

# Inverse trigonometric functions

## Lesson #19

### MAT 1375 Precalculus

New York City College of Technology CUNY



The inverse function of  $y = \tan(x)$

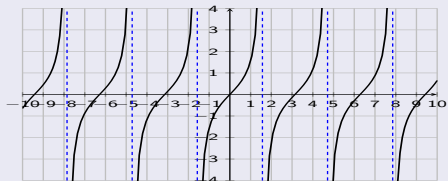
Graph  $y = \tan(x)$ .



# The inverse function of $y = \tan(x)$

## Graph $y = \tan(x)$ .

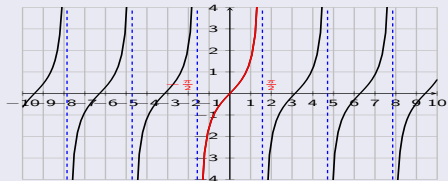
x	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
y	undef	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undef



# The inverse function of $y = \tan(x)$

## Graph $y = \tan(x)$ .

x	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
y	undef	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undef

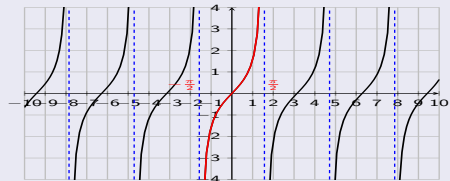


$y = \tan(x)$  is one-to-one when restricted to the domain  $D = (-\frac{\pi}{2}, \frac{\pi}{2})$  with range  $R = \mathbb{R}$ .

# The inverse function of $y = \tan(x)$

## Graph $y = \tan(x)$ .

x	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
y	undef	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undef



$y = \tan(x)$  is one-to-one when restricted to the domain  $D = (-\frac{\pi}{2}, \frac{\pi}{2})$  with range  $R = \mathbb{R}$ .

## Inverse tangent function

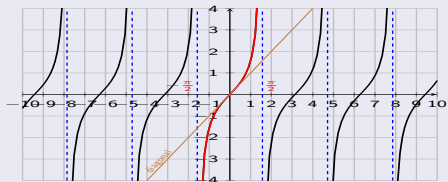
Define the inverse tangent function as:

$$x = \tan(y) \Leftrightarrow y = \tan^{-1}(x) = \arctan(x)$$

# The inverse function of $y = \tan(x)$

## Graph $y = \tan(x)$ .

x	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
y	undef	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undef



$y = \tan(x)$  is one-to-one when restricted to the domain  $D = (-\frac{\pi}{2}, \frac{\pi}{2})$  with range  $R = \mathbb{R}$ .

## Inverse tangent function

Define the inverse tangent function as:

$$x = \tan(y) \Leftrightarrow y = \tan^{-1}(x) = \arctan(x)$$

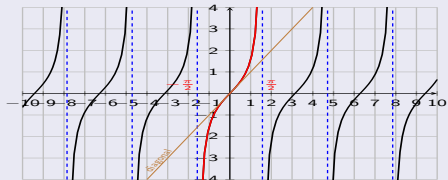
Domain:  $D = \mathbb{R}$ , Range  $R = (-\frac{\pi}{2}, \frac{\pi}{2})$



# The inverse function of $y = \tan(x)$

## Graph $y = \tan(x)$ .

x	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
y	undef	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undef



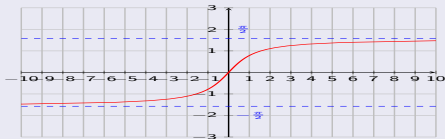
$y = \tan(x)$  is one-to-one when restricted to the domain  $D = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$  with range  $R = \mathbb{R}$ .

## Inverse tangent function

Define the inverse tangent function as:

$$x = \tan(y) \Leftrightarrow y = \tan^{-1}(x) = \arctan(x)$$

Domain:  $D = \mathbb{R}$ , Range  $R = \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

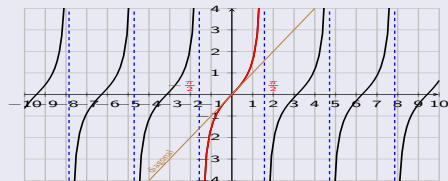


**Note:**  $\arctan$  is an odd function:  
 $\arctan(-x) = -\arctan(x)$

# The inverse function of $y = \tan(x)$

## Graph $y = \tan(x)$ .

$x$	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$y$	undef	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undef



$y = \tan(x)$  is one-to-one when restricted to the domain  $D = (-\frac{\pi}{2}, \frac{\pi}{2})$  with range  $R = \mathbb{R}$ .

1  $\arctan(\sqrt{3}) =$       4  $\arctan(2 + \sqrt{3}) =$

2  $\arctan(-\frac{\sqrt{3}}{3}) =$

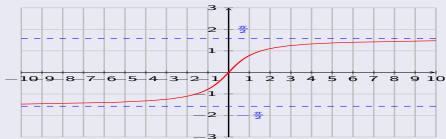
3  $\arctan(-1) =$

## Inverse tangent function

Define the inverse tangent function as:

$$x = \tan(y) \Leftrightarrow y = \tan^{-1}(x) = \arctan(x)$$

Domain:  $D = \mathbb{R}$ , Range  $R = (-\frac{\pi}{2}, \frac{\pi}{2})$



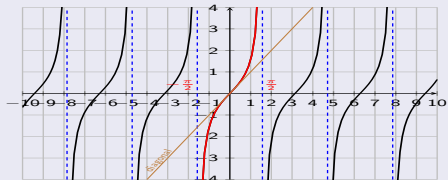
**Note:**  $\arctan$  is an odd function:  
 $\arctan(-x) = -\arctan(x)$



# The inverse function of $y = \tan(x)$

## Graph $y = \tan(x)$ .

$x$	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$y$	undef	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undef



$y = \tan(x)$  is one-to-one when restricted to the domain  $D = (-\frac{\pi}{2}, \frac{\pi}{2})$  with range  $R = \mathbb{R}$ .

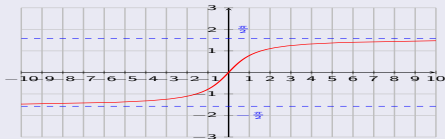
- $\arctan(\sqrt{3}) = \frac{\pi}{3} = 60^\circ$
- $\arctan(-\frac{\sqrt{3}}{3}) =$
- $\arctan(-1) =$
- $\arctan(2 + \sqrt{3}) =$

## Inverse tangent function

Define the inverse tangent function as:

$$x = \tan(y) \Leftrightarrow y = \tan^{-1}(x) = \arctan(x)$$

Domain:  $D = \mathbb{R}$ , Range  $R = (-\frac{\pi}{2}, \frac{\pi}{2})$

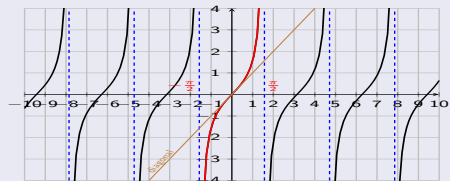


**Note:**  $\arctan$  is an odd function:  
 $\arctan(-x) = -\arctan(x)$

# The inverse function of $y = \tan(x)$

## Graph $y = \tan(x)$ .

$x$	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$y$	undef	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undef



$y = \tan(x)$  is one-to-one when restricted to the domain  $D = (-\frac{\pi}{2}, \frac{\pi}{2})$  with range  $R = \mathbb{R}$ .

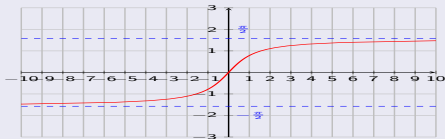
- $\arctan(\sqrt{3}) = \frac{\pi}{3} = 60^\circ$
- $\arctan(-\frac{\sqrt{3}}{3}) = -\frac{\pi}{6} = -30^\circ$
- $\arctan(-1) =$
- $\arctan(2 + \sqrt{3}) =$

## Inverse tangent function

Define the inverse tangent function as:

$$x = \tan(y) \Leftrightarrow y = \tan^{-1}(x) = \arctan(x)$$

Domain:  $D = \mathbb{R}$ , Range  $R = (-\frac{\pi}{2}, \frac{\pi}{2})$



**Note:**  $\arctan$  is an odd function:  
 $\arctan(-x) = -\arctan(x)$

# The inverse function of $y = \tan(x)$

## Graph $y = \tan(x)$ .

$x$	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$y$	undef	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undef



$y = \tan(x)$  is one-to-one when restricted to the domain  $D = (-\frac{\pi}{2}, \frac{\pi}{2})$  with range  $R = \mathbb{R}$ .

