Graphs of trigonometric functions

Lesson #18

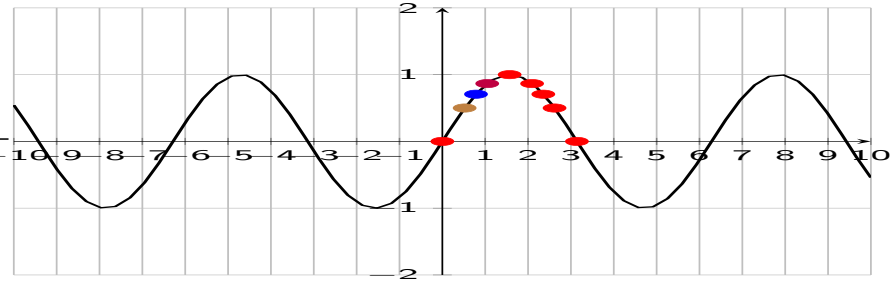
MAT 1375 Precalculus

New York City College of Technology CUNY

Graphs of sin, cos, and tan

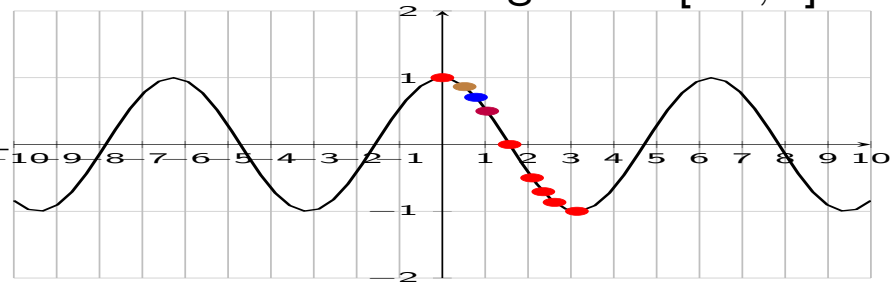
- Graph the function $y = \sin(x)$ domain $D = \mathbb{R}$ range $R = [-1, 1]$

x	0°	30°	45°	60°	90°	120°	135°	150°	180°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
y	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\approx$	0	.5	.707	.866	1	.866	.707	.5	0



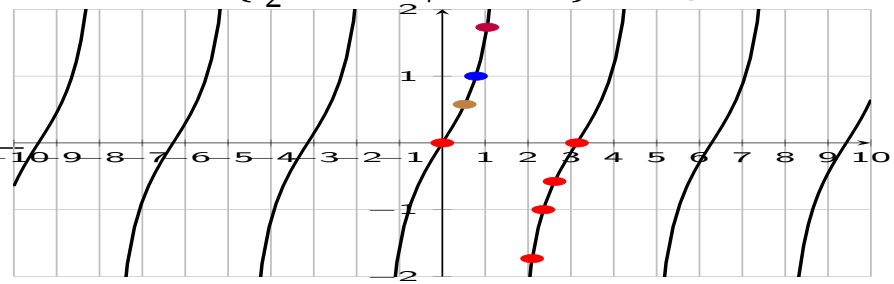
- Graph the function $y = \cos(x)$ domain $D = \mathbb{R}$ range $R = [-1, 1]$

x	0°	30°	45°	60°	90°	120°	135°	150°	180°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
y	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
$\approx$	1	.866	.707	.5	0	$-.5$	$-.707$	$-.866$	-1



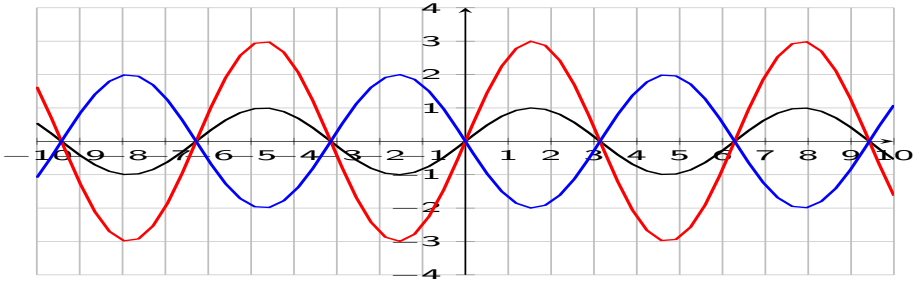
- Graph the function $y = \tan(x)$ domain $D = \mathbb{R} - \{\frac{\pi}{2} + k\pi \mid k \in \mathbb{Z}\}$ range $R = \mathbb{R}$

x	0°	30°	45°	60°	90°	120°	135°	150°	180°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
y	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undef	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0
$\approx$	0	.577	1	1.73		-1.73	-1	$-.577$	0



