = \frac{\sqrt{2}}{2}$

   Solution:

   $$\arccos\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$$

   $$-\arccos\left(\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$$

Solve $\cos(x) = c$

Solution:

$$x = \arccos(c) + n \cdot 2\pi, \quad \text{for } n \in \mathbb{Z}$$
$$x = -\arccos(c) + n \cdot 2\pi, \quad \text{for } n \in \mathbb{Z}$$

1. Solve for x: $\cos(x) = \frac{\sqrt{2}}{2}$

   Solution:

   $\arccos(\frac{\sqrt{2}}{2}) = \frac{\pi}{4}$

   $-\arccos(\frac{\sqrt{2}}{2}) = -\frac{\pi}{4}$

   $\Rightarrow x = \frac{\pi}{4} + n2\pi$, for $n \in \mathbb{Z}$

   or $x = -\frac{\pi}{4} + n2\pi$, for $n \in \mathbb{Z}$

Solving equations with cos - exercises

Solve $\cos(x) = c$

Solution:

$$\begin{aligned} x &= \arccos(c) + n \cdot 2\pi, && \text{for } n \in \mathbb{Z} \\ x &= -\arccos(c) + n \cdot 2\pi, && \text{for } n \in \mathbb{Z} \end{aligned}$$

1. Solve for x: $\cos(x) = \frac{\sqrt{2}}{2}$
   Solution:
   $\arccos(\frac{\sqrt{2}}{2}) = \frac{\pi}{4}$
   $-\arccos(\frac{\sqrt{2}}{2}) = -\frac{\pi}{4}$
   $\Rightarrow x = \frac{\pi}{4} + n2\pi$, for $n \in \mathbb{Z}$
   or $x = -\frac{\pi}{4} + n2\pi$, for $n \in \mathbb{Z}$
2. Solve for x: $\cos(x) = \frac{\sqrt{3}}{2}$
3. Solve for x: $2\cos(x) + 1 = 0$
4. Solve for x: $\cos^2(x) + \cos(x) = 0$

Solve $\cos(x) = c$

Solution:

$$\begin{aligned} x &= \arccos(c) + n \cdot 2\pi, \quad \text{for } n \in \mathbb{Z} \\ x &= -\arccos(c) + n \cdot 2\pi, \quad \text{for } n \in \mathbb{Z} \end{aligned}$$

1. Solve for x: $\cos(x) = \frac{\sqrt{2}}{2}$
   Solution:
   $\arccos(\frac{\sqrt{2}}{2}) = \frac{\pi}{4}$
   $-\arccos(\frac{\sqrt{2}}{2}) = -\frac{\pi}{4}$
   $\Rightarrow x = \frac{\pi}{4} + n2\pi$, for $n \in \mathbb{Z}$
   or $x = -\frac{\pi}{4} + n2\pi$, for $n \in \mathbb{Z}$
2. Solve for x: $\cos(x) = \frac{\sqrt{3}}{2}$
   Solution:
   $\arccos(\frac{\sqrt{3}}{2}) = \frac{\pi}{6}$
   $-\arccos(\frac{\sqrt{3}}{2}) = -\frac{\pi}{6}$
   $\Rightarrow x = \frac{\pi}{6} + 2n\pi$, for $n \in \mathbb{Z}$
   or $x = -\frac{\pi}{6} + 2n\pi$, for $n \in \mathbb{Z}$
3. Solve for x: $2\cos(x) + 1 = 0$
4. Solve for x: $\cos^2(x) + \cos(x) = 0$

Solve $\cos(x) = c$

Solution:

$$\begin{array}{ll} x = \arccos(c) + n\cdot 2\pi, & \text{for } n\in\mathbb{Z} \\ x = -\arccos(c) + n\cdot 2\pi, & \text{for } n\in\mathbb{Z} \end{array}$$

1. Solve for x: $\cos(x) = \frac{\sqrt{2}}{2}$

   Solution:

   $\arccos(\frac{\sqrt{2}}{2}) = \frac{\pi}{4}$

   $-\arccos(\frac{\sqrt{2}}{2}) = -\frac{\pi}{4}$

   $\Rightarrow x = \frac{\pi}{4} + n2\pi$, for $n \in \mathbb{Z}$

   or $x = -\frac{\pi}{4} + n2\pi$, for $n \in \mathbb{Z}$

2. Solve for x: $\cos(x) = \frac{\sqrt{3}}{2}$

   Solution:

   $\arccos(\frac{\sqrt{3}}{2}) = \frac{\pi}{6}$

   $-\arccos(\frac{\sqrt{3}}{2}) = -\frac{\pi}{6}$

   $\Rightarrow x = \frac{\pi}{6} + 2n\pi$, for $n \in \mathbb{Z}$

   or $x = -\frac{\pi}{6} + 2n\pi$, for $n \in \mathbb{Z}$

3. Solve for x: $2\cos(x) + 1 = 0$

   Solution:

   $\Rightarrow 2\cos(x) = -1 \Rightarrow \cos(x) = -\frac{1}{2}$

   $\arccos(-\frac{1}{2}) = 120^\circ = \frac{2\pi}{3}$

   $-\arccos(-\frac{1}{2}) = -\frac{2\pi}{3}$

   $\Rightarrow x = \frac{2\pi}{3} + 2n\pi$, for $n \in \mathbb{Z}$

   or $x = -\frac{2\pi}{3} + 2n\pi$, for $n \in \mathbb{Z}$

4. Solve for x: $\cos^2(x) + \cos(x) = 0$

Solve $\cos(x) = c$

Solution:

$$\begin{aligned} x &= \arccos(c) + n \cdot 2\pi, \quad \text{for } n \in \mathbb{Z} \\ x &= -\arccos(c) + n \cdot 2\pi, \quad \text{for } n \in \mathbb{Z} \end{aligned}$$