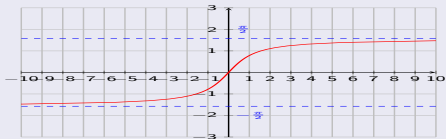
- $\arctan(\sqrt{3}) = \frac{\pi}{3} = 60^\circ$
- $\arctan(-\frac{\sqrt{3}}{3}) = -\frac{\pi}{6} = -30^\circ$
- $\arctan(-1) = -\frac{\pi}{4} = -45^\circ$
- $\arctan(2 + \sqrt{3}) =$

## Inverse tangent function

Define the inverse tangent function as:

$$x = \tan(y) \Leftrightarrow y = \tan^{-1}(x) = \arctan(x)$$

Domain:  $D = \mathbb{R}$ , Range  $R = (-\frac{\pi}{2}, \frac{\pi}{2})$



**Note:**  $\arctan$  is an odd function:  
 $\arctan(-x) = -\arctan(x)$

# The inverse function of $y = \tan(x)$

## Graph $y = \tan(x)$ .

$x$	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$y$	undef	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undef



$y = \tan(x)$  is one-to-one when restricted to the domain  $D = (-\frac{\pi}{2}, \frac{\pi}{2})$  with range  $R = \mathbb{R}$ .

$$\textcircled{1} \arctan(\sqrt{3}) = \frac{\pi}{3} = 60^\circ$$

$$\textcircled{2} \arctan(-\frac{\sqrt{3}}{3}) = -\frac{\pi}{6} = -30^\circ$$

$$\textcircled{3} \arctan(-1) = -\frac{\pi}{4} = -45^\circ$$

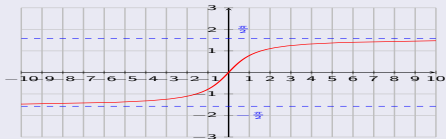
$$\textcircled{4} \arctan(2 + \sqrt{3}) = \text{(use a calculator!)} \\ = 75^\circ = 75 \cdot \frac{\pi}{180} = \frac{5\pi}{12}$$

## Inverse tangent function

Define the inverse tangent function as:

$$x = \tan(y) \Leftrightarrow y = \tan^{-1}(x) = \arctan(x)$$

Domain:  $D = \mathbb{R}$ , Range  $R = (-\frac{\pi}{2}, \frac{\pi}{2})$

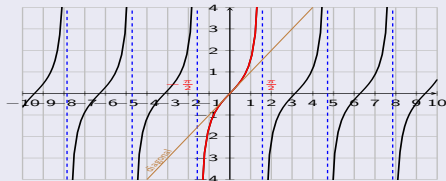


**Note:** arctan is an odd function:  
 $\arctan(-x) = -\arctan(x)$

# The inverse function of $y = \tan(x)$

## Graph $y = \tan(x)$ .

$x$	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$y$	undef	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undef



$y = \tan(x)$  is one-to-one when restricted to the domain  $D = (-\frac{\pi}{2}, \frac{\pi}{2})$  with range  $R = \mathbb{R}$ .

①  $\arctan(\sqrt{3}) =$   
 $= \frac{\pi}{3} = 60^\circ$

②  $\arctan(-\frac{\sqrt{3}}{3}) =$   
 $= -\frac{\pi}{6} = -30^\circ$

③  $\arctan(-1) =$   
 $= -\frac{\pi}{4} = -45^\circ$

④  $\arctan(2 + \sqrt{3}) =$   
(use a calculator!)  
 $= 75^\circ = 75 \cdot \frac{\pi}{180} = \frac{5\pi}{12}$

⑤  $\arctan(\sqrt{3} - 2) =$

⑥  $\arctan(2) =$

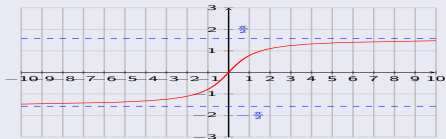
⑦  $\arctan(-5) =$

## Inverse tangent function

Define the inverse tangent function as:

$$x = \tan(y) \Leftrightarrow y = \tan^{-1}(x) = \arctan(x)$$

Domain:  $D = \mathbb{R}$ , Range  $R = (-\frac{\pi}{2}, \frac{\pi}{2})$

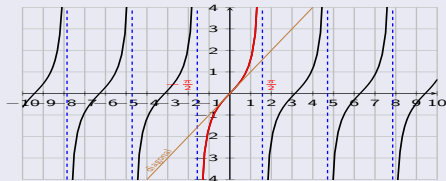


**Note:**  $\arctan$  is an odd function:  
 $\arctan(-x) = -\arctan(x)$

# The inverse function of $y = \tan(x)$

## Graph $y = \tan(x)$ .

$x$	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$y$	undef	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undef



$y = \tan(x)$  is one-to-one when restricted to the domain  $D = (-\frac{\pi}{2}, \frac{\pi}{2})$  with range  $R = \mathbb{R}$ .

$$\textcircled{1} \arctan(\sqrt{3}) = \frac{\pi}{3} = 60^\circ$$

$$\textcircled{2} \arctan(-\frac{\sqrt{3}}{3}) = -\frac{\pi}{6} = -30^\circ$$

$$\textcircled{3} \arctan(-1) = -\frac{\pi}{4} = -45^\circ$$

$$\textcircled{4} \arctan(2 + \sqrt{3}) = 75^\circ = 75 \cdot \frac{\pi}{180} = \frac{5\pi}{12}$$

$$\textcircled{5} \arctan(\sqrt{3} - 2) = -15^\circ = -15 \cdot \frac{\pi}{180} = -\frac{\pi}{12}$$

$$\textcircled{6} \arctan(2) =$$

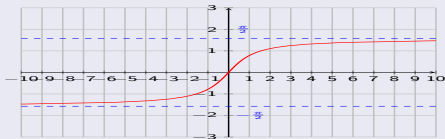
$$\textcircled{7} \arctan(-5) =$$

## Inverse tangent function

Define the inverse tangent function as:

$$x = \tan(y) \Leftrightarrow y = \tan^{-1}(x) = \arctan(x)$$

Domain:  $D = \mathbb{R}$ , Range  $R = (-\frac{\pi}{2}, \frac{\pi}{2})$

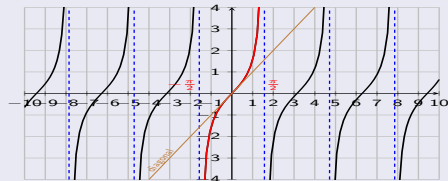


**Note:** arctan is an odd function:  
 $\arctan(-x) = -\arctan(x)$

# The inverse function of $y = \tan(x)$

## Graph $y = \tan(x)$ .

$x$	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$y$	undef	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undef



$y = \tan(x)$  is one-to-one when restricted to the domain  $D = (-\frac{\pi}{2}, \frac{\pi}{2})$  with range  $R = \mathbb{R}$ .

$$\textcircled{1} \arctan(\sqrt{3}) = \frac{\pi}{3} = 60^\circ$$

$$\textcircled{2} \arctan(-\frac{\sqrt{3}}{3}) = -\frac{\pi}{6} = -30^\circ$$

$$\textcircled{3} \arctan(-1) = -\frac{\pi}{4} = -45^\circ$$

$$\textcircled{4} \arctan(2 + \sqrt{3}) = 75^\circ = 75 \cdot \frac{\pi}{180} = \frac{5\pi}{12}$$

(use a calculator!)

