

Graphs of trigonometric functions

Lesson #18

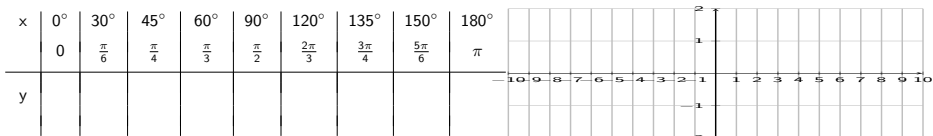
MAT 1375 Precalculus

New York City College of Technology CUNY



Graphs of sin, cos, and tan

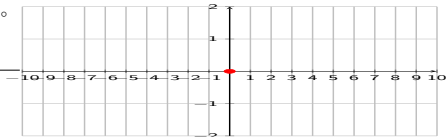
- Graph the function $y = \sin(x)$



Graphs of sin, cos, and tan

- Graph the function $y = \sin(x)$

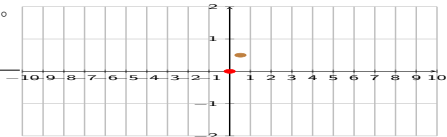
x	0°	30°	45°	60°	90°	120°	135°	150°	180°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
y	0								
\approx	0								



Graphs of sin, cos, and tan

- Graph the function $y = \sin(x)$

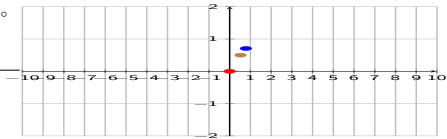
x	0°	30°	45°	60°	90°	120°	135°	150°	180°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
y	0	$\frac{1}{2}$							
\approx	0	.5							



Graphs of sin, cos, and tan

- Graph the function $y = \sin(x)$

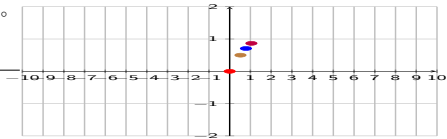
x	0°	30°	45°	60°	90°	120°	135°	150°	180°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
y	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$						
\approx	0	.5	.707						



Graphs of sin, cos, and tan

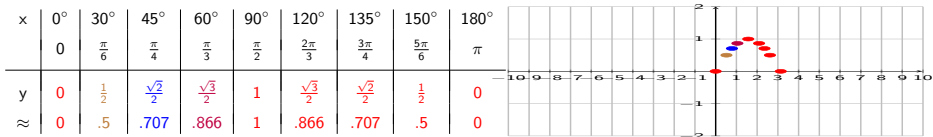
- Graph the function $y = \sin(x)$

x	0°	30°	45°	60°	90°	120°	135°	150°	180°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
y	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$					
\approx	0	.5	.707	.866					



Graphs of sin, cos, and tan

- Graph the function $y = \sin(x)$

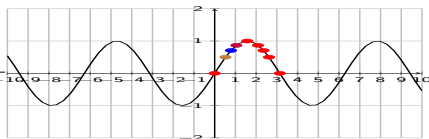


Graphs of sin, cos, and tan

• Graph the function $y = \sin(x)$

domain $D = \mathbb{R}$ range $R = [-1, 1]$

x	0°	30°	45°	60°	90°	120°	135°	150°	180°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
y	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
≈	0	.5	.707	.866	1	.866	.707	.5	0

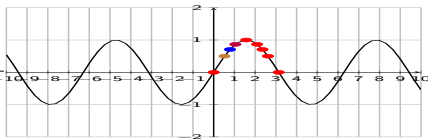


Graphs of sin, cos, and tan

• Graph the function $y = \sin(x)$

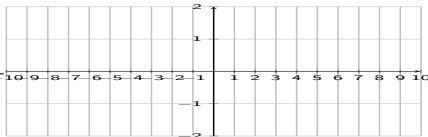
domain $D = \mathbb{R}$ range $R = [-1, 1]$

x	0°	30°	45°	60°	90°	120°	135°	150°	180°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
y	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
\approx	0	.5	.707	.866	1	.866	.707	.5	0



• Graph the function $y = \cos(x)$

x	0°	30°	45°	60°	90°	120°	135°	150°	180°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
y									

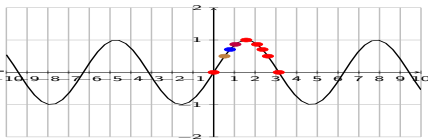


Graphs of sin, cos, and tan

• Graph the function $y = \sin(x)$

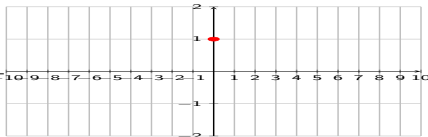
domain $D = \mathbb{R}$ range $R = [-1, 1]$

x	0°	30°	45°	60°	90°	120°	135°	150°	180°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
y	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
≈	0	.5	.707	.866	1	.866	.707	.5	0



• Graph the function $y = \cos(x)$

x	0°	30°	45°	60°	90°	120°	135°	150°	180°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
y	1								
≈	1								