1. Solve for x: $\cos(x) = \frac{\sqrt{2}}{2}$
   Solution:
   $\arccos(\frac{\sqrt{2}}{2}) = \frac{\pi}{4}$
   $-\arccos(\frac{\sqrt{2}}{2}) = -\frac{\pi}{4}$
   $\Rightarrow x = \frac{\pi}{4} + n2\pi$, for $n \in \mathbb{Z}$
   or $x = -\frac{\pi}{4} + n2\pi$, for $n \in \mathbb{Z}$
2. Solve for x: $\cos(x) = \frac{\sqrt{3}}{2}$
   Solution:
   $\arccos(\frac{\sqrt{3}}{2}) = \frac{\pi}{6}$
   $-\arccos(\frac{\sqrt{3}}{2}) = -\frac{\pi}{6}$
   $\Rightarrow x = \frac{\pi}{6} + 2n\pi$, for $n \in \mathbb{Z}$
   or $x = -\frac{\pi}{6} + 2n\pi$, for $n \in \mathbb{Z}$
3. Solve for x: $2\cos(x) + 1 = 0$
   Solution:
   $\Rightarrow 2\cos(x) = -1 \Rightarrow \cos(x) = -\frac{1}{2}$
   $\arccos(-\frac{1}{2}) = 120^\circ = \frac{2\pi}{3}$
   $-\arccos(-\frac{1}{2}) = -\frac{2\pi}{3}$
   $\Rightarrow x = \frac{2\pi}{3} + 2n\pi$, for $n \in \mathbb{Z}$
   or $x = -\frac{2\pi}{3} + 2n\pi$, for $n \in \mathbb{Z}$
4. Solve for x: $\cos^2(x) + \cos(x) = 0$
   Solution:
   $\Rightarrow \cos(x) \cdot (\cos(x) + 1) = 0$
   $\Rightarrow \cos(x) = 0$ or $\cos(x) = -1$
   $\pm\arccos(0) = \pm\frac{\pi}{2}$
   $\pm\arccos(-1) = \pm\pi$
   $\Rightarrow x = \frac{\pi}{2} + 2n\pi$, for $n \in \mathbb{Z}$
   or $x = -\frac{\pi}{2} + 2n\pi$, for $n \in \mathbb{Z}$
   or $x = \pi + 2n\pi$, for $n \in \mathbb{Z}$
   or $x = -\pi + 2n\pi$, for $n \in \mathbb{Z}$

Solve $\cos(x) = c$

Solution:

$$x = \arccos(c) + n \cdot 2\pi, \quad \text{for } n \in \mathbb{Z}$$
$$x = -\arccos(c) + n \cdot 2\pi, \quad \text{for } n \in \mathbb{Z}$$

5. Solve for x:
$$\cos^2(x) - 2\cos(x) - 3 = 0$$

6. Solve for x: $4\cos^2(x) - 3 = 0$

Solve $\cos(x) = c$

Solution:

$$x = \arccos(c) + n \cdot 2\pi, \quad \text{for } n \in \mathbb{Z}$$
$$x = -\arccos(c) + n \cdot 2\pi, \quad \text{for } n \in \mathbb{Z}$$

5. Solve for x:
   $$\cos^2(x) - 2\cos(x) - 3 = 0$$
   Solution:
   Substitute $u = \cos(x)$
   $\Rightarrow u^2 - 2u - 3 = 0$
   $\Rightarrow$

6. Solve for x: $4\cos^2(x) - 3 = 0$

Solve $\cos(x) = c$

Solution:

$$x = \arccos(c) + n \cdot 2\pi, \quad \text{for } n \in \mathbb{Z}$$
$$x = -\arccos(c) + n \cdot 2\pi, \quad \text{for } n \in \mathbb{Z}$$

5. Solve for x:

   $$\cos^2(x) - 2\cos(x) - 3 = 0$$

   Solution:
   Substitute $u = \cos(x)$
   $\Rightarrow u^2 - 2u - 3 = 0$
   $\Rightarrow (u-3)(u+1) = 0$
   $\Rightarrow u = 3$ or $u = -1$
   $\Rightarrow \cos(x) = 3 \quad$ or $\quad \cos(x) = -1$

6. Solve for x: $\quad 4\cos^2(x) - 3 = 0$

Solve $\cos(x) = c$

Solution:

$$\begin{aligned} x &= \arccos(c) + n \cdot 2\pi, \quad \text{for } n \in \mathbb{Z} \\ x &= -\arccos(c) + n \cdot 2\pi, \quad \text{for } n \in \mathbb{Z} \end{aligned}$$

5. Solve for x:

   $$\cos^2(x) - 2\cos(x) - 3 = 0$$

   Solution:

   Substitute $u = \cos(x)$

   $\Rightarrow u^2 - 2u - 3 = 0$

   $\Rightarrow (u-3)(u+1) = 0$

   $\Rightarrow u = 3$ or $u = -1$

   $\Rightarrow \cos(x) = 3 \quad$ or $\quad \cos(x) = -1$

   Note: $\cos(x) = 3$ has no solutions, since the range of cos is $[-1, 1]$.

   $$\arccos(-1) = \pi$$
   $$-\arccos(-1) = -\pi$$

   $\Rightarrow x = \pi + 2n\pi$, for $n \in \mathbb{Z}$

   or $x = -\pi + 2n\pi$, for $n \in \mathbb{Z}$

6. Solve for x: $\quad 4\cos^2(x) - 3 = 0$

Solve $\cos(x) = c$

Solution:

$$x = \arccos(c) + n \cdot 2\pi, \quad \text{for } n \in \mathbb{Z}$$
$$x = -\arccos(c) + n \cdot 2\pi, \quad \text{for } n \in \mathbb{Z}$$

5. Solve for x:

$$\cos^2(x) - 2\cos(x) - 3 = 0$$

Solution:
Substitute $u = \cos(x)$
$\Rightarrow u^2 - 2u - 3 = 0$
$\Rightarrow (u-3)(u+1) = 0$
$\Rightarrow u = 3$ or $u = -1$
$\Rightarrow \cos(x) = 3$ or $\cos(x) = -1$
Note: $\cos(x) = 3$ has no solutions, since the range of cos is $[-1, 1]$.