$$\textcircled{5} \arctan(\sqrt{3} - 2) = -15^\circ = -15 \cdot \frac{\pi}{180} = -\frac{\pi}{12}$$

$$\textcircled{6} \arctan(2) = ?$$

$$\arctan(2) \approx 63.43^\circ$$

$$\approx 63.43 \cdot \frac{\pi}{180} \approx 1.107$$

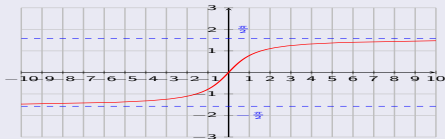
$$\textcircled{7} \arctan(-5) =$$

## Inverse tangent function

Define the inverse tangent function as:

$$x = \tan(y) \Leftrightarrow y = \tan^{-1}(x) = \arctan(x)$$

Domain:  $D = \mathbb{R}$ , Range  $R = (-\frac{\pi}{2}, \frac{\pi}{2})$

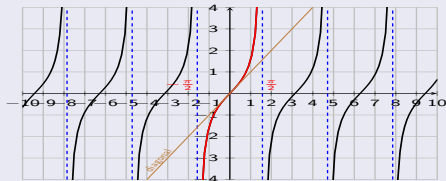


**Note:** arctan is an odd function:  
 $\arctan(-x) = -\arctan(x)$

# The inverse function of $y = \tan(x)$

## Graph $y = \tan(x)$ .

$x$	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$y$	undef	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undef



$y = \tan(x)$  is one-to-one when restricted to the domain  $D = (-\frac{\pi}{2}, \frac{\pi}{2})$  with range  $R = \mathbb{R}$ .

$$\textcircled{1} \arctan(\sqrt{3}) = \frac{\pi}{3} = 60^\circ$$

$$\textcircled{2} \arctan(-\frac{\sqrt{3}}{3}) = -\frac{\pi}{6} = -30^\circ$$

$$\textcircled{3} \arctan(-1) = -\frac{\pi}{4} = -45^\circ$$

$$\textcircled{4} \arctan(2 + \sqrt{3}) = 75^\circ = 75 \cdot \frac{\pi}{180} = \frac{5\pi}{12}$$

(use a calculator!)

$$\textcircled{5} \arctan(\sqrt{3} - 2) = -15^\circ = -15 \cdot \frac{\pi}{180} = -\frac{\pi}{12}$$

$$\textcircled{6} \arctan(2) = 63.43^\circ \approx 63.43 \cdot \frac{\pi}{180} \approx 1.107$$

$$\textcircled{7} \arctan(-5) = -78.69^\circ \approx -78.69 \cdot \frac{\pi}{180} \approx -1.373$$

## Inverse tangent function

Define the inverse tangent function as:

$$x = \tan(y) \Leftrightarrow y = \tan^{-1}(x) = \arctan(x)$$

Domain:  $D = \mathbb{R}$ , Range  $R = (-\frac{\pi}{2}, \frac{\pi}{2})$



**Note:** arctan is an odd function:  
 $\arctan(-x) = -\arctan(x)$



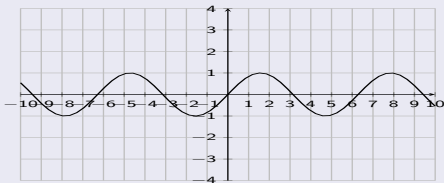
The inverse function of  $y = \sin(x)$

Graph  $y = \sin(x)$ .

# The inverse function of $y = \sin(x)$

Graph  $y = \sin(x)$ .

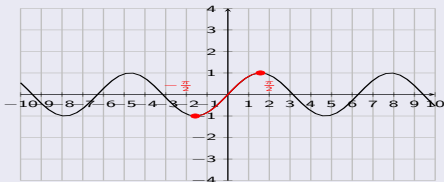
$x$	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	$0$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$y$	$-1$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	$0$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$1$



# The inverse function of $y = \sin(x)$

## Graph $y = \sin(x)$ .

$x$	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	$0$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$y$	$-1$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	$0$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$1$

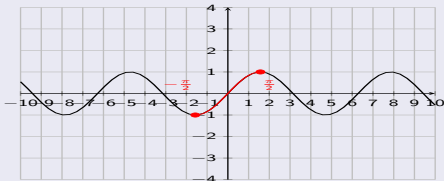


$y = \sin(x)$  is one-to-one when restricted to the domain  $D = [-\frac{\pi}{2}, \frac{\pi}{2}]$  with range  $R = [-1, 1]$ .

# The inverse function of $y = \sin(x)$

## Graph $y = \sin(x)$ .

$x$	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	$0$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$y$	$-1$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	$0$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$1$



$y = \sin(x)$  is one-to-one when restricted to the domain  $D = [-\frac{\pi}{2}, \frac{\pi}{2}]$  with range  $R = [-1, 1]$ .

## Inverse sine function

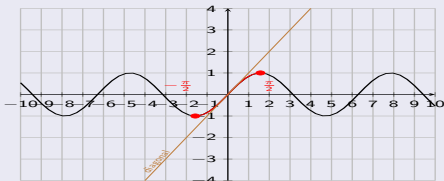
Define the inverse sine function as:

$$x = \sin(y) \Leftrightarrow y = \sin^{-1}(x) = \arcsin(x)$$

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$y$	$-1$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	$0$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$1$



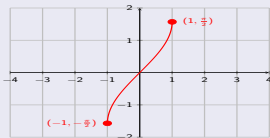
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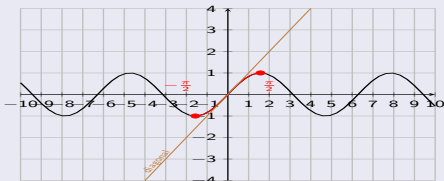
Domain:  $D = [-1, 1]$ , Range  $R = [-\frac{\pi}{2}, \frac{\pi}{2}]$



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## Graph $y = \sin(x)$ .

$x$	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	$0$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$y$	$-1$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	$0$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$1$



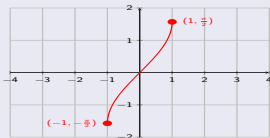
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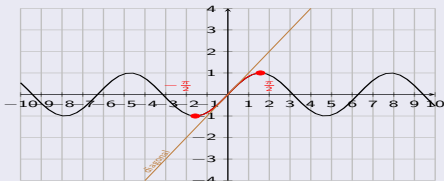


**Note:** arcsin is an odd function:  
 $\arcsin(-x) = -\arcsin(x)$

# The inverse function of $y = \sin(x)$

## Graph $y = \sin(x)$ .

$x$	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	$0$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$y$	$-1$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	$0$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$1$



$y = \sin(x)$  is one-to-one when restricted to the domain  $D = [-\frac{\pi}{2}, \frac{\pi}{2}]$  with range  $R = [-1, 1]$ .

1  $\arcsin(\frac{\sqrt{2}}{2}) =$       4  $\arcsin(\frac{\sqrt{6}-\sqrt{2}}{4}) =$

2  $\arcsin(-\frac{\sqrt{3}}{2}) =$

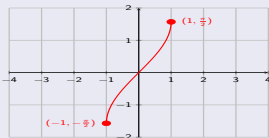
3  $\arcsin(1) =$

## Inverse sine function

Define the inverse sine function as:

$$x = \sin(y) \Leftrightarrow y = \sin^{-1}(x) = \arcsin(x)$$

Domain:  $D = [-1, 1]$ , Range  $R = [-\frac{\pi}{2}, \frac{\pi}{2}]$

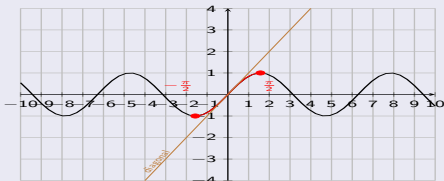


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# The inverse function of $y = \sin(x)$

## Graph $y = \sin(x)$ .

$x$	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	$0$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$y$	$-1$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	$0$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$1$