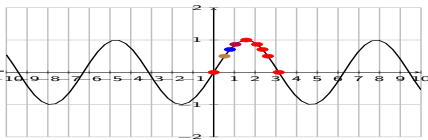


Graphs of sin, cos, and tan

• Graph the function $y = \sin(x)$

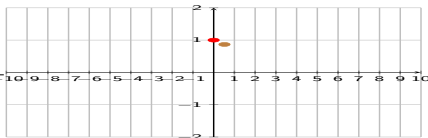
domain $D = \mathbb{R}$ range $R = [-1, 1]$

x	0°	30°	45°	60°	90°	120°	135°	150°	180°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
y	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
≈	0	.5	.707	.866	1	.866	.707	.5	0



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x	0°	30°	45°	60°	90°	120°	135°	150°	180°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
y	1	$\frac{\sqrt{3}}{2}$							
≈	1	.866							

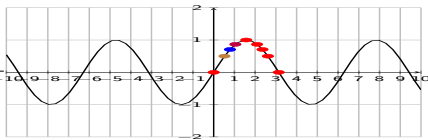


Graphs of sin, cos, and tan

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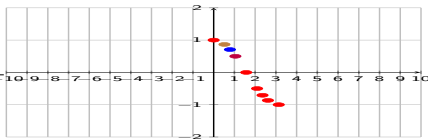
domain $D = \mathbb{R}$ range $R = [-1, 1]$

x	0°	30°	45°	60°	90°	120°	135°	150°	180°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
y	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
≈	0	.5	.707	.866	1	.866	.707	.5	0



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x	0°	30°	45°	60°	90°	120°	135°	150°	180°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
y	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
≈	1	.866	.707	.5	0	-.5	-.707	-.866	-1

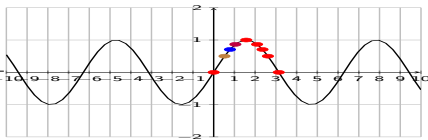


Graphs of sin, cos, and tan

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domain $D = \mathbb{R}$ range $R = [-1, 1]$

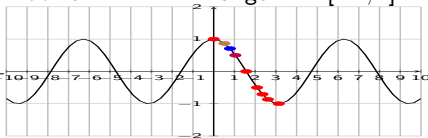
x	0°	30°	45°	60°	90°	120°	135°	150°	180°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
y	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
≈	0	.5	.707	.866	1	.866	.707	.5	0



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domain $D = \mathbb{R}$ range $R = [-1, 1]$

x	0°	30°	45°	60°	90°	120°	135°	150°	180°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
y	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
≈	1	.866	.707	.5	0	-.5	-.707	-.866	-1

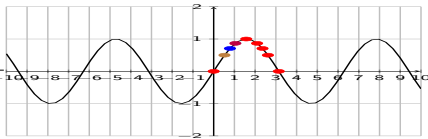


Graphs of sin, cos, and tan

• Graph the function $y = \sin(x)$

domain $D = \mathbb{R}$ range $R = [-1, 1]$

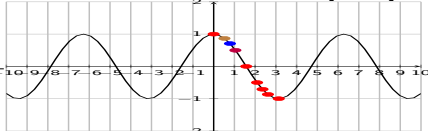
x	0°	30°	45°	60°	90°	120°	135°	150°	180°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
y	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
≈	0	.5	.707	.866	1	.866	.707	.5	0



• Graph the function $y = \cos(x)$

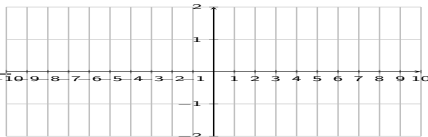
domain $D = \mathbb{R}$ range $R = [-1, 1]$

x	0°	30°	45°	60°	90°	120°	135°	150°	180°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
y	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
≈	1	.866	.707	.5	0	-.5	-.707	-.866	-1



• Graph the function $y = \tan(x)$

x	0°	30°	45°	60°	90°	120°	135°	150°	180°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
y									

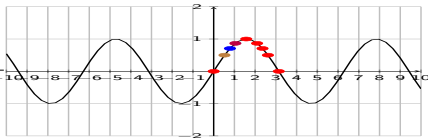


Graphs of sin, cos, and tan

• Graph the function $y = \sin(x)$

domain $D = \mathbb{R}$ range $R = [-1, 1]$

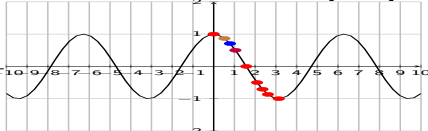
x	0°	30°	45°	60°	90°	120°	135°	150°	180°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
y	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
≈	0	.5	.707	.866	1	.866	.707	.5	0



• Graph the function $y = \cos(x)$

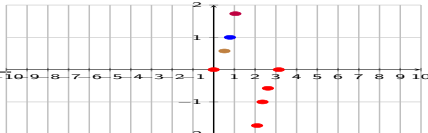
domain $D = \mathbb{R}$ range $R = [-1, 1]$

x	0°	30°	45°	60°	90°	120°	135°	150°	180°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
y	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
≈	1	.866	.707	.5	0	-.5	-.707	-.866	-1