$$\arccos(-1) = \pi$$
$$-\arccos(-1) = -\pi$$

$\Rightarrow x = \pi + 2n\pi$, for $n \in \mathbb{Z}$
or $x = -\pi + 2n\pi$, for $n \in \mathbb{Z}$

6. Solve for x: $4\cos^2(x) - 3 = 0$
Solution:
Substitute $u = \cos(x)$
$\Rightarrow 4u^2 - 3 = 0 \quad \Rightarrow 4u^2 = 3$
$\Rightarrow u^2 = \frac{3}{4} \quad \Rightarrow u = \pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{2}$
$\Rightarrow \cos(x) = \frac{\sqrt{3}}{2}$ or $\cos(x) = -\frac{\sqrt{3}}{2}$

Solve $\cos(x) = c$

Solution:

$$\begin{aligned} x &= \arccos(c) + n \cdot 2\pi, \quad \text{for } n \in \mathbb{Z} \\ x &= -\arccos(c) + n \cdot 2\pi, \quad \text{for } n \in \mathbb{Z} \end{aligned}$$

5. Solve for x:

$$\cos^2(x) - 2\cos(x) - 3 = 0$$

Solution:

Substitute $u = \cos(x)$

$\Rightarrow u^2 - 2u - 3 = 0$

$\Rightarrow (u-3)(u+1) = 0$

$\Rightarrow u = 3$ or $u = -1$

$\Rightarrow \cos(x) = 3$ or $\cos(x) = -1$

Note: $\cos(x) = 3$ has no solutions, since the range of cos is $[-1, 1]$.

$$\arccos(-1) = \pi$$

$$-\arccos(-1) = -\pi$$

$\Rightarrow x = \pi + 2n\pi$, for $n \in \mathbb{Z}$

or $x = -\pi + 2n\pi$, for $n \in \mathbb{Z}$

6. Solve for x: $4\cos^2(x) - 3 = 0$

Solution:

Substitute $u = \cos(x)$

$\Rightarrow 4u^2 - 3 = 0 \quad \Rightarrow 4u^2 = 3$

$\Rightarrow u^2 = \frac{3}{4} \quad \Rightarrow u = \pm\sqrt{\frac{3}{4}} = \pm\frac{\sqrt{3}}{2}$

$\Rightarrow \cos(x) = \frac{\sqrt{3}}{2}$ or $\cos(x) = -\frac{\sqrt{3}}{2}$

$$\arccos\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$$

$$-\arccos\left(\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{6}$$

$$\arccos\left(-\frac{\sqrt{3}}{2}\right) = 150^\circ = \frac{5\pi}{6}$$

$$-\arccos\left(-\frac{\sqrt{3}}{2}\right) = -\frac{5\pi}{6}$$

$\Rightarrow x = \frac{\pi}{6} + 2n\pi$, for $n \in \mathbb{Z}$

or $x = -\frac{\pi}{6} + 2n\pi$, for $n \in \mathbb{Z}$

or $x = \frac{5\pi}{6} + 2n\pi$, for $n \in \mathbb{Z}$

or $x = -\frac{5\pi}{6} + 2n\pi$, for $n \in \mathbb{Z}$

Solving equations $\sin(x) = c$

Example

Solve for x: $\sin(x) = \frac{1}{2}$

Solving equations $\sin(x) = c$

Example

Solve for x: $\sin(x) = \frac{1}{2}$

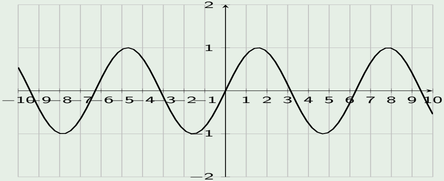
One solution comes from

$$x = \arcsin(\tfrac{1}{2}) = \frac{\pi}{6}$$

Solving equations $\sin(x) = c$

Example

Solve for x: $\sin(x) = \frac{1}{2}$

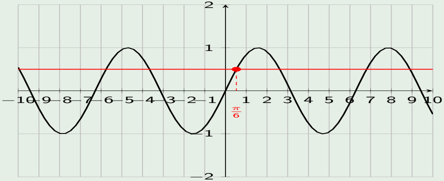


One solution comes from

$$x = \arcsin(\tfrac{1}{2}) = \frac{\pi}{6}$$

Example

Solve for x: $\sin(x) = \frac{1}{2}$

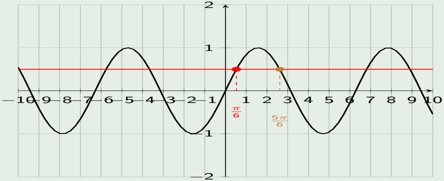


One solution comes from

$$x = \arcsin(\tfrac{1}{2}) = \frac{\pi}{6}$$

Example

Solve for x: $\sin(x) = \frac{1}{2}$



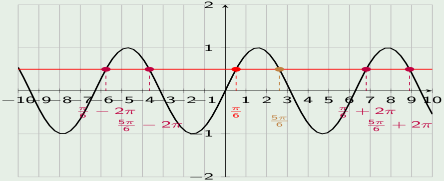
One solution comes from

$$x = \arcsin(\tfrac{1}{2}) = \frac{\pi}{6}$$

A second solution is: $x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$.

Example

Solve for x: $\sin(x) = \frac{1}{2}$



One solution comes from

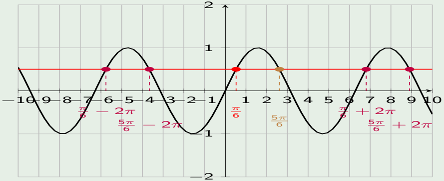
$$x = \arcsin(\tfrac{1}{2}) = \frac{\pi}{6}$$

A second solution is: $x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$.
All other solutions come from adding or subtracting 2πs. The general solution is:

$$\begin{cases} x = \frac{\pi}{6} + n \cdot 2\pi, & \text{for } n \in \mathbb{Z} \\ x = \frac{5\pi}{6} + n \cdot 2\pi, & \text{for } n \in \mathbb{Z} \end{cases}$$

Example

Solve for x: $\sin(x) = \frac{1}{2}$



One solution comes from

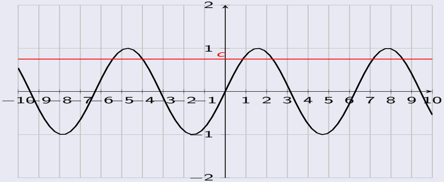
$$x = \arcsin(\tfrac{1}{2}) = \frac{\pi}{6}$$

A second solution is: $x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$.