$y = \sin(x)$  is one-to-one when restricted to the domain  $D = [-\frac{\pi}{2}, \frac{\pi}{2}]$  with range  $R = [-1, 1]$ .

$$1 \quad \arcsin\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4} = 45^\circ$$

$$4 \quad \arcsin\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right) =$$

$$2 \quad \arcsin\left(-\frac{\sqrt{3}}{2}\right) =$$

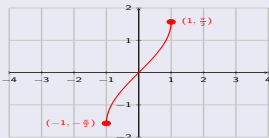
$$3 \quad \arcsin(1) =$$

## Inverse sine function

Define the inverse sine function as:

$$x = \sin(y) \Leftrightarrow y = \sin^{-1}(x) = \arcsin(x)$$

Domain:  $D = [-1, 1]$ , Range  $R = [-\frac{\pi}{2}, \frac{\pi}{2}]$



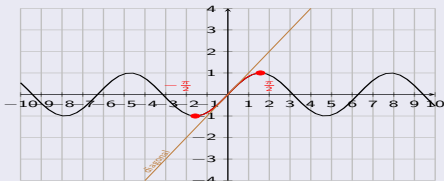
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# The inverse function of $y = \sin(x)$

## Graph $y = \sin(x)$ .

$x$	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	$0$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$y$	$-1$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	$0$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$1$



$y = \sin(x)$  is one-to-one when restricted to the domain  $D = [-\frac{\pi}{2}, \frac{\pi}{2}]$  with range  $R = [-1, 1]$ .

①  $\arcsin(\frac{\sqrt{2}}{2}) =$   
 $= \frac{\pi}{4} = 45^\circ$

④  $\arcsin(\frac{\sqrt{6}-\sqrt{2}}{4}) =$

②  $\arcsin(-\frac{\sqrt{3}}{2}) =$   
 $= -\frac{\pi}{3} = -60^\circ$

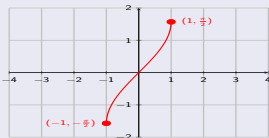
③  $\arcsin(1) =$

## Inverse sine function

Define the inverse sine function as:

$$x = \sin(y) \Leftrightarrow y = \sin^{-1}(x) = \arcsin(x)$$

Domain:  $D = [-1, 1]$ , Range  $R = [-\frac{\pi}{2}, \frac{\pi}{2}]$

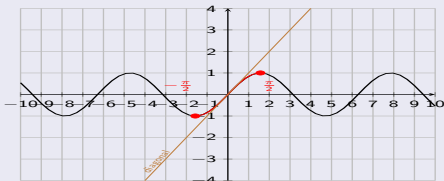


**Note:** arcsin is an odd function:  
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# The inverse function of $y = \sin(x)$

## Graph $y = \sin(x)$ .

$x$	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	$0$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$y$	$-1$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	$0$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$1$



$y = \sin(x)$  is one-to-one when restricted to the domain  $D = [-\frac{\pi}{2}, \frac{\pi}{2}]$  with range  $R = [-1, 1]$ .

$$\textcircled{1} \arcsin\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4} = 45^\circ$$

$$\textcircled{4} \arcsin\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right) =$$

$$\textcircled{2} \arcsin\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3} = -60^\circ$$

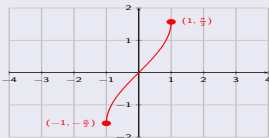
$$\textcircled{3} \arcsin(1) = \frac{\pi}{2} = 90^\circ$$

## Inverse sine function

Define the inverse sine function as:

$$x = \sin(y) \Leftrightarrow y = \sin^{-1}(x) = \arcsin(x)$$

Domain:  $D = [-1, 1]$ , Range  $R = [-\frac{\pi}{2}, \frac{\pi}{2}]$

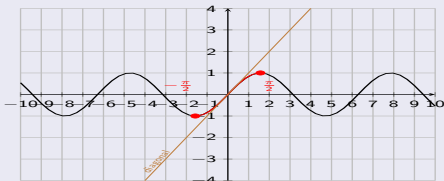


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# The inverse function of $y = \sin(x)$

## Graph $y = \sin(x)$ .

$x$	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	$0$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$y$	$-1$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	$0$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$1$



$y = \sin(x)$  is one-to-one when restricted to the domain  $D = [-\frac{\pi}{2}, \frac{\pi}{2}]$  with range  $R = [-1, 1]$ .

$$\textcircled{1} \arcsin\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4} = 45^\circ$$

$$\textcircled{2} \arcsin\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3} = -60^\circ$$

$$\textcircled{3} \arcsin(1) = \frac{\pi}{2} = 90^\circ$$

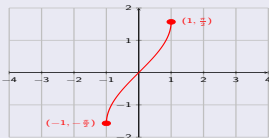
$$\textcircled{4} \arcsin\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right) = \text{(use a calculator!)} \\ = 15^\circ = 15 \cdot \frac{\pi}{180} = \frac{\pi}{12}$$

## Inverse sine function

Define the inverse sine function as:

$$x = \sin(y) \Leftrightarrow y = \sin^{-1}(x) = \arcsin(x)$$

Domain:  $D = [-1, 1]$ , Range  $R = [-\frac{\pi}{2}, \frac{\pi}{2}]$

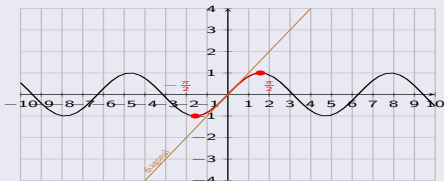


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 $\arcsin(-x) = -\arcsin(x)$

# The inverse function of $y = \sin(x)$

## Graph $y = \sin(x)$ .

$x$	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	$0$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$y$	$-1$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	$0$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$1$



$y = \sin(x)$  is one-to-one when restricted to the domain  $D = [-\frac{\pi}{2}, \frac{\pi}{2}]$  with range  $R = [-1, 1]$ .

1  $\arcsin(\frac{\sqrt{2}}{2}) =$   
 $= \frac{\pi}{4} = 45^\circ$

2  $\arcsin(-\frac{\sqrt{3}}{2}) =$   
 $= -\frac{\pi}{3} = -60^\circ$

3  $\arcsin(1) =$   
 $= \frac{\pi}{2} = 90^\circ$

4  $\arcsin(\frac{\sqrt{6}-\sqrt{2}}{4}) =$   
(use a calculator!)  
 $= 15^\circ = 15 \cdot \frac{\pi}{180} = \frac{\pi}{12}$

5  $\arcsin(\frac{\sqrt{2}-\sqrt{2}}{2}) =$

6  $\arcsin(-0.3) =$

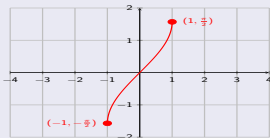
7  $\arcsin(7) =$

## Inverse sine function

Define the inverse sine function as:

$$x = \sin(y) \Leftrightarrow y = \sin^{-1}(x) = \arcsin(x)$$

Domain:  $D = [-1, 1]$ , Range  $R = [-\frac{\pi}{2}, \frac{\pi}{2}]$

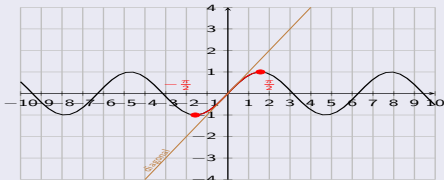


**Note:** arcsin is an odd function:  
 $\arcsin(-x) = -\arcsin(x)$

# The inverse function of $y = \sin(x)$

## Graph $y = \sin(x)$ .

$x$	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	$0$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$y$	$-1$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	$0$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$1$



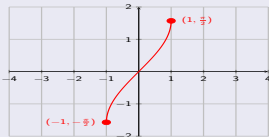
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 $\arcsin(-x) = -\arcsin(x)$

$$\textcircled{1} \arcsin\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4} = 45^\circ$$

$$\textcircled{2} \arcsin\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3} = -60^\circ$$

$$\textcircled{3} \arcsin(1) = \frac{\pi}{2} = 90^\circ$$

$$\textcircled{4} \arcsin\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right) = \text{(use a calculator!)} \\ = 15^\circ = 15 \cdot \frac{\pi}{180} = \frac{\pi}{12}$$

$$\textcircled{5} \arcsin\left(\frac{\sqrt{2}-\sqrt{2}}{2}\right) = 22.5^\circ = 22.5 \cdot \frac{\pi}{180} = \frac{\pi}{8}$$