• Graph the function $y = \tan(x)$

x	0°	30°	45°	60°	90°	120°	135°	150°	180°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
y	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undef	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0
≈	0	.577	1	1.73		-1.73	-1	-.577	0

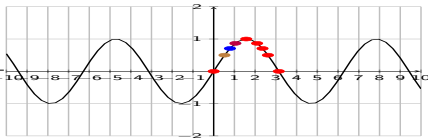


Graphs of sin, cos, and tan

• Graph the function $y = \sin(x)$

domain $D = \mathbb{R}$ range $R = [-1, 1]$

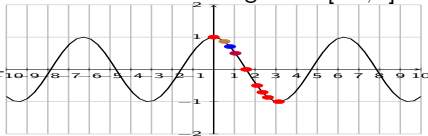
x	0°	30°	45°	60°	90°	120°	135°	150°	180°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
y	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
≈	0	.5	.707	.866	1	.866	.707	.5	0



• Graph the function $y = \cos(x)$

domain $D = \mathbb{R}$ range $R = [-1, 1]$

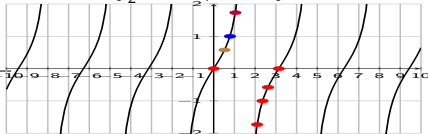
x	0°	30°	45°	60°	90°	120°	135°	150°	180°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
y	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1
≈	1	.866	.707	.5	0	-.5	-.707	-.866	-1



• Graph the function $y = \tan(x)$

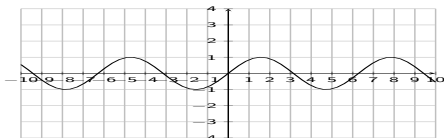
domain $D = \mathbb{R} - \{\frac{\pi}{2} + k\pi \mid k \in \mathbb{Z}\}$ range $R = \mathbb{R}$

x	0°	30°	45°	60°	90°	120°	135°	150°	180°
	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
y	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undef	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0
≈	0	.577	1	1.73		-1.73	-1	-.577	0