Amplitude, period, and phase shift for sin, cos

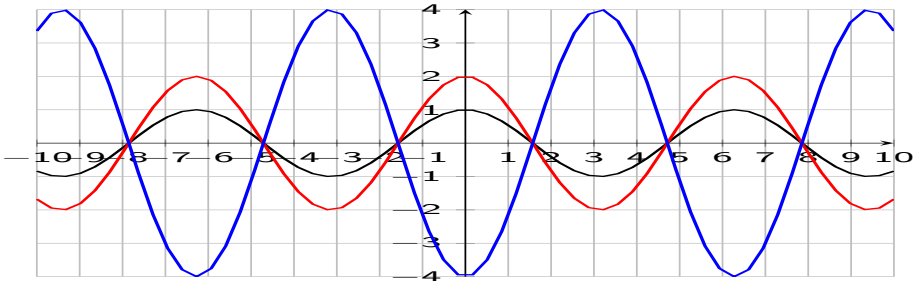
- Amplitude A



$y = \sin(x)$ $\quad A = 1$

$y = 3\sin(x)$ $\quad A = 3$

$y = -2\sin(x)$ $\quad A = 2$

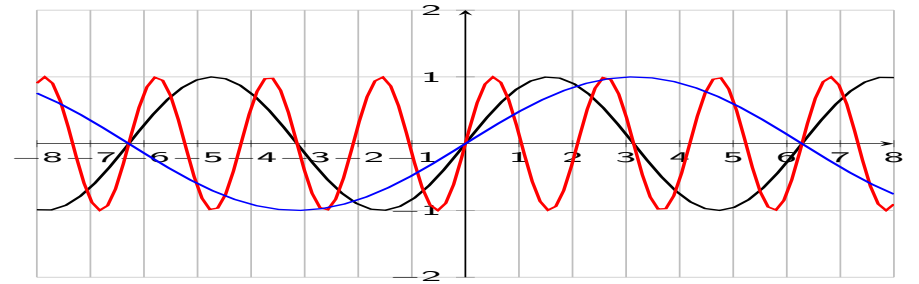


$y = \cos(x)$ $\quad A = 1$

$y = 2\cos(x)$ $\quad A = 2$

$y = -4\cos(x)$ $\quad A = 4$

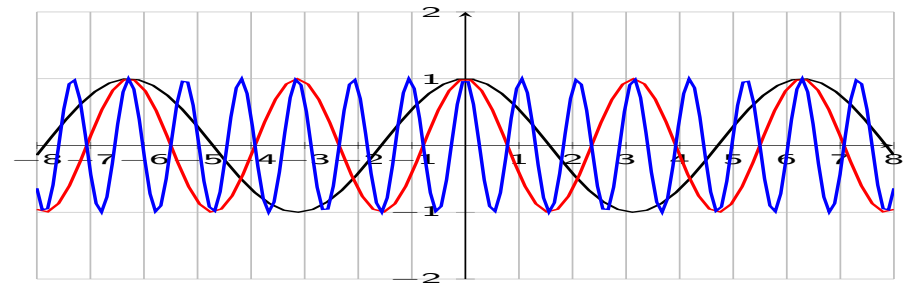
- Period P



$y = \sin(x)$ $\quad P = 2\pi$

$y = \sin(3x)$ $\quad P = \frac{2\pi}{3}$

$y = \sin(\frac{1}{2}x)$ $\quad P = \frac{2\pi}{\frac{1}{2}} = 2\pi \cdot \frac{2}{1} = 4\pi$