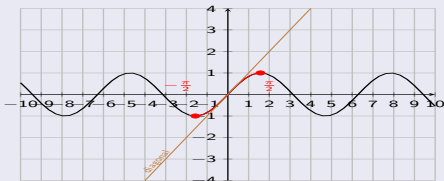
$$\textcircled{6} \arcsin(-0.3) =$$

$$\textcircled{7} \arcsin(7) =$$

# The inverse function of $y = \sin(x)$

## Graph $y = \sin(x)$ .

$x$	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	$0$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$y$	$-1$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	$0$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$1$



$y = \sin(x)$  is one-to-one when restricted to the domain  $D = [-\frac{\pi}{2}, \frac{\pi}{2}]$  with range  $R = [-1, 1]$ .

$$\textcircled{1} \arcsin\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4} = 45^\circ$$

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$$\textcircled{5} \arcsin\left(\frac{\sqrt{2}-\sqrt{2}}{2}\right) = 22.5^\circ = 22.5 \cdot \frac{\pi}{180} = \frac{\pi}{8}$$

$$\textcircled{6} \arcsin(-0.3) = ? \\ \arcsin(-0.3) \approx -17.46^\circ \\ \approx -0.3047$$

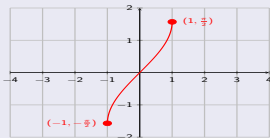
$$\textcircled{7} \arcsin(7) =$$

## Inverse sine function

Define the inverse sine function as:

$$x = \sin(y) \Leftrightarrow y = \sin^{-1}(x) = \arcsin(x)$$

Domain:  $D = [-1, 1]$ , Range  $R = [-\frac{\pi}{2}, \frac{\pi}{2}]$

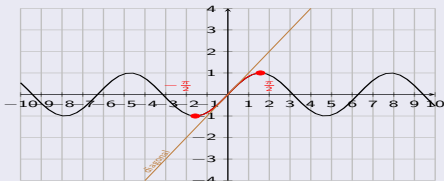


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 $\arcsin(-x) = -\arcsin(x)$

# The inverse function of $y = \sin(x)$

## Graph $y = \sin(x)$ .

$x$	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	$0$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$y$	$-1$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	$0$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$1$



$y = \sin(x)$  is one-to-one when restricted to the domain  $D = [-\frac{\pi}{2}, \frac{\pi}{2}]$  with range  $R = [-1, 1]$ .

$$\textcircled{1} \arcsin\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4} = 45^\circ$$

$$\textcircled{2} \arcsin\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3} = -60^\circ$$

$$\textcircled{3} \arcsin(1) = \frac{\pi}{2} = 90^\circ$$

$$\textcircled{4} \arcsin\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right) = 15^\circ = 15 \cdot \frac{\pi}{180} = \frac{\pi}{12}$$

(use a calculator!)

$$\textcircled{5} \arcsin\left(\frac{\sqrt{2}-\sqrt{2}}{2}\right) = 22.5^\circ = 22.5 \cdot \frac{\pi}{180} = \frac{\pi}{8}$$

$$\textcircled{6} \arcsin(-0.3) = ?$$

$$\arcsin(-0.3) \approx -17.46^\circ$$

$$\approx -0.3047$$

$$\textcircled{7} \arcsin(7) = ?$$

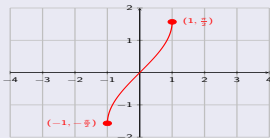
$\arcsin(7)$  is undefined, since the domain of  $\arcsin$  is  $D = [-1, 1]$ .

## Inverse sine function

Define the inverse sine function as:

$$x = \sin(y) \Leftrightarrow y = \sin^{-1}(x) = \arcsin(x)$$

Domain:  $D = [-1, 1]$ , Range  $R = [-\frac{\pi}{2}, \frac{\pi}{2}]$



**Note:**  $\arcsin$  is an odd function:  
 $\arcsin(-x) = -\arcsin(x)$

The inverse function of  $y = \cos(x)$

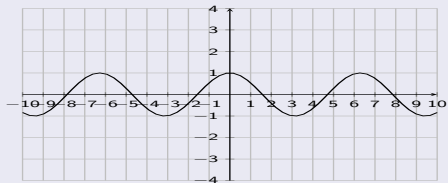
Graph  $y = \cos(x)$ .



# The inverse function of $y = \cos(x)$

Graph  $y = \cos(x)$ .

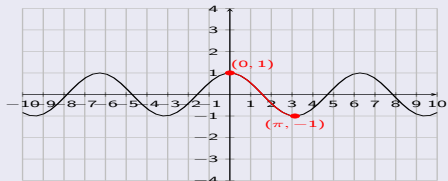
x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$
y	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1



# The inverse function of $y = \cos(x)$

## Graph $y = \cos(x)$ .

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$
y	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1

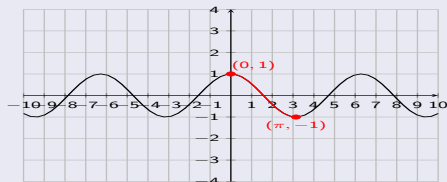


$y = \cos(x)$  is one-to-one when restricted to the domain  $D = [0, \pi]$  with range  $R = [-1, 1]$ .

# The inverse function of $y = \cos(x)$

## Graph $y = \cos(x)$ .

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$
y	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1



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## Inverse cosine function

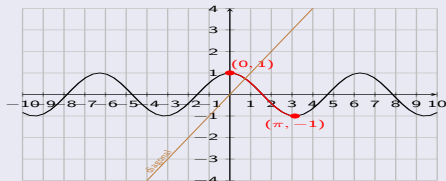
Define the inverse cosine function as:

$$x = \cos(y) \Leftrightarrow y = \cos^{-1}(x) = \arccos(x)$$

# The inverse function of $y = \cos(x)$

## Graph $y = \cos(x)$ .

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$
y	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1



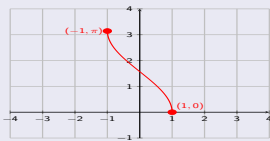
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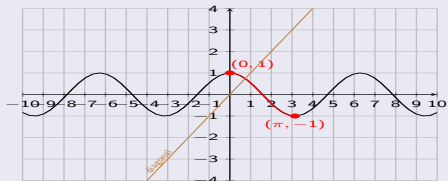
Domain:  $D = [-1, 1]$ , Range  $R = [0, \pi]$



# The inverse function of $y = \cos(x)$

## Graph $y = \cos(x)$ .

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$
y	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1



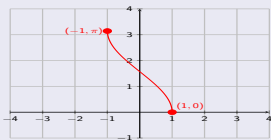
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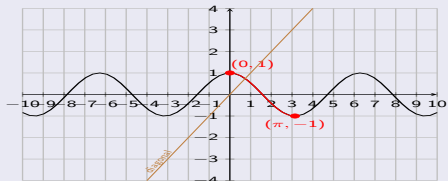


**Note:** arccos is neither even nor odd!  
But:  $\arccos(-x) = \pi - \arccos(x)$

# The inverse function of $y = \cos(x)$

## Graph $y = \cos(x)$ .

$x$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$
$y$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1



$y = \cos(x)$  is one-to-one when restricted to the domain  $D = [0, \pi]$  with range  $R = [-1, 1]$ .