Amplitude, period, and phase shift for sin, cos

- Amplitude A

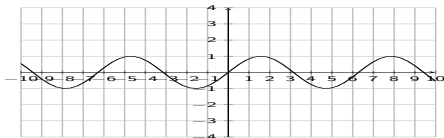


$$y = \sin(x)$$

$$A = 1$$

Amplitude, period, and phase shift for sin, cos

- Amplitude A



$$y = \sin(x)$$

$$y = 3\sin(x)$$

$$A = 1$$

Amplitude, period, and phase shift for sin, cos

• Amplitude A



$$y = \sin(x)$$

$$y = 3\sin(x)$$

$$A = 1$$

$$A = 3$$

Amplitude, period, and phase shift for sin, cos

• Amplitude A



$$y = \sin(x)$$

$$A = 1$$

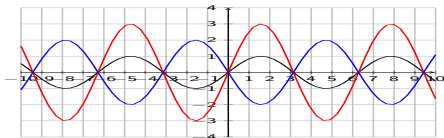
$$y = 3 \sin(x)$$

$$A = 3$$

$$y = -2 \sin(x)$$

Amplitude, period, and phase shift for sin, cos

• Amplitude A



$$y = \sin(x)$$

$$A = 1$$

$$y = 3 \sin(x)$$

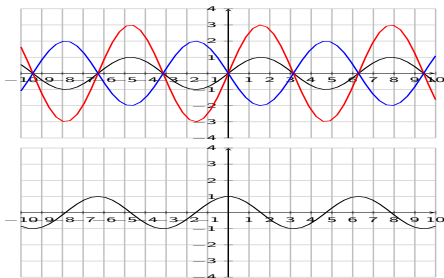
$$A = 3$$

$$y = -2 \sin(x)$$

$$A = 2$$

Amplitude, period, and phase shift for sin, cos

• Amplitude A



$$y = \sin(x) \quad A = 1$$

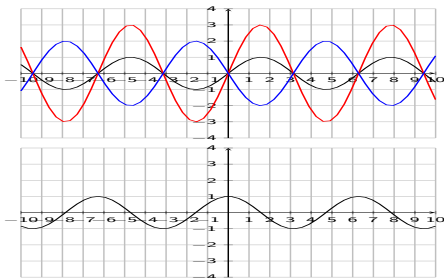
$$y = 3 \sin(x) \quad A = 3$$

$$y = -2 \sin(x) \quad A = 2$$

$$y = \cos(x) \quad A = 1$$

Amplitude, period, and phase shift for sin, cos

• Amplitude A



$$y = \sin(x) \quad A = 1$$

$$y = 3 \sin(x) \quad A = 3$$

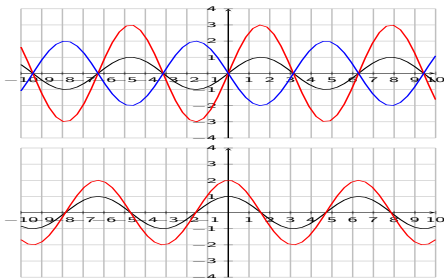
$$y = -2 \sin(x) \quad A = 2$$

$$y = \cos(x) \quad A = 1$$

$$y = 2 \cos(x)$$

Amplitude, period, and phase shift for sin, cos

• Amplitude A



$$y = \sin(x) \quad A = 1$$

$$y = 3 \sin(x) \quad A = 3$$

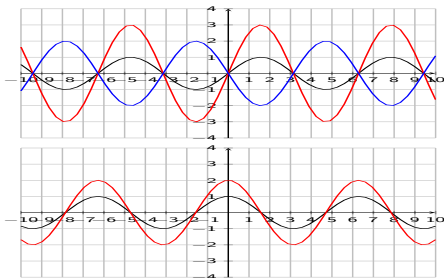
$$y = -2 \sin(x) \quad A = 2$$

$$y = \cos(x) \quad A = 1$$

$$y = 2 \cos(x) \quad A = 2$$

Amplitude, period, and phase shift for sin, cos

• Amplitude A



$$y = \sin(x) \quad A = 1$$

$$y = 3 \sin(x) \quad A = 3$$

$$y = -2 \sin(x) \quad A = 2$$

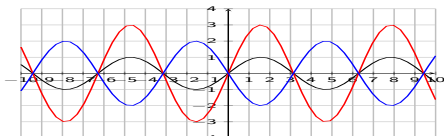
$$y = \cos(x) \quad A = 1$$

$$y = 2 \cos(x) \quad A = 2$$

$$y = -4 \cos(x)$$

Amplitude, period, and phase shift for sin, cos

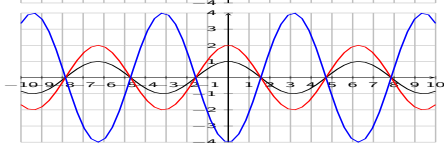
• Amplitude A



$$y = \sin(x) \quad A = 1$$

$$y = 3\sin(x) \quad A = 3$$

$$y = -2\sin(x) \quad A = 2$$

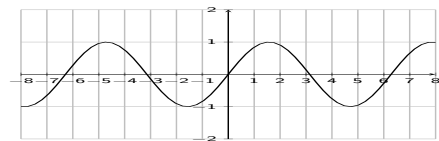


$$y = \cos(x) \quad A = 1$$

$$y = 2\cos(x) \quad A = 2$$

$$y = -4\cos(x) \quad A = 4$$

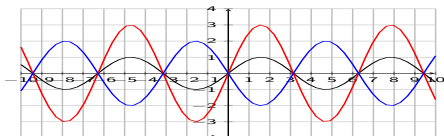
• Period P



$$y = \sin(x) \quad P = 2\pi$$

Amplitude, period, and phase shift for sin, cos

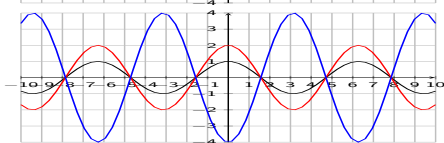
• Amplitude A



$$y = \sin(x) \quad A = 1$$

$$y = 3 \sin(x) \quad A = 3$$

$$y = -2 \sin(x) \quad A = 2$$

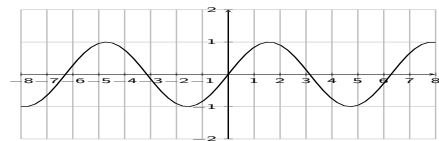


$$y = \cos(x) \quad A = 1$$