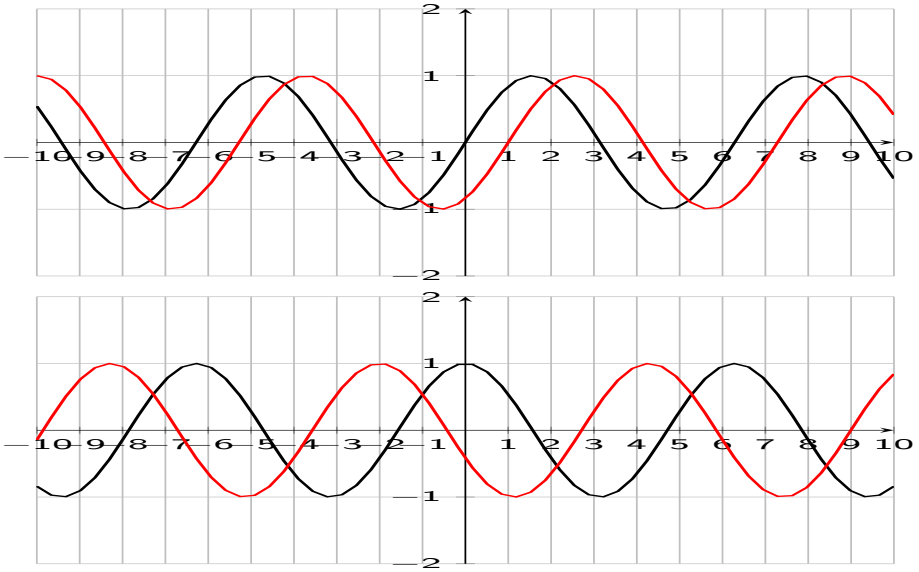
$y = \cos(x)$ $\quad P = 2\pi$

$y = \cos(2x)$ $\quad P = \frac{2\pi}{2} = \pi$

$y = \cos(6x)$ $\quad P = \frac{2\pi}{6} = \frac{\pi}{3}$

Amplitude, period, and phase shift for sin, cos

- Phase shift S



$y = \sin(x)$ $\quad S = 0$

$y = \sin(x - 1)$ $\quad S = 1$

$y = \cos(x)$ $\quad S = 0$

$y = \cos(x + 2)$ $\quad S = -2$

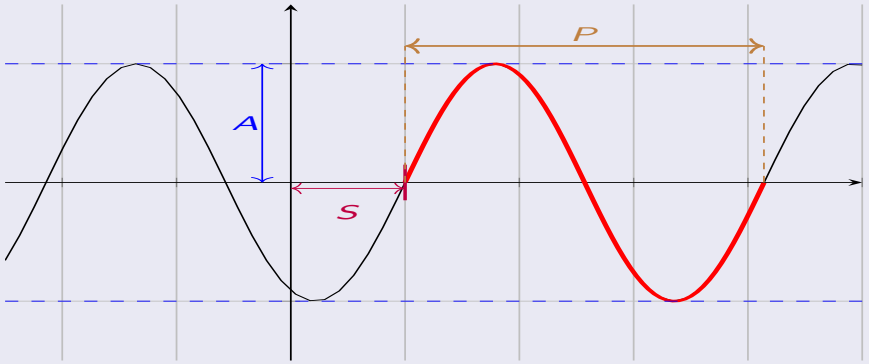
Amplitude, period, phase shift

When $y = a \cdot \sin(bx + c)$ or $y = a \cdot \cos(bx + c)$

Amplitude $A = |a|$

Period $P = \left|\frac{2\pi}{b}\right|$

Phase shift $S = \frac{-c}{b}$

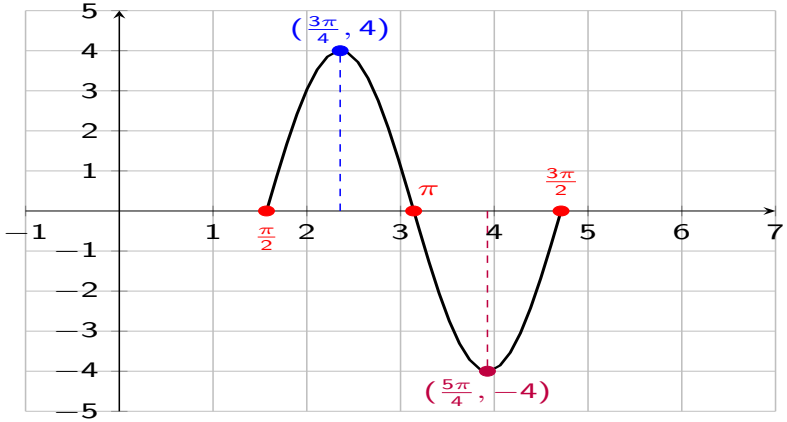


Graphing sin and cos - exercises

Find amplitude, period, and phase shift. Graph the function over one period. Label all maxima, minima, and roots.

1. $y = 4\sin(2x - \pi)$

$A = 4,\ P = \frac{2\pi}{2} = \pi,\ S = \frac{-(-\pi)}{2} = \frac{\pi}{2}$