1  $\arccos\left(\frac{1}{2}\right) =$       4  $\arccos\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right) =$

2  $\arccos\left(-\frac{\sqrt{2}}{2}\right) =$

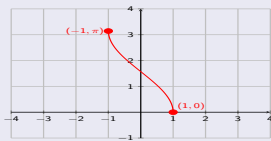
3  $\arccos\left(-\frac{1}{2}\right) =$

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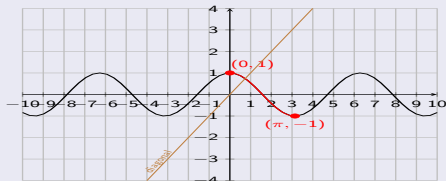
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$y$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1



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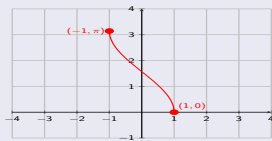
- $\arccos\left(\frac{1}{2}\right) = \frac{\pi}{3} = 60^\circ$
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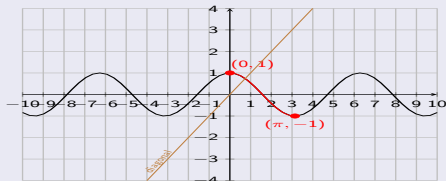


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$y$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1



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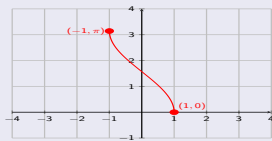
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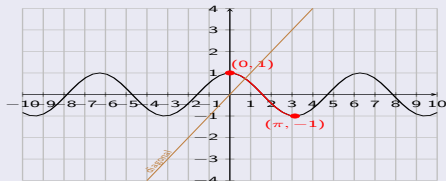
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$x$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$
$y$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1



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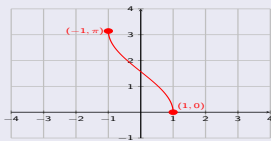
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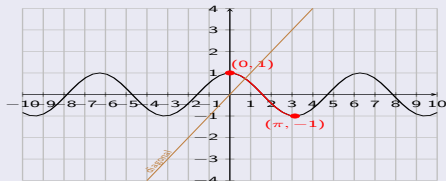


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$y = \cos(x)$  is one-to-one when restricted to the domain  $D = [0, \pi]$  with range  $R = [-1, 1]$ .

$$\begin{aligned} \textcircled{1} \arccos\left(\frac{1}{2}\right) &= \\ &= \frac{\pi}{3} = 60^\circ \end{aligned}$$

$$\begin{aligned} \textcircled{2} \arccos\left(-\frac{\sqrt{2}}{2}\right) &= \\ &= \frac{3\pi}{4} = 135^\circ \end{aligned}$$

$$\begin{aligned} \textcircled{3} \arccos\left(-\frac{1}{2}\right) &= \\ &= \frac{2\pi}{3} = 120^\circ \end{aligned}$$

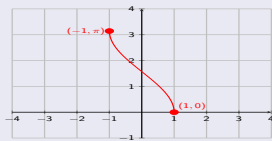
$$\begin{aligned} \textcircled{4} \arccos\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right) &= \\ & \text{(use a calculator!)} \\ &= 75^\circ = 75 \cdot \frac{\pi}{180} = \frac{5\pi}{12} \end{aligned}$$

## Inverse cosine function

Define the inverse cosine function as:

$$x = \cos(y) \Leftrightarrow y = \cos^{-1}(x) = \arccos(x)$$

Domain:  $D = [-1, 1]$ , Range  $R = [0, \pi]$



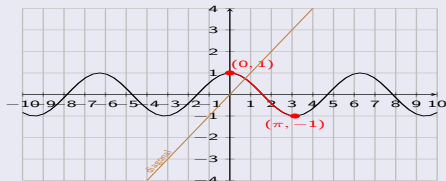
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# The inverse function of $y = \cos(x)$

## Graph $y = \cos(x)$ .

$x$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$
$y$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1



$y = \cos(x)$  is one-to-one when restricted to the domain  $D = [0, \pi]$  with range  $R = [-1, 1]$ .

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$$\textcircled{4} \arccos\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right) = 75^\circ = 75 \cdot \frac{\pi}{180} = \frac{5\pi}{12}$$

(use a calculator!)

$$\textcircled{5} \arccos\left(-\frac{1+\sqrt{3}}{2\sqrt{2}}\right) =$$

$$\textcircled{6} \arccos(0.55) =$$

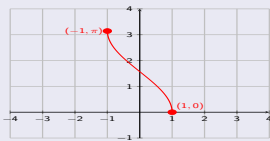
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Domain:  $D = [-1, 1]$ , Range  $R = [0, \pi]$



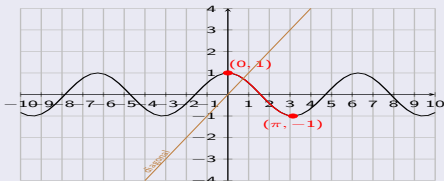
**Note:** arccos is neither even nor odd!

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# The inverse function of $y = \cos(x)$

## Graph $y = \cos(x)$ .

$x$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$
$y$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1



$y = \cos(x)$  is one-to-one when restricted to the domain  $D = [0, \pi]$  with range  $R = [-1, 1]$ .

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$$\textcircled{4} \arccos\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right) = \text{(use a calculator!)} = 75^\circ = 75 \cdot \frac{\pi}{180} = \frac{5\pi}{12}$$

$$\textcircled{5} \arccos\left(-\frac{1+\sqrt{3}}{2\sqrt{2}}\right) = 165^\circ = 165 \cdot \frac{\pi}{180} = \frac{11\pi}{12}$$

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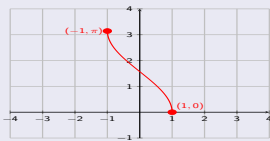
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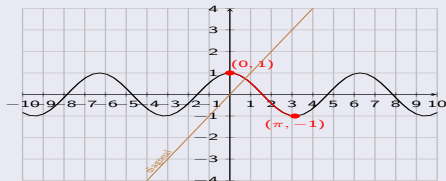


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# The inverse function of $y = \cos(x)$

## Graph $y = \cos(x)$ .

$x$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$
$y$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1



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(use a calculator!)

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$$\textcircled{6} \arccos(0.55) = ?$$

$$\arccos(0.55) \approx 56.63^\circ$$

$$\approx 0.988$$

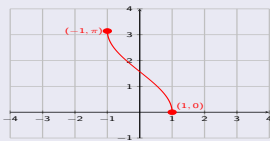
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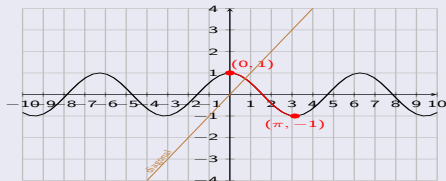


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# The inverse function of $y = \cos(x)$

## Graph $y = \cos(x)$ .

$x$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\pi$
$y$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1



$y = \cos(x)$  is one-to-one when restricted to the domain  $D = [0, \pi]$  with range  $R = [-1, 1]$ .

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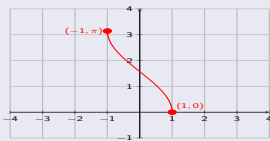
$\arccos(-2.3)$  is undefined, since the domain of  $\arccos$  is  $D = [-1, 1]$ .