$$y = 2 \cos(x) \quad A = 2$$

$$y = -4 \cos(x) \quad A = 4$$

• Period P

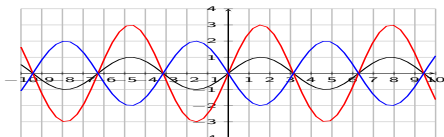


$$y = \sin(x) \quad P = 2\pi$$

$$y = \sin(3x)$$

Amplitude, period, and phase shift for sin, cos

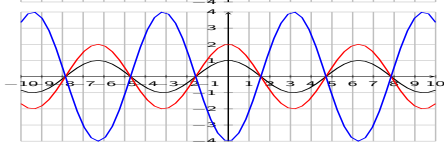
• Amplitude A



$$y = \sin(x) \quad A = 1$$

$$y = 3\sin(x) \quad A = 3$$

$$y = -2\sin(x) \quad A = 2$$

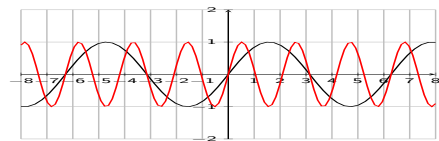


$$y = \cos(x) \quad A = 1$$

$$y = 2\cos(x) \quad A = 2$$

$$y = -4\cos(x) \quad A = 4$$

• Period P

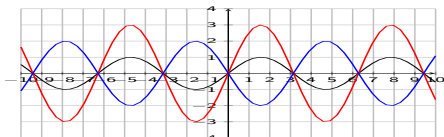


$$y = \sin(x) \quad P = 2\pi$$

$$y = \sin(3x) \quad P = \frac{2\pi}{3}$$

Amplitude, period, and phase shift for sin, cos

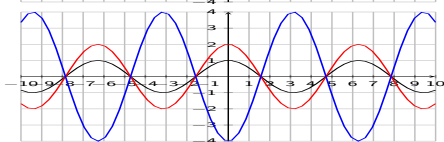
• Amplitude A



$$y = \sin(x) \quad A = 1$$

$$y = 3 \sin(x) \quad A = 3$$

$$y = -2 \sin(x) \quad A = 2$$



$$y = \cos(x) \quad A = 1$$

$$y = 2 \cos(x) \quad A = 2$$

$$y = -4 \cos(x) \quad A = 4$$

• Period P



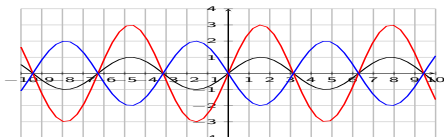
$$y = \sin(x) \quad P = 2\pi$$

$$y = \sin(3x) \quad P = \frac{2\pi}{3}$$

$$y = \sin\left(\frac{1}{2}x\right)$$

Amplitude, period, and phase shift for sin, cos

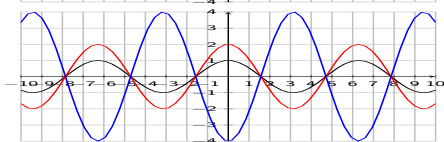
• Amplitude A



$$y = \sin(x) \quad A = 1$$

$$y = 3 \sin(x) \quad A = 3$$

$$y = -2 \sin(x) \quad A = 2$$

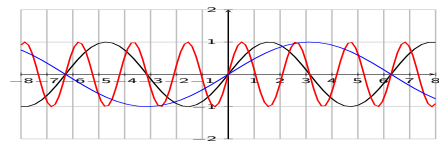


$$y = \cos(x) \quad A = 1$$

$$y = 2 \cos(x) \quad A = 2$$

$$y = -4 \cos(x) \quad A = 4$$

• Period P



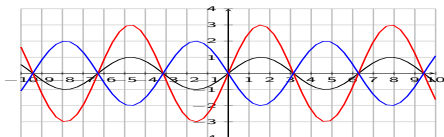
$$y = \sin(x) \quad P = 2\pi$$

$$y = \sin(3x) \quad P = \frac{2\pi}{3}$$

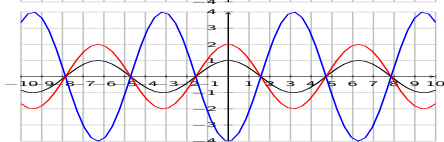
$$y = \sin\left(\frac{1}{2}x\right) \quad P = \frac{2\pi}{\frac{1}{2}} = 2\pi \cdot \frac{2}{1} = 4\pi$$

Amplitude, period, and phase shift for sin, cos

• Amplitude A

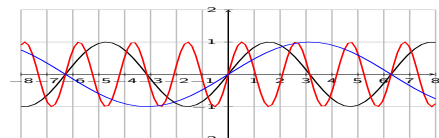


$$\begin{array}{ll} y = \sin(x) & A = 1 \\ y = 3 \sin(x) & A = 3 \\ y = -2 \sin(x) & A = 2 \end{array}$$

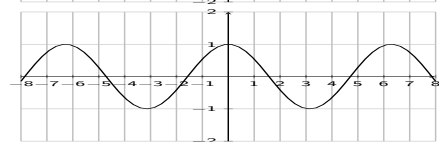


$$\begin{array}{ll} y = \cos(x) & A = 1 \\ y = 2 \cos(x) & A = 2 \\ y = -4 \cos(x) & A = 4 \end{array}$$

• Period P



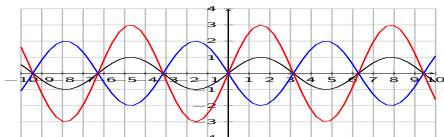
$$\begin{array}{ll} y = \sin(x) & P = 2\pi \\ y = \sin(3x) & P = \frac{2\pi}{3} \\ y = \sin\left(\frac{1}{2}x\right) & P = \frac{2\pi}{\frac{1}{2}} = 2\pi \cdot \frac{2}{1} \\ & = 4\pi \end{array}$$



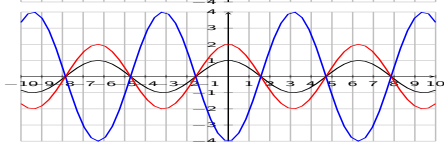
$$\begin{array}{ll} y = \cos(x) & P = 2\pi \\ y = \cos(2x) & \\ y = \cos(6x) & \end{array}$$