All other solutions come from adding or subtracting 2πs. The general solution is:

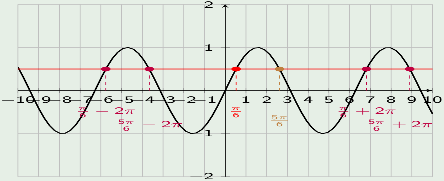
$$\begin{cases} x = \frac{\pi}{6} + n \cdot 2\pi, & \text{for } n \in \mathbb{Z} \\ x = \frac{5\pi}{6} + n \cdot 2\pi, & \text{for } n \in \mathbb{Z} \end{cases}$$

Solving $\sin(x) = c$



Example

Solve for x: $\sin(x) = \frac{1}{2}$



One solution comes from

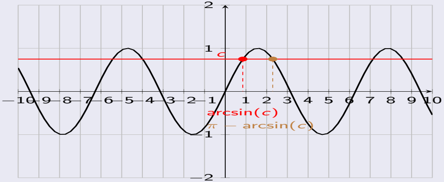
$$x = \arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6}$$

A second solution is: $x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$.

All other solutions come from adding or subtracting 2πs. The general solution is:

$$\begin{cases} x = \frac{\pi}{6} + n \cdot 2\pi, & \text{for } n \in \mathbb{Z} \\ x = \frac{5\pi}{6} + n \cdot 2\pi, & \text{for } n \in \mathbb{Z} \end{cases}$$

Solving $\sin(x) = c$



One solution comes from

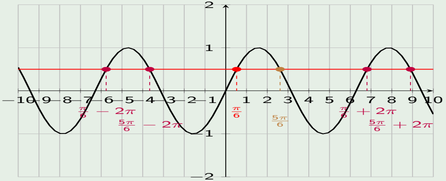
$$x = \arcsin(c)$$

A second solution is: $x = \pi - \arcsin(c)$.

Solving equations $\sin(x) = c$

Example

Solve for x: $\sin(x) = \frac{1}{2}$



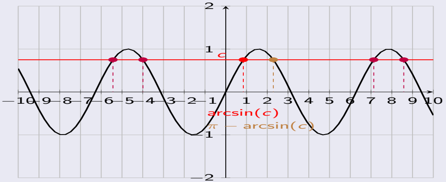
One solution comes from

$$x = \arcsin\left(\tfrac{1}{2}\right) = \frac{\pi}{6}$$

A second solution is: $x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$.
All other solutions come from adding or subtracting 2πs. The general solution is:

$$\begin{cases} x = \frac{\pi}{6} + n \cdot 2\pi, & \text{for } n \in \mathbb{Z} \\ x = \frac{5\pi}{6} + n \cdot 2\pi, & \text{for } n \in \mathbb{Z} \end{cases}$$

Solving $\sin(x) = c$



One solution comes from

$$x = \arcsin(c)$$

A second solution is: $x = \pi - \arcsin(c)$.

All other solutions come from adding or subtracting multiple of 2πs:

$$\begin{array}{|ll|} \hline x = \arcsin(c) + n \cdot 2\pi, & \text{for } n \in \mathbb{Z} \\ x = \pi - \arcsin(c) + n \cdot 2\pi, & \text{for } n \in \mathbb{Z} \\ \hline \end{array}$$

Solve $\sin(x) = c$

Solution:

$$\begin{aligned} x &= \arcsin(c) + n \cdot 2\pi, \quad \text{for } n \in \mathbb{Z} \\ x &= \pi - \arcsin(c) + n \cdot 2\pi, \quad \text{for } n \in \mathbb{Z} \end{aligned}$$

1. Solve for x: $\sin(x) = -\frac{\sqrt{2}}{2}$

Solve $\sin(x) = c$

Solution:

$$\begin{aligned} x &= \arcsin(c) + n \cdot 2\pi, \quad \text{for } n \in \mathbb{Z} \\ x &= \pi - \arcsin(c) + n \cdot 2\pi, \quad \text{for } n \in \mathbb{Z} \end{aligned}$$

1. Solve for x: $\sin(x) = -\frac{\sqrt{2}}{2}$
   Solution:
   $$\arcsin(-\tfrac{\sqrt{2}}{2}) = -\tfrac{\pi}{4}$$
   $$\pi - \arcsin(\tfrac{\sqrt{2}}{2}) = \pi - (-\tfrac{\pi}{4}) = \pi + \tfrac{\pi}{4} = \tfrac{5\pi}{4}$$

Solve $\sin(x) = c$

Solution:

$$\begin{aligned} x &= \quad\;\; \arcsin(c) + n \cdot 2\pi, \quad \text{for } n \in \mathbb{Z} \\ x &= \pi - \arcsin(c) + n \cdot 2\pi, \quad \text{for } n \in \mathbb{Z} \end{aligned}$$

1. Solve for x: $\sin(x) = -\frac{\sqrt{2}}{2}$
   Solution:
   $\arcsin(-\frac{\sqrt{2}}{2}) = -\frac{\pi}{4}$
   $\pi - \arcsin(\frac{\sqrt{2}}{2}) = \pi - (-\frac{\pi}{4}) = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$
   $\Rightarrow x = -\frac{\pi}{4} + 2n\pi$, for $n \in \mathbb{Z}$
   or $x = \frac{5\pi}{4} + 2n\pi$, for $n \in \mathbb{Z}$