## Inverse cosine function

Define the inverse cosine function as:

$$x = \cos(y) \Leftrightarrow y = \cos^{-1}(x) = \arccos(x)$$

Domain:  $D = [-1, 1]$ , Range  $R = [0, \pi]$



**Note:**  $\arccos$  is neither even nor odd!  
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# Solving equations $\tan(x) = c$

## Example

Solve for  $x$ :  $\tan(x) = 1$

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One solution comes from

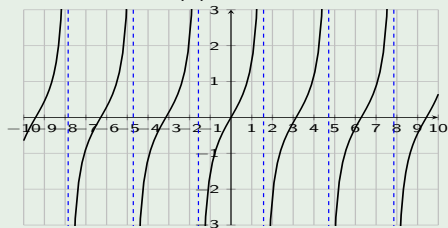
$$x = \arctan(1) = \frac{\pi}{4}$$



# Solving equations $\tan(x) = c$

## Example

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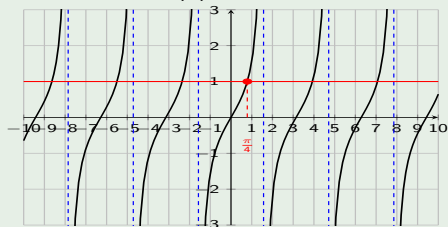
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# Solving equations $\tan(x) = c$

## Example

Solve for  $x$ :  $\tan(x) = 1$



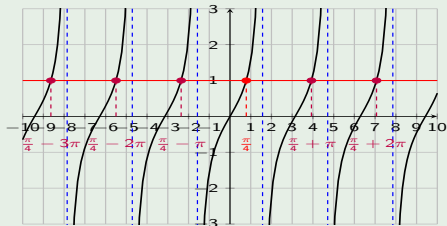
One solution comes from

$$x = \arctan(1) = \frac{\pi}{4}$$

# Solving equations $\tan(x) = c$

## Example

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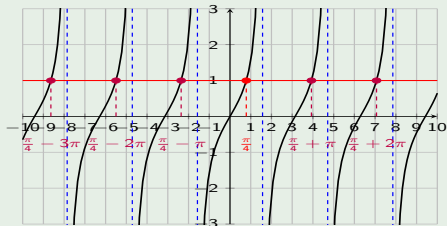
All other solutions come from adding or subtracting  $\pi$ s:

$$\dots, \frac{\pi}{4} - 2\pi, \frac{\pi}{4} - \pi, \frac{\pi}{4}, \frac{\pi}{4} + \pi, \frac{\pi}{4} + 2\pi, \dots$$

# Solving equations $\tan(x) = c$

## Example

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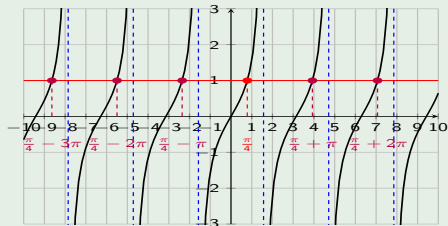
General solution:

$$x = \frac{\pi}{4} + n \cdot \pi, \quad \text{for } n \in \mathbb{Z}$$

# Solving equations $\tan(x) = c$

## Example

Solve for  $x$ :  $\tan(x) = 1$



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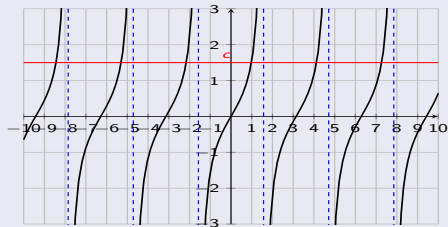
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## Solving $\tan(x) = c$



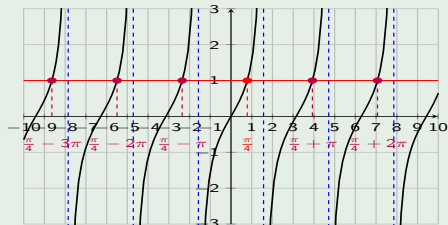
One solution comes from

$$x = \arctan(c)$$

# Solving equations $\tan(x) = c$

## Example

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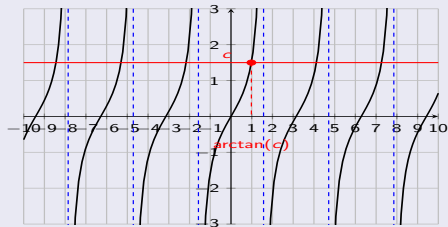
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## Solving $\tan(x) = c$



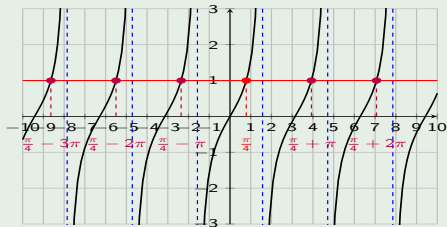
One solution comes from

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# Solving equations $\tan(x) = c$

## Example

Solve for  $x$ :  $\tan(x) = 1$



One solution comes from

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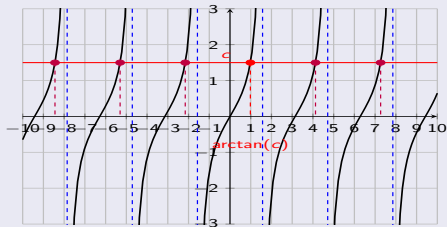
All other solutions come from adding or subtracting  $\pi$ s:

$$\dots, \frac{\pi}{4} - 2\pi, \frac{\pi}{4} - \pi, \frac{\pi}{4}, \frac{\pi}{4} + \pi, \frac{\pi}{4} + 2\pi, \dots$$

General solution:

$$x = \frac{\pi}{4} + n \cdot \pi, \quad \text{for } n \in \mathbb{Z}$$

## Solving $\tan(x) = c$



One solution comes from

$$x = \arctan(c)$$

All other solutions come from adding or subtracting multiple of  $\pi$ s:

$$x = \arctan(c) + n \cdot \pi \quad \text{for } n \in \mathbb{Z}$$

## Solving equations with $\tan$ - exercises

Solve  $\tan(x) = c$

Solution:  $x = \arctan(c) + n \cdot \pi$  for  $n \in \mathbb{Z}$

1 Solve for  $x$ :  $\tan(x) = -\sqrt{3}$



## Solving equations with $\tan$ - exercises

Solve  $\tan(x) = c$

Solution:  $x = \arctan(c) + n \cdot \pi$  for  $n \in \mathbb{Z}$

④ Solve for  $x$ :  $\tan(x) = -\sqrt{3}$

Solution:

$$\arctan(-\sqrt{3}) = -\frac{\pi}{3}$$

Solve  $\tan(x) = c$

Solution:  $x = \arctan(c) + n \cdot \pi$  for  $n \in \mathbb{Z}$

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## Solving equations with tan - exercises

Solve  $\tan(x) = c$

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$$\arctan(-\sqrt{3}) = -\frac{\pi}{3}$$

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2 Solve for  $x$ :  $\tan(x) = \frac{\sqrt{3}}{3}$

3 Solve for  $x$ :  $\tan(x) = 0$

4 Solve for  $x$ :  $\tan(x) + 1 = 0$

5 Solve for  $x$ :  $\tan(x) - \sqrt{3} = 0$

## Solving equations with $\tan$ - exercises

Solve  $\tan(x) = c$

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Solution:

$$\arctan(-\sqrt{3}) = -\frac{\pi}{3}$$

$$\Rightarrow x = -\frac{\pi}{3} + n\pi, \text{ for } n \in \mathbb{Z}$$

2 Solve for  $x$ :  $\tan(x) = \frac{\sqrt{3}}{3}$

Solution:

$$\arctan\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$$

$$\Rightarrow x = \frac{\pi}{6} + n\pi, \text{ for } n \in \mathbb{Z}$$

3 Solve for  $x$ :  $\tan(x) = 0$

4 Solve for  $x$ :  $\tan(x) + 1 = 0$

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## Solving equations with $\tan$ - exercises

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Solution:

$$\arctan\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$$

$$\Rightarrow x = \frac{\pi}{6} + n\pi, \text{ for } n \in \mathbb{Z}$$

3 Solve for  $x$ :  $\tan(x) = 0$

Solution:

$$\arctan(0) = 0$$

$$\Rightarrow x = 0 + n\pi, \text{ for } n \in \mathbb{Z}$$

4 Solve for  $x$ :  $\tan(x) + 1 = 0$

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## Solving equations with $\tan$ - exercises

### Solve $\tan(x) = c$

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① Solve for  $x$ :  $\tan(x) = -\sqrt{3}$

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$$\Rightarrow x = -\frac{\pi}{3} + n\pi, \text{ for } n \in \mathbb{Z}$$

② Solve for  $x$ :  $\tan(x) = \frac{\sqrt{3}}{3}$

Solution:

$$\arctan\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$$

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④ Solve for  $x$ :  $\tan(x) + 1 = 0$