Amplitude, period, and phase shift for sin, cos

• Amplitude A

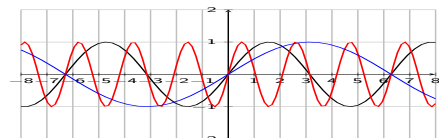


$$\begin{array}{ll} y = \sin(x) & A = 1 \\ y = 3 \sin(x) & A = 3 \\ y = -2 \sin(x) & A = 2 \end{array}$$

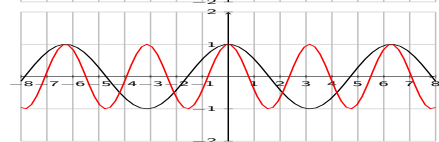


$$\begin{array}{ll} y = \cos(x) & A = 1 \\ y = 2 \cos(x) & A = 2 \\ y = -4 \cos(x) & A = 4 \end{array}$$

• Period P



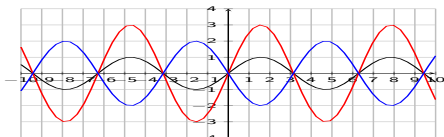
$$\begin{array}{ll} y = \sin(x) & P = 2\pi \\ y = \sin(3x) & P = \frac{2\pi}{3} \\ y = \sin\left(\frac{1}{2}x\right) & P = \frac{2\pi}{\frac{1}{2}} = 2\pi \cdot \frac{2}{1} \\ & = 4\pi \end{array}$$



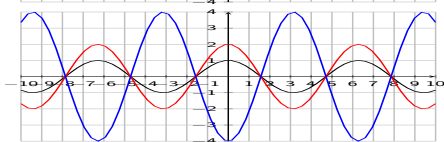
$$\begin{array}{ll} y = \cos(x) & P = 2\pi \\ y = \cos(2x) & P = \frac{2\pi}{2} = \pi \\ y = \cos(6x) & P = \frac{2\pi}{6} = \frac{\pi}{3} \end{array}$$

Amplitude, period, and phase shift for sin, cos

• Amplitude A

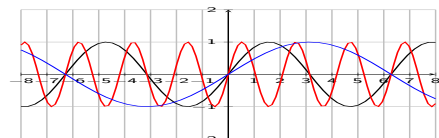


$$\begin{array}{ll} y = \sin(x) & A = 1 \\ y = 3 \sin(x) & A = 3 \\ y = -2 \sin(x) & A = 2 \end{array}$$

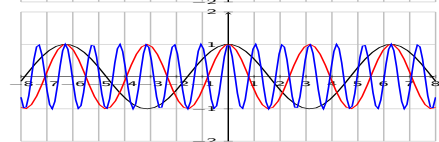


$$\begin{array}{ll} y = \cos(x) & A = 1 \\ y = 2 \cos(x) & A = 2 \\ y = -4 \cos(x) & A = 4 \end{array}$$

• Period P



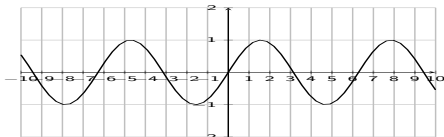
$$\begin{array}{ll} y = \sin(x) & P = 2\pi \\ y = \sin(3x) & P = \frac{2\pi}{3} \\ y = \sin\left(\frac{1}{2}x\right) & P = \frac{2\pi}{\frac{1}{2}} = 2\pi \cdot \frac{2}{1} \\ & = 4\pi \end{array}$$



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Amplitude, period, and phase shift for sin, cos