Solve $\sin(x) = c$

Solution:

$$\begin{aligned} x &= \arcsin(c) + n\cdot 2\pi, \quad \text{for } n \in \mathbb{Z} \\ x &= \pi - \arcsin(c) + n\cdot 2\pi, \quad \text{for } n \in \mathbb{Z} \end{aligned}$$

1. Solve for x: $\sin(x) = -\frac{\sqrt{2}}{2}$
   Solution:
   $\arcsin(-\frac{\sqrt{2}}{2}) = -\frac{\pi}{4}$
   $\pi - \arcsin(\frac{\sqrt{2}}{2}) = \pi - (-\frac{\pi}{4}) = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$
   $\Rightarrow x = -\frac{\pi}{4} + 2n\pi$, for $n \in \mathbb{Z}$
   or $x = \frac{5\pi}{4} + 2n\pi$, for $n \in \mathbb{Z}$
2. Solve for x: $\sin(x) = -\frac{1}{2}$
3. Solve for x: $2\sin(x) - \sqrt{3} = 0$
4. Solve for x: $2\sin^2(x) - 1 = 0$

Solving equations with sin - exercises

Solve $\sin(x) = c$

Solution:

$$\begin{aligned} x &= \arcsin(c) + n \cdot 2\pi, \quad \text{for } n \in \mathbb{Z} \\ x &= \pi - \arcsin(c) + n \cdot 2\pi, \quad \text{for } n \in \mathbb{Z} \end{aligned}$$

1. Solve for x: $\sin(x) = -\frac{\sqrt{2}}{2}$
   Solution:
   $\arcsin(-\frac{\sqrt{2}}{2}) = -\frac{\pi}{4}$
   $\pi - \arcsin(\frac{\sqrt{2}}{2}) = \pi - (-\frac{\pi}{4}) = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$
   $\Rightarrow x = -\frac{\pi}{4} + 2n\pi$, for $n \in \mathbb{Z}$
   or $x = \frac{5\pi}{4} + 2n\pi$, for $n \in \mathbb{Z}$
2. Solve for x: $\sin(x) = -\frac{1}{2}$
   Solution:
   $\arcsin(-\frac{1}{2}) = -\frac{\pi}{6}$
   $\pi - \arcsin(\frac{1}{2}) = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$
   $\Rightarrow x = -\frac{\pi}{6} + 2n\pi$, for $n \in \mathbb{Z}$
   or $x = \frac{7\pi}{6} + 2n\pi$, for $n \in \mathbb{Z}$
3. Solve for x: $2\sin(x) - \sqrt{3} = 0$
4. Solve for x: $2\sin^2(x) - 1 = 0$

Solve $\sin(x) = c$

Solution:

$$x = \arcsin(c) + n \cdot 2\pi, \quad \text{for } n \in \mathbb{Z}$$
$$x = \pi - \arcsin(c) + n \cdot 2\pi, \quad \text{for } n \in \mathbb{Z}$$

1. Solve for x: $\sin(x) = -\frac{\sqrt{2}}{2}$
   Solution:
   $\arcsin(-\frac{\sqrt{2}}{2}) = -\frac{\pi}{4}$
   $\pi - \arcsin(\frac{\sqrt{2}}{2}) = \pi - (-\frac{\pi}{4}) = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$
   $\Rightarrow x = -\frac{\pi}{4} + 2n\pi$, for $n \in \mathbb{Z}$
   or $x = \frac{5\pi}{4} + 2n\pi$, for $n \in \mathbb{Z}$
2. Solve for x: $\sin(x) = -\frac{1}{2}$
   Solution:
   $\arcsin(-\frac{1}{2}) = -\frac{\pi}{6}$
   $\pi - \arcsin(\frac{1}{2}) = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$
   $\Rightarrow x = -\frac{\pi}{6} + 2n\pi$, for $n \in \mathbb{Z}$
   or $x = \frac{7\pi}{6} + 2n\pi$, for $n \in \mathbb{Z}$
3. Solve for x: $2\sin(x) - \sqrt{3} = 0$
   Solution:
   $\Rightarrow 2\sin(x) = \sqrt{3} \Rightarrow \sin(x) = \frac{\sqrt{3}}{2}$
   $\arcsin(\frac{\sqrt{3}}{2}) = \frac{\pi}{3}$
   $\pi - \arcsin(\frac{\sqrt{3}}{2}) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$
   $\Rightarrow x = \frac{\pi}{3} + 2n\pi$, for $n \in \mathbb{Z}$
   or $x = \frac{2\pi}{3} + 2n\pi$, for $n \in \mathbb{Z}$
4. Solve for x: $2\sin^2(x) - 1 = 0$

Solving equations with sin - exercises

Solve $\sin(x) = c$

Solution:

$$x = \arcsin(c) + n \cdot 2\pi, \quad \text{for } n \in \mathbb{Z}$$
$$x = \pi - \arcsin(c) + n \cdot 2\pi, \quad \text{for } n \in \mathbb{Z}$$

1. Solve for x: $\sin(x) = -\frac{\sqrt{2}}{2}$
   Solution:
   $\arcsin(-\frac{\sqrt{2}}{2}) = -\frac{\pi}{4}$
   $\pi - \arcsin(\frac{\sqrt{2}}{2}) = \pi - (-\frac{\pi}{4}) = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$
   $\Rightarrow x = -\frac{\pi}{4} + 2n\pi$, for $n \in \mathbb{Z}$
   or $x = \frac{5\pi}{4} + 2n\pi$, for $n \in \mathbb{Z}$
2. Solve for x: $\sin(x) = -\frac{1}{2}$
   Solution:
   $\arcsin(-\frac{1}{2}) = -\frac{\pi}{6}$
   $\pi - \arcsin(\frac{1}{2}) = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$
   $\Rightarrow x = -\frac{\pi}{6} + 2n\pi$, for $n \in \mathbb{Z}$
   or $x = \frac{7\pi}{6} + 2n\pi$, for $n \in \mathbb{Z}$
3. Solve for x: $2\sin(x) - \sqrt{3} = 0$
   Solution:
   $\Rightarrow 2\sin(x) = \sqrt{3} \Rightarrow \sin(x) = \frac{\sqrt{3}}{2}$
   $\arcsin(\frac{\sqrt{3}}{2}) = \frac{\pi}{3}$
   $\pi - \arcsin(\frac{\sqrt{3}}{2}) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$
   $\Rightarrow x = \frac{\pi}{3} + 2n\pi$, for $n \in \mathbb{Z}$
   or $x = \frac{2\pi}{3} + 2n\pi$, for $n \in \mathbb{Z}$
4. Solve for x: $2\sin^2(x) - 1 = 0$
   Solution:
   Substitute $u = \sin(x) \Rightarrow 2u^2 - 1 = 0$
   $\Rightarrow 2u^2 = 1 \Rightarrow u^2 = \frac{1}{2} \Rightarrow u = \pm\sqrt{\frac{1}{2}} = \pm\frac{\sqrt{2}}{2}$
   $\Rightarrow \sin(x) = \frac{\sqrt{2}}{2}$ or $\sin(x) = -\frac{\sqrt{2}}{2}$

Solving equations with sin - exercises

Solve $\sin(x) = c$

Solution:

$$x = \arcsin(c) + n \cdot 2\pi, \quad \text{for } n \in \mathbb{Z}$$
$$x = \pi - \arcsin(c) + n \cdot 2\pi, \quad \text{for } n \in \mathbb{Z}$$

1. Solve for x: $\sin(x) = -\frac{\sqrt{2}}{2}$
   Solution:
   $\arcsin(-\frac{\sqrt{2}}{2}) = -\frac{\pi}{4}$
   $\pi - \arcsin(\frac{\sqrt{2}}{2}) = \pi - (-\frac{\pi}{4}) = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$
   $\Rightarrow x = -\frac{\pi}{4} + 2n\pi$, for $n \in \mathbb{Z}$
   or $x = \frac{5\pi}{4} + 2n\pi$, for $n \in \mathbb{Z}$
2. Solve for x: $\sin(x) = -\frac{1}{2}$
   Solution:
   $\arcsin(-\frac{1}{2}) = -\frac{\pi}{6}$
   $\pi - \arcsin(\frac{1}{2}) = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$
   $\Rightarrow x = -\frac{\pi}{6} + 2n\pi$, for $n \in \mathbb{Z}$
   or $x = \frac{7\pi}{6} + 2n\pi$, for $n \in \mathbb{Z}$
3. Solve for x: $2\sin(x) - \sqrt{3} = 0$
   Solution:
   $\Rightarrow 2\sin(x) = \sqrt{3} \Rightarrow \sin(x) = \frac{\sqrt{3}}{2}$
   $\arcsin(\frac{\sqrt{3}}{2}) = \frac{\pi}{3}$
   $\pi - \arcsin(\frac{\sqrt{3}}{2}) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$
   $\Rightarrow x = \frac{\pi}{3} + 2n\pi$, for $n \in \mathbb{Z}$
   or $x = \frac{2\pi}{3} + 2n\pi$, for $n \in \mathbb{Z}$
4. Solve for x: $2\sin^2(x) - 1 = 0$
   Solution:
   Substitute $u = \sin(x) \Rightarrow 2u^2 - 1 = 0$
   $\Rightarrow 2u^2 = 1 \Rightarrow u^2 = \frac{1}{2} \Rightarrow u = \pm\sqrt{\frac{1}{2}} = \pm\frac{\sqrt{2}}{2}$
   $\Rightarrow \sin(x) = \frac{\sqrt{2}}{2}$ or $\sin(x) = -\frac{\sqrt{2}}{2}$
   $\arcsin(\frac{\sqrt{2}}{2}) = \frac{\pi}{4}$
   $\pi - \arcsin(\frac{\sqrt{2}}{2}) = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$
   $\arcsin(-\frac{\sqrt{2}}{2}) = -\frac{\pi}{4}$
   $\pi - \arcsin(-\frac{\sqrt{2}}{2}) = \pi - (-\frac{\pi}{4}) = \frac{5\pi}{4}$
   $\Rightarrow x = \frac{\pi}{4} + 2n\pi$, for $n \in \mathbb{Z}$
   or $x = \frac{3\pi}{4} + 2n\pi$, for $n \in \mathbb{Z}$
   or $x = -\frac{\pi}{4} + 2n\pi$, for $n \in \mathbb{Z}$
   or $x = \frac{5\pi}{4} + 2n\pi$, for $n \in \mathbb{Z}$

Summary for solving trigonometric equations

$\sin(x) = c$	$\cos(x) = c$	$\tan(x) = c$	
$x = \arcsin(c) + 2n\pi$	$x = \arccos(c) + 2n\pi$		for $n \in \mathbb{Z}$
$x = \pi - \arcsin(c) + 2n\pi$	$x = -\arccos(c) + 2n\pi$	$x = \arctan(c) + n\pi$	

1. Solve: $\tan^2(x) + \tan(x) = 0$
2. Solve: $2\sin^2(x) + \sqrt{3}\sin(x) = 0$

Summary for solving trigonometric equations

$\sin(x) = c$	$\cos(x) = c$	$\tan(x) = c$	
$x = \arcsin(c) + 2n\pi$	$x = \arccos(c) + 2n\pi$		for $n \in \mathbb{Z}$
$x = \pi - \arcsin(c) + 2n\pi$	$x = -\arccos(c) + 2n\pi$	$x = \arctan(c) + n\pi$	

1. Solve: $\tan^2(x) + \tan(x) = 0$
   Solution:
   Substitute $u = \tan(x)$
   $\Rightarrow u^2 + u = 0 \Rightarrow u(u+1) = 0$
   $\Rightarrow u = 0 \quad \text{or} \quad u = -1$
   $\Rightarrow \tan(x) = 0 \quad \text{or} \quad \tan(x) = -1$