Solution:

$$\Rightarrow \tan(x) = -1$$

$$\arctan(-1) = -\frac{\pi}{4}$$

$$\Rightarrow x = -\frac{\pi}{4} + n\pi, \text{ for } n \in \mathbb{Z}$$

⑤ Solve for  $x$ :  $\tan(x) - \sqrt{3} = 0$

# Solving equations with $\tan$ - exercises

## Solve $\tan(x) = c$

Solution:  $x = \arctan(c) + n \cdot \pi$  for  $n \in \mathbb{Z}$

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$$\arctan(-\sqrt{3}) = -\frac{\pi}{3}$$

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② Solve for  $x$ :  $\tan(x) = \frac{\sqrt{3}}{3}$

Solution:

$$\arctan\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$$

$$\Rightarrow x = \frac{\pi}{6} + n\pi, \text{ for } n \in \mathbb{Z}$$

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⑤ Solve for  $x$ :  $\tan(x) - \sqrt{3} = 0$

Solution:

$$\Rightarrow \tan(x) = \sqrt{3}$$

$$\arctan(\sqrt{3}) = \frac{\pi}{3}$$

$$\Rightarrow x = \frac{\pi}{3} + n\pi, \text{ for } n \in \mathbb{Z}$$

## Solve $\tan(x) = c$

Solution:  $x = \arctan(c) + n \cdot \pi$  for  $n \in \mathbb{Z}$

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$$\Rightarrow x = -\frac{\pi}{3} + n\pi, \text{ for } n \in \mathbb{Z}$$

2 Solve for  $x$ :  $\tan(x) = \frac{\sqrt{3}}{3}$

Solution:

$$\arctan\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$$

$$\Rightarrow x = \frac{\pi}{6} + n\pi, \text{ for } n \in \mathbb{Z}$$

3 Solve for  $x$ :  $\tan(x) = 0$

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$$\Rightarrow x = -\frac{\pi}{4} + n\pi, \text{ for } n \in \mathbb{Z}$$

5 Solve for  $x$ :  $\tan(x) - \sqrt{3} = 0$

Solution:

$$\Rightarrow \tan(x) = \sqrt{3}$$

$$\arctan(\sqrt{3}) = \frac{\pi}{3}$$

$$\Rightarrow x = \frac{\pi}{3} + n\pi, \text{ for } n \in \mathbb{Z}$$

6 Solve for  $x$ :  $\tan^2(x) - \tan(x) = 0$



## Solve $\tan(x) = c$

Solution:  $x = \arctan(c) + n \cdot \pi$  for  $n \in \mathbb{Z}$

1 Solve for  $x$ :  $\tan(x) = -\sqrt{3}$

Solution:

$$\arctan(-\sqrt{3}) = -\frac{\pi}{3}$$

$$\Rightarrow x = -\frac{\pi}{3} + n\pi, \text{ for } n \in \mathbb{Z}$$

2 Solve for  $x$ :  $\tan(x) = \frac{\sqrt{3}}{3}$

Solution:

$$\arctan\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$$

$$\Rightarrow x = \frac{\pi}{6} + n\pi, \text{ for } n \in \mathbb{Z}$$

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5 Solve for  $x$ :  $\tan(x) - \sqrt{3} = 0$

Solution:

$$\Rightarrow \tan(x) = \sqrt{3}$$

$$\arctan(\sqrt{3}) = \frac{\pi}{3}$$

$$\Rightarrow x = \frac{\pi}{3} + n\pi, \text{ for } n \in \mathbb{Z}$$

6 Solve for  $x$ :  $\tan^2(x) - \tan(x) = 0$

Solution:

$$\Rightarrow \tan(x) \cdot (\tan(x) - 1) = 0$$

$$\Rightarrow \tan(x) = 0 \text{ or } \tan(x) = 1$$

$$\arctan(0) = 0$$

$$\arctan(1) = \frac{\pi}{4}$$

$$\Rightarrow x = 0 + n\pi, \text{ for } n \in \mathbb{Z}$$

$$\text{or } x = \frac{\pi}{4} + n\pi, \text{ for } n \in \mathbb{Z}$$

## Solving equations with $\tan$ - exercises

Solve  $\tan(x) = c$

Solution:  $x = \arctan(c) + n \cdot \pi$  for  $n \in \mathbb{Z}$

• Solve for  $x$ :  $\tan^2(x) + \sqrt{3}\tan(x) = 0$

## Solving equations with $\tan$ - exercises

Solve  $\tan(x) = c$

Solution:  $x = \arctan(c) + n \cdot \pi$  for  $n \in \mathbb{Z}$

• Solve for  $x$ :  $\tan^2(x) + \sqrt{3}\tan(x) = 0$

Solution:

Substitute  $u = \tan(x)$

$$\Rightarrow u^2 + \sqrt{3}u = 0 \Rightarrow u(u + \sqrt{3}) = 0$$

$$\Rightarrow u = 0 \text{ or } u = -\sqrt{3}$$

$$\Rightarrow \tan(x) = 0 \text{ or } \tan(x) = -\sqrt{3}$$

## Solving equations with $\tan$ - exercises

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## Solving equations with $\tan$ - exercises

Solve  $\tan(x) = c$

Solution:  $x = \arctan(c) + n \cdot \pi$  for  $n \in \mathbb{Z}$

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Solution:

Substitute  $u = \tan(x)$

$$\Rightarrow u^2 + \sqrt{3}u = 0 \Rightarrow u(u + \sqrt{3}) = 0$$

$$\Rightarrow u = 0 \text{ or } u = -\sqrt{3}$$

$$\Rightarrow \tan(x) = 0 \text{ or } \tan(x) = -\sqrt{3}$$

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$$\text{or } x = -\frac{\pi}{3} + n\pi, \text{ for } n \in \mathbb{Z}$$

8 Solve for  $x$ :  $\tan^2(x) - 1 = 0$

9 Solve for  $x$ :  $3\tan^2(x) - 1 = 0$

## Solving equations with $\tan$ - exercises

Solve  $\tan(x) = c$

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8 Solve for  $x$ :  $\tan^2(x) - 1 = 0$

Solution:

Substitute  $u = \tan(x)$

$$\Rightarrow u^2 - 1 = 0 \Rightarrow u^2 = 1 \Rightarrow u = \pm 1$$

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## Solving equations with $\tan$ - exercises

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Solution:  $x = \arctan(c) + n \cdot \pi$  for  $n \in \mathbb{Z}$

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Substitute  $u = \tan(x)$

$$\Rightarrow 3u^2 - 1 = 0 \Rightarrow 3u^2 = 1$$

$$\Rightarrow u^2 = \frac{1}{3} \Rightarrow u = \pm \frac{1}{\sqrt{3}} = \pm \frac{\sqrt{3}}{3}$$

$$\Rightarrow \tan(x) = \frac{\sqrt{3}}{3} \text{ or } \tan(x) = -\frac{\sqrt{3}}{3}$$



# Solving equations with $\tan$ - exercises

Solve  $\tan(x) = c$

Solution:  $x = \arctan(c) + n \cdot \pi$  for  $n \in \mathbb{Z}$

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Solution:

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$$\Rightarrow \tan(x) = 0 \text{ or } \tan(x) = -\sqrt{3}$$

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9 Solve for  $x$ :  $3\tan^2(x) - 1 = 0$

Solution:

Substitute  $u = \tan(x)$

$$\Rightarrow 3u^2 - 1 = 0 \Rightarrow 3u^2 = 1$$

$$\Rightarrow u^2 = \frac{1}{3} \Rightarrow u = \pm \frac{1}{\sqrt{3}} = \pm \frac{\sqrt{3}}{3}$$

$$\Rightarrow \tan(x) = \frac{\sqrt{3}}{3} \text{ or } \tan(x) = -\frac{\sqrt{3}}{3}$$

$$\arctan\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$$

$$\arctan\left(-\frac{\sqrt{3}}{3}\right) = -\frac{\pi}{6}$$

$$\Rightarrow x = \frac{\pi}{6} + n\pi, \text{ for } n \in \mathbb{Z}$$

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