- Phase shift S

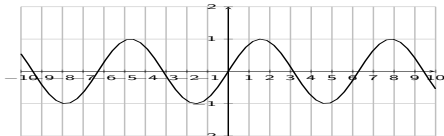


$$y = \sin(x)$$

$$S = 0$$

Amplitude, period, and phase shift for sin, cos

- Phase shift S



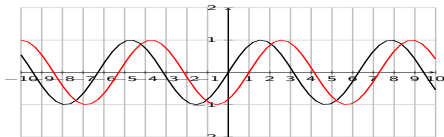
$$y = \sin(x)$$

$$y = \sin(x - 1)$$

$$S = 0$$

Amplitude, period, and phase shift for sin, cos

• Phase shift S



$$y = \sin(x)$$

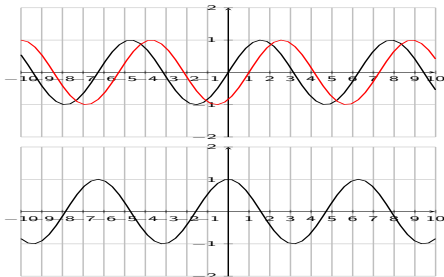
$$y = \sin(x - 1)$$

$$S = 0$$

$$S = 1$$

Amplitude, period, and phase shift for sin, cos

• Phase shift S



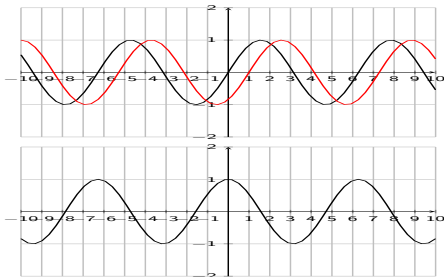
$$y = \sin(x) \quad S = 0$$

$$y = \sin(x - 1) \quad S = 1$$

$$y = \cos(x) \quad S = 0$$

Amplitude, period, and phase shift for sin, cos

• Phase shift S



$$y = \sin(x) \quad S = 0$$

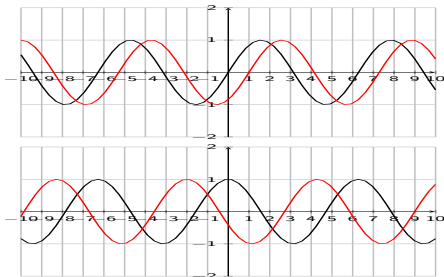
$$y = \sin(x - 1) \quad S = 1$$

$$y = \cos(x) \quad S = 0$$

$$y = \cos(x + 2)$$

Amplitude, period, and phase shift for sin, cos

• Phase shift S



$$y = \sin(x) \quad S = 0$$

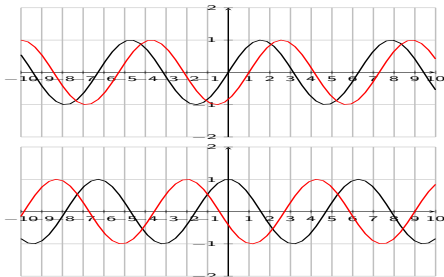
$$y = \sin(x - 1) \quad S = 1$$

$$y = \cos(x) \quad S = 0$$

$$y = \cos(x + 2) \quad S = -2$$

Amplitude, period, and phase shift for sin, cos

• Phase shift S



$$y = \sin(x) \quad S = 0$$

$$y = \sin(x - 1) \quad S = 1$$

$$y = \cos(x) \quad S = 0$$

$$y = \cos(x + 2) \quad S = -2$$

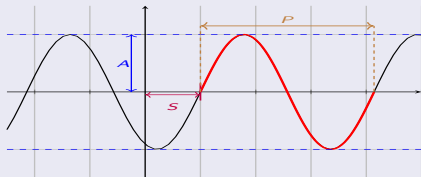
Amplitude, period, phase shift

When $y = a \cdot \sin(bx + c)$ or $y = a \cdot \cos(bx + c)$

Amplitude $A = |a|$

Period $P = \left| \frac{2\pi}{b} \right|$

Phase shift $S = \frac{-c}{b}$



Graphing sin and cos - exercises

Find amplitude, period, and phase shift. Graph the function over one period. Label all maxima, minima, and roots.

1 $y = 4 \sin(2x - \pi)$

2 $y = 3 \sin(5x - \pi)$

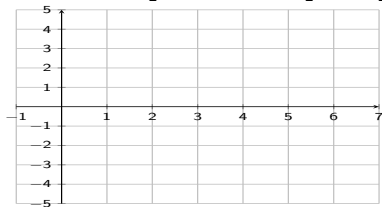
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1 $y = 4 \sin(2x - \pi)$

2 $y = 3 \sin(5x - \pi)$

$$A = 4, P = \frac{2\pi}{2} = \pi, S = \frac{-(-\pi)}{2} = \frac{\pi}{2}$$



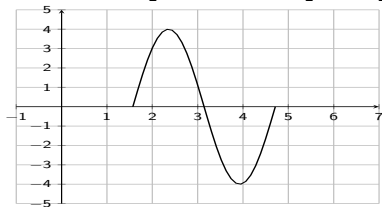
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Roots:

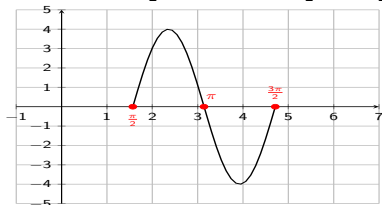
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2 $y = 3 \sin(5x - \pi)$

$$A = 4, P = \frac{2\pi}{2} = \pi, S = \frac{-(-\pi)}{2} = \frac{\pi}{2}$$



Roots:

- $S = \frac{\pi}{2}$
- $S + P = \frac{\pi}{2} + \pi = \frac{3\pi}{2}$
- $(\frac{\pi}{2} + \frac{3\pi}{2}) \div 2 = \frac{4\pi}{2} \cdot \frac{1}{2} = \frac{4\pi}{4} = \pi$

Maximum:

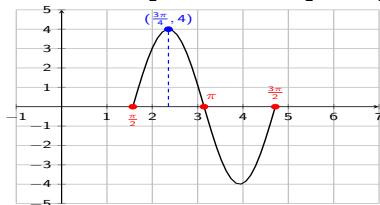
Graphing sin and cos - exercises

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Roots:

- $S = \frac{\pi}{2}$
- $S + P = \frac{\pi}{2} + \pi = \frac{3\pi}{2}$
- $(\frac{\pi}{2} + \frac{3\pi}{2}) \div 2 = \frac{4\pi}{2} \cdot \frac{1}{2} = \frac{4\pi}{4} = \pi$

Maximum:

- $(\frac{\pi}{2} + \pi) \div 2 = \frac{3\pi}{2} \cdot \frac{1}{2} = \frac{3\pi}{4}$

Minimum:

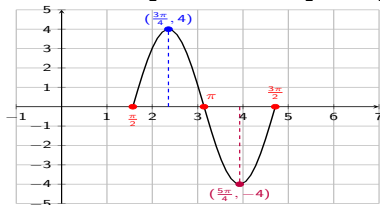
Graphing sin and cos - exercises

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2 $y = 3 \sin(5x - \pi)$

$$A = 4, P = \frac{2\pi}{2} = \pi, S = \frac{-(-\pi)}{2} = \frac{\pi}{2}$$



Roots:

- $S = \frac{\pi}{2}$
- $S + P = \frac{\pi}{2} + \pi = \frac{3\pi}{2}$
- $(\frac{\pi}{2} + \frac{3\pi}{2}) \div 2 = \frac{4\pi}{2} \cdot \frac{1}{2} = \frac{4\pi}{4} = \pi$

Maximum:

- $(\frac{\pi}{2} + \pi) \div 2 = \frac{3\pi}{2} \cdot \frac{1}{2} = \frac{3\pi}{4}$

Minimum:

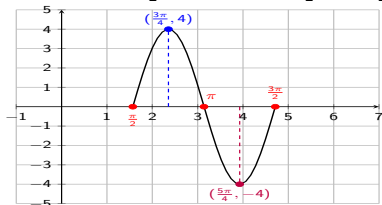
- $(\pi + \frac{3\pi}{2}) \div 2 = \frac{5\pi}{2} \cdot \frac{1}{2} = \frac{5\pi}{4}$

Graphing sin and cos - exercises

Find amplitude, period, and phase shift. Graph the function over one period. Label all maxima, minima, and roots.

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$$A = 4, P = \frac{2\pi}{2} = \pi, S = \frac{-(-\pi)}{2} = \frac{\pi}{2}$$



Roots:

- $S = \frac{\pi}{2}$
- $S + P = \frac{\pi}{2} + \pi = \frac{3\pi}{2}$
- $(\frac{\pi}{2} + \frac{3\pi}{2}) \div 2 = \frac{4\pi}{2} \cdot \frac{1}{2} = \frac{4\pi}{4} = \pi$

Maximum:

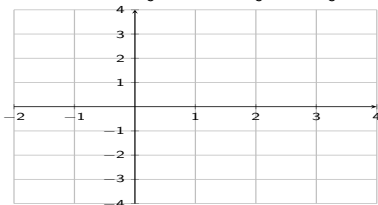
- $(\frac{\pi}{2} + \pi) \div 2 = \frac{3\pi}{2} \cdot \frac{1}{2} = \frac{3\pi}{4}$

Minimum:

- $(\pi + \frac{3\pi}{2}) \div 2 = \frac{5\pi}{2} \cdot \frac{1}{2} = \frac{5\pi}{4}$

2 $y = 3 \sin(5x - \pi)$

$$A = 3, P = \frac{2\pi}{5}, S = \frac{-(-\pi)}{5} = \frac{\pi}{5}$$

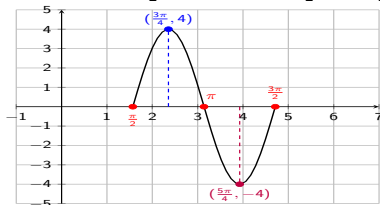


Graphing sin and cos - exercises

Find amplitude, period, and phase shift. Graph the function over one period. Label all maxima, minima, and roots.

① $y = 4 \sin(2x - \pi)$

$$A = 4, P = \frac{2\pi}{2} = \pi, S = \frac{-(-\pi)}{2} = \frac{\pi}{2}$$



Roots:

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- $S + P = \frac{\pi}{2} + \pi = \frac{3\pi}{2}$
- $(\frac{\pi}{2} + \frac{3\pi}{2}) \div 2 = \frac{4\pi}{2} \cdot \frac{1}{2} = \frac{4\pi}{4} = \pi$

Maximum:

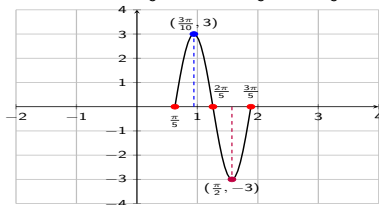
- $(\frac{\pi}{2} + \pi) \div 2 = \frac{3\pi}{2} \cdot \frac{1}{2} = \frac{3\pi}{4}$

Minimum:

- $(\pi + \frac{3\pi}{2}) \div 2 = \frac{5\pi}{2} \cdot \frac{1}{2} = \frac