2. Solve: $2\sin^2(x) + \sqrt{3}\sin(x) = 0$

Solving equations with sin, cos, and tan - exercises

Summary for solving trigonometric equations

$\sin(x) = c$	$\cos(x) = c$	$\tan(x) = c$	
$x = \arcsin(c) + 2n\pi$	$x = \arccos(c) + 2n\pi$		for $n \in \mathbb{Z}$
$x = \pi - \arcsin(c) + 2n\pi$	$x = -\arccos(c) + 2n\pi$	$x = \arctan(c) + n\pi$	

1. Solve: $\tan^2(x) + \tan(x) = 0$
   Solution:
   Substitute $u = \tan(x)$
   $\Rightarrow u^2 + u = 0 \Rightarrow u(u+1) = 0$
   $\Rightarrow u = 0 \quad \text{or} \quad u = -1$
   $\Rightarrow \tan(x) = 0 \quad \text{or} \quad \tan(x) = -1$

   $\arctan(0) = 0$
   $\arctan(-1) = -\frac{\pi}{4}$

   $\Rightarrow x = 0 + n\pi$, for $n \in \mathbb{Z}$
   $\Rightarrow x = -\frac{\pi}{4} + n\pi$, for $n \in \mathbb{Z}$

2. Solve: $2\sin^2(x) + \sqrt{3}\sin(x) = 0$

Summary for solving trigonometric equations

$\sin(x) = c$	$\cos(x) = c$	$\tan(x) = c$	
$x = \arcsin(c) + 2n\pi$ $x = \pi - \arcsin(c) + 2n\pi$	$x = \arccos(c) + 2n\pi$ $x = -\arccos(c) + 2n\pi$	$x = \arctan(c) + n\pi$	for $n \in \mathbb{Z}$

1. Solve: $\tan^2(x) + \tan(x) = 0$
   Solution:
   Substitute $u = \tan(x)$
   $\Rightarrow u^2 + u = 0 \Rightarrow u(u+1) = 0$
   $\Rightarrow u = 0 \quad \text{or} \quad u = -1$
   $\Rightarrow \tan(x) = 0 \quad \text{or} \quad \tan(x) = -1$

   $\arctan(0) = 0$
   $\arctan(-1) = -\frac{\pi}{4}$

   $\Rightarrow x = 0 + n\pi$, for $n \in \mathbb{Z}$
   $\Rightarrow x = -\frac{\pi}{4} + n\pi$, for $n \in \mathbb{Z}$

2. Solve: $2\sin^2(x) + \sqrt{3}\sin(x) = 0$
   Solution:
   Substitute $u = \sin(x)$
   $\Rightarrow 2u^2 + \sqrt{3}u = 0 \Rightarrow u(2u + \sqrt{3}) = 0$
   $\Rightarrow u = 0 \quad \text{or} \quad u = -\frac{\sqrt{3}}{2}$
   $\Rightarrow \sin(x) = 0 \quad \text{or} \quad \sin(x) = -\frac{\sqrt{3}}{2}$

Summary for solving trigonometric equations

$\sin(x) = c$	$\cos(x) = c$	$\tan(x) = c$	
$x = \arcsin(c) + 2n\pi$	$x = \arccos(c) + 2n\pi$		for $n \in \mathbb{Z}$
$x = \pi - \arcsin(c) + 2n\pi$	$x = -\arccos(c) + 2n\pi$	$x = \arctan(c) + n\pi$	

1. Solve: $\tan^2(x) + \tan(x) = 0$
   Solution:
   Substitute $u = \tan(x)$
   $\Rightarrow u^2 + u = 0 \Rightarrow u(u+1) = 0$
   $\Rightarrow u = 0 \quad \text{or} \quad u = -1$
   $\Rightarrow \tan(x) = 0 \quad \text{or} \quad \tan(x) = -1$

   $\arctan(0) = 0$
   $\arctan(-1) = -\frac{\pi}{4}$

   $\Rightarrow x = 0 + n\pi$, for $n \in \mathbb{Z}$
   $\Rightarrow x = -\frac{\pi}{4} + n\pi$, for $n \in \mathbb{Z}$

2. Solve: $2\sin^2(x) + \sqrt{3}\sin(x) = 0$
   Solution:
   Substitute $u = \sin(x)$
   $\Rightarrow 2u^2 + \sqrt{3}u = 0 \Rightarrow u(2u + \sqrt{3}) = 0$
   $\Rightarrow u = 0 \quad \text{or} \quad u = -\frac{\sqrt{3}}{2}$
   $\Rightarrow \sin(x) = 0 \quad \text{or} \quad \sin(x) = -\frac{\sqrt{3}}{2}$

   $\arcsin(0) = 0$
   $\pi - \arcsin(0) = \pi - 0 = \pi$
   $\arcsin(-\frac{\sqrt{3}}{2}) = -\frac{\pi}{3}$
   $\pi - \arcsin(-\frac{\sqrt{3}}{2}) = \pi - (-\frac{\pi}{3}) = \frac{4\pi}{3}$

   $\Rightarrow x = 0 + 2n\pi$, for $n \in \mathbb{Z}$
   $\Rightarrow x = \pi + 2n\pi$, for $n \in \mathbb{Z}$
   $\Rightarrow x = -\frac{\pi}{3} + 2n\pi$, for $n \in \mathbb{Z}$
   $\Rightarrow x = \frac{4\pi}{3} + 2n\pi$, for $n \in \mathbb{Z}$

Summary for solving trigonometric equations

$\sin(x) = c$	$\cos(x) = c$	$\tan(x) = c$	
$x = \arcsin(c) + 2n\pi$	$x = \arccos(c) + 2n\pi$		for $n \in \mathbb{Z}$
$x = \pi - \arcsin(c) + 2n\pi$	$x = -\arccos(c) + 2n\pi$	$x = \arctan(c) + n\pi$	

3. Solve: $\cos(x)\tan(x) + \tan(x) = 0$

4. Solve: $\cos(x) - 2\cos(x)\sin(x) = 0$

Summary for solving trigonometric equations

$\sin(x) = c$	$\cos(x) = c$	$\tan(x) = c$	
$x = \arcsin(c) + 2n\pi$	$x = \arccos(c) + 2n\pi$		for $n \in \mathbb{Z}$
$x = \pi - \arcsin(c) + 2n\pi$	$x = -\arccos(c) + 2n\pi$	$x = \arctan(c) + n\pi$	

3. Solve: $\cos(x)\tan(x) + \tan(x) = 0$
   Solution:
   $\Rightarrow \tan(x) \cdot (\cos(x) + 1) = 0$
   $\Rightarrow \tan(x) = 0 \quad \text{or} \quad \cos(x) = -1$

4. Solve: $\cos(x) - 2\cos(x)\sin(x) = 0$

Solving equations with sin, cos, and tan - exercises

Summary for solving trigonometric equations

$\sin(x) = c$	$\cos(x) = c$	$\tan(x) = c$	
$x = \arcsin(c) + 2n\pi$	$x = \arccos(c) + 2n\pi$		for $n \in \mathbb{Z}$
$x = \pi - \arcsin(c) + 2n\pi$	$x = -\arccos(c) + 2n\pi$	$x = \arctan(c) + n\pi$	

3. Solve: $\cos(x)\tan(x) + \tan(x) = 0$

   Solution:

   $\Rightarrow \tan(x) \cdot (\cos(x) + 1) = 0$

   $\Rightarrow \tan(x) = 0 \quad \text{or} \quad \cos(x) = -1$

   $$\arctan(0) = 0$$
   $$\arccos(-1) = \pi$$
   $$-\arccos(-1) = -\pi$$

   $\Rightarrow x = 0 + n\pi$, for $n \in \mathbb{Z}$

   $\Rightarrow x = \pi + 2n\pi$, for $n \in \mathbb{Z}$

   $\Rightarrow x = -\pi + 2n\pi$, for $n \in \mathbb{Z}$

4. Solve: $\cos(x) - 2\cos(x)\sin(x) = 0$

Solving equations with sin, cos, and tan - exercises

Summary for solving trigonometric equations

$\sin(x) = c$	$\cos(x) = c$	$\tan(x) = c$	
$x = \arcsin(c) + 2n\pi$	$x = \arccos(c) + 2n\pi$		for $n \in \mathbb{Z}$
$x = \pi - \arcsin(c) + 2n\pi$	$x = -\arccos(c) + 2n\pi$	$x = \arctan(c) + n\pi$	

3. Solve: $\cos(x)\tan(x) + \tan(x) = 0$
   Solution:
   $\Rightarrow \tan(x) \cdot (\cos(x) + 1) = 0$
   $\Rightarrow \tan(x) = 0 \quad \text{or} \quad \cos(x) = -1$

   $\arctan(0) = 0$
   $\arccos(-1) = \pi$
   $-\arccos(-1) = -\pi$

   $\Rightarrow x = 0 + n\pi$, for $n \in \mathbb{Z}$
   $\Rightarrow x = \pi + 2n\pi$, for $n \in \mathbb{Z}$
   $\Rightarrow x = -\pi + 2n\pi$, for $n \in \mathbb{Z}$

4. Solve: $\cos(x) - 2\cos(x)\sin(x) = 0$
   Solution:
   $\Rightarrow \cos(x) \cdot (1 - 2\sin(x)) = 0$
   $\Rightarrow \cos(x) = 0 \quad \text{or} \quad 1 - 2\sin(x) = 0$
   $\Rightarrow \cos(x) = 0 \quad \text{or} \quad 2\sin(x) = 1$
   $\Rightarrow \cos(x) = 0 \quad \text{or} \quad \sin(x) = \frac{1}{2}$

Solving equations with sin, cos, and tan - exercises

Summary for solving trigonometric equations

$\sin(x) = c$	$\cos(x) = c$	$\tan(x) = c$	
$x = \arcsin(c) + 2n\pi$ $x = \pi - \arcsin(c) + 2n\pi$	$x = \arccos(c) + 2n\pi$ $x = -\arccos(c) + 2n\pi$	$x = \arctan(c) + n\pi$	for $n \in \mathbb{Z}$

3. Solve: $\cos(x)\tan(x) + \tan(x) = 0$
   Solution:
   $\Rightarrow \tan(x) \cdot (\cos(x) + 1) = 0$
   $\Rightarrow \tan(x) = 0 \quad \text{or} \quad \cos(x) = -1$

   $$\arctan(0) = 0$$
   $$\arccos(-1) = \pi$$
   $$-\arccos(-1) = -\pi$$

   $\Rightarrow x = 0 + n\pi$, for $n \in \mathbb{Z}$
   $\Rightarrow x = \pi + 2n\pi$, for $n \in \mathbb{Z}$
   $\Rightarrow x = -\pi + 2n\pi$, for $n \in \mathbb{Z}$

4. Solve: $\cos(x) - 2\cos(x)\sin(x) = 0$
   Solution:
   $\Rightarrow \cos(x) \cdot (1 - 2\sin(x)) = 0$
   $\Rightarrow \cos(x) = 0 \quad \text{or} \quad 1 - 2\sin(x) = 0$
   $\Rightarrow \cos(x) = 0 \quad \text{or} \quad 2\sin(x) = 1$
   $\Rightarrow \cos(x) = 0 \quad \text{or} \quad \sin(x) = \frac{1}{2}$

   $$\arccos(0) = \frac{\pi}{2}$$
   $$-\arccos(0) = -\frac{\pi}{2}$$
   $$\arcsin(\tfrac{1}{2}) = \frac{\pi}{6}$$
   $$\pi - \arcsin(\tfrac{1}{2}) = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$$

   $\Rightarrow x = \frac{\pi}{2} + 2n\pi$, for $n \in \mathbb{Z}$
   $\Rightarrow x = -\frac{\pi}{2} + 2n\pi$, for $n \in \mathbb{Z}$
   $\Rightarrow x = \frac{\pi}{6} + 2n\pi$, for $n \in \mathbb{Z}$
   $\Rightarrow x = \frac{5\pi}{6} + 2n\pi$, for $n \in \mathbb{Z}$