Roots:

- $S = \frac{\pi}{2}$
- $S + P = \frac{\pi}{2} + \pi = \frac{3\pi}{2}$
- $\left(\frac{\pi}{2} + \frac{3\pi}{2}\right) \div 2 = \frac{4\pi}{2} \cdot \frac{1}{2} = \frac{4\pi}{4} = \pi$

Maximum:

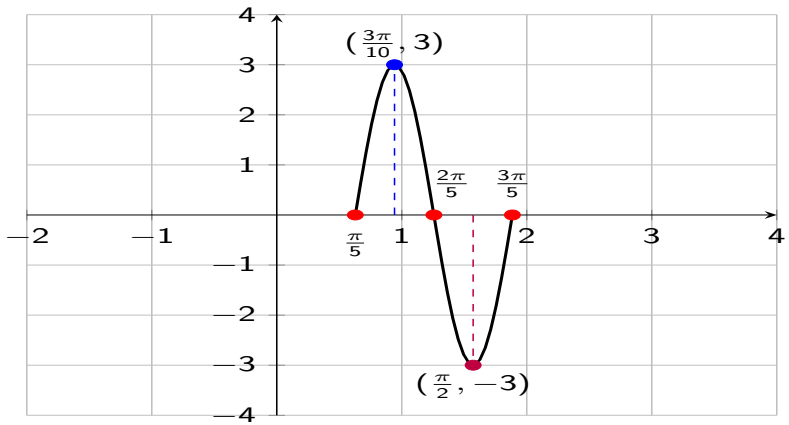
- $\left(\frac{\pi}{2} + \pi\right) \div 2 = \frac{3\pi}{2} \cdot \frac{1}{2} = \frac{3\pi}{4}$

Minimum:

- $\left(\pi + \frac{3\pi}{2}\right) \div 2 = \frac{5\pi}{2} \cdot \frac{1}{2} = \frac{5\pi}{4}$

2. $y = 3\sin(5x - \pi)$

$A = 3,\ P = \frac{2\pi}{5},\ S = \frac{-(-\pi)}{5} = \frac{\pi}{5}$



Roots:

- $S = \frac{\pi}{5}$
- $S + P = \frac{\pi}{5} + \frac{2\pi}{5} = \frac{3\pi}{5}$
- $\left(\frac{\pi}{5} + \frac{3\pi}{5}\right) \div 2 = \frac{4\pi}{5} \cdot \frac{1}{2} = \frac{2\pi}{5}$

Maximum:

- $\left(\frac{\pi}{5} + \frac{2\pi}{5}\right) \div 2 = \frac{3\pi}{5} \cdot \frac{1}{2} = \frac{3\pi}{10}$

Minimum:

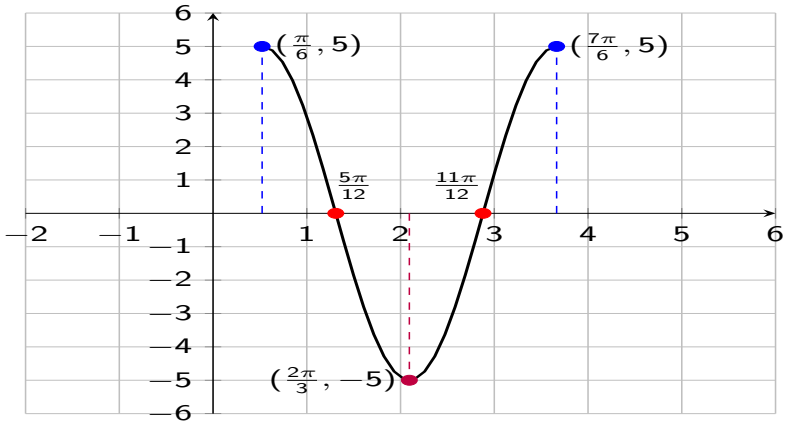
- $\left(\frac{2\pi}{5} + \frac{3\pi}{5}\right) \div 2 = \frac{5\pi}{5} \cdot \frac{1}{2} = \frac{\pi}{2}$

Graphing sin and cos - exercises

Find amplitude, period, and phase shift. Graph the function over one period. Label all maxima, minima, and roots.

3. $y = 5\cos(2x - \frac{\pi}{3})$

$A = 5,\ P = \frac{2\pi}{2} = \pi,\ S = \frac{-(-\frac{\pi}{3})}{2} = \frac{\pi}{6}$



Maxima:

- $S = \frac{\pi}{6}$
- $S + P = \frac{\pi}{6} + \pi = \frac{7\pi}{6}$

Minimum:

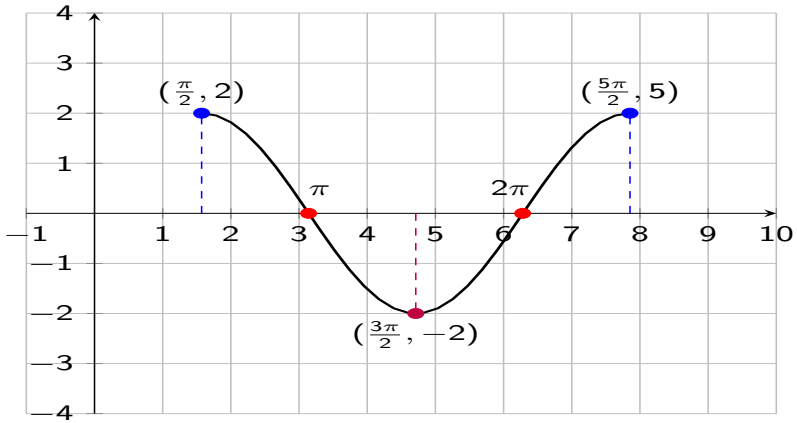
- $(\frac{\pi}{6} + \frac{7\pi}{6}) \div 2 = \frac{8\pi}{6} \cdot \frac{1}{2} = \frac{8\pi}{12} = \frac{2\pi}{3}$

Roots:

- $(\frac{\pi}{6} + \frac{2\pi}{3}) \div 2 = \frac{5\pi}{6} \cdot \frac{1}{2} = \frac{5\pi}{12}$
- $(\frac{2\pi}{3} + \frac{7\pi}{6}) \div 2 = \frac{11\pi}{6} \cdot \frac{1}{2} = \frac{11\pi}{12}$

4. $y = 2\cos(x - \frac{\pi}{2})$

$A = 2,\ P = 2\pi,\ S = \frac{-(-\frac{\pi}{2})}{1} = \frac{\pi}{2}$



Maxima:

- $S = \frac{\pi}{2}$
- $S + P = \frac{\pi}{2} + 2\pi = \frac{5\pi}{2}$

Minimum:

- $(\frac{\pi}{2} + \frac{5\pi}{2}) \div 2 = \frac{6\pi}{2} \cdot \frac{1}{2} = \frac{3\pi}{2}$

Roots:

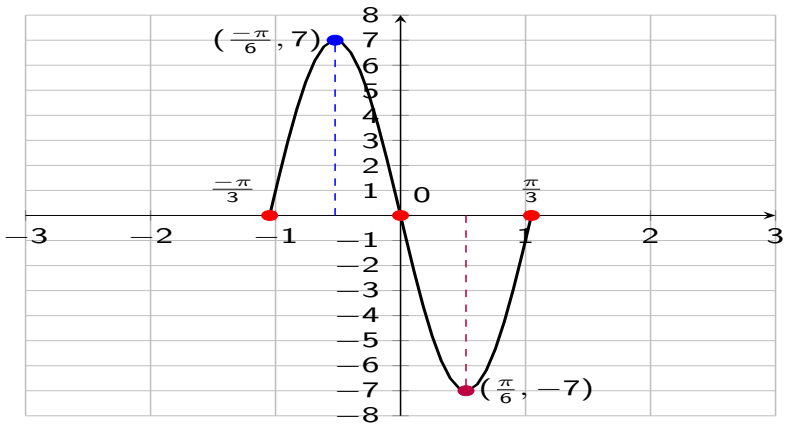
- $(\frac{\pi}{2} + \frac{3\pi}{2}) \div 2 = \frac{4\pi}{2} \cdot \frac{1}{2} = \frac{4\pi}{4} = \pi$
- $(\frac{3\pi}{2} + \frac{5\pi}{2}) \div 2 = \frac{8\pi}{2} \cdot \frac{1}{2} = \frac{8\pi}{4} = 2\pi$

Graphing sin and cos - exercises

Find amplitude, period, and phase shift. Graph the function over one period. Label all maxima, minima, and roots.

5. $y = 7\sin(3x + \pi)$

$A = 7,\ P = \frac{2\pi}{3},\ S = \frac{-\pi}{3}$



Roots:

- $S = \frac{-\pi}{3}$
- $S + P = \frac{-\pi}{3} + \frac{2\pi}{3} = \frac{\pi}{3}$
- $\left(\frac{-\pi}{3} + \frac{\pi}{3}\right) \div 2 = 0 \cdot \frac{1}{2} = 0$

Maximum:

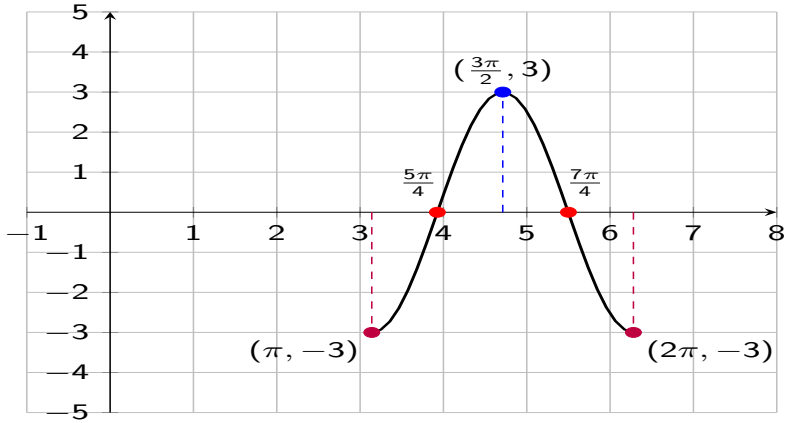
- $\left(\frac{-\pi}{3} + 0\right) \div 2 = \frac{-\pi}{3} \cdot \frac{1}{2} = \frac{-\pi}{6}$

Minimum:

- $\left(0 + \frac{\pi}{3}\right) \div 2 = \frac{\pi}{3} \cdot \frac{1}{2} = \frac{\pi}{6}$

6. $y = -3\cos(2x - 2\pi)$

$A = |-3| = 3,\ P = \frac{2\pi}{2} = \pi,\ S = \frac{-(-2\pi)}{2} = \pi$



Minima:

- $S = \pi$
- $S + P = \pi + \pi = 2\pi$

Maximum:

- $\left(\pi + 2\pi\right) \div 2 = 3\pi \cdot \frac{1}{2} = \frac{3\pi}{2}$

Roots:

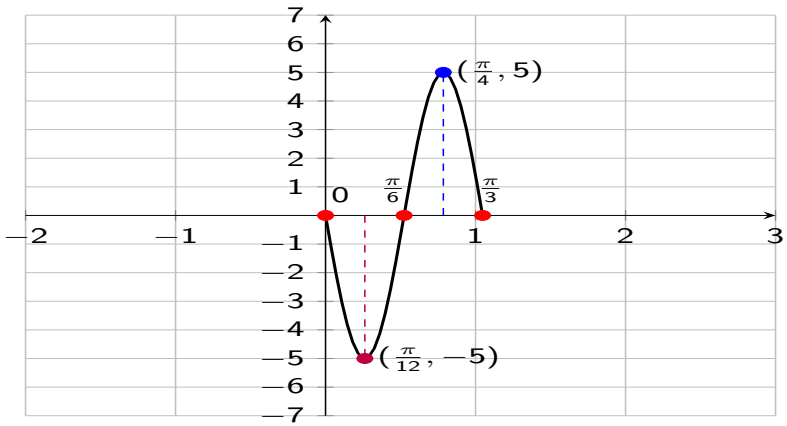
- $\left(\pi + \frac{3\pi}{2}\right) \div 2 = \frac{5\pi}{2} \cdot \frac{1}{2} = \frac{5\pi}{4}$
- $\left(\frac{3\pi}{2} + 2\pi\right) \div 2 = \frac{7\pi}{2} \cdot \frac{1}{2} = \frac{7\pi}{4}$

Graphing sin and cos - exercises

Find amplitude, period, and phase shift. Graph the function over one period. Label all maxima, minima, and roots.

7. $y = -5\sin(6x)$

$A = 5,\ P = \frac{2\pi}{6} = \frac{\pi}{3},\ S = \frac{-0}{6} = 0$



Roots:

- $S = 0$
- $S + P = 0 + \frac{\pi}{3} = \frac{\pi}{3}$
- $\left(0 + \frac{\pi}{3}\right) \div 2 = \frac{\pi}{3} \cdot \frac{1}{2} = \frac{\pi}{6}$

Minimum:

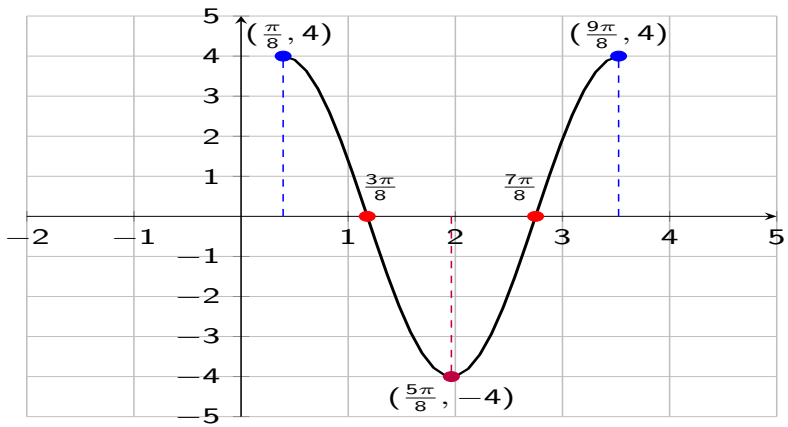
- $\left(0 + \frac{\pi}{6}\right) \div 2 = \frac{\pi}{6} \cdot \frac{1}{2} = \frac{\pi}{12}$

Maximum:

- $\left(\frac{\pi}{6} + \frac{\pi}{3}\right) \div 2 = \frac{3\pi}{6} \cdot \frac{1}{2} = \frac{3\pi}{12} = \frac{\pi}{4}$

8. $y = 4\cos(2x - \frac{\pi}{4})$

$A = 4,\ P = \frac{2\pi}{2} = \pi,\ S = \frac{-(-\frac{\pi}{4})}{2} = \frac{\pi}{8}$



Maxima:

- $S = \frac{\pi}{8}$
- $S + P = \frac{\pi}{8} + \pi = \frac{9\pi}{8}$

Minimum:

- $\left(\frac{\pi}{8} + \frac{9\pi}{8}\right) \div 2 = \frac{10\pi}{8} \cdot \frac{1}{2} = \frac{10\pi}{16} = \frac{5\pi}{8}$

Roots:

- $\left(\frac{\pi}{8} + \frac{5\pi}{8}\right) \div 2 = \frac{6\pi}{8} \cdot \frac{1}{2} = \frac{3\pi}{8}$
- $\left(\frac{5\pi}{8} + \frac{9\pi}{8}\right) \div 2 = \frac{14\pi}{8} \cdot \frac{1}{2} = \frac{7\pi}{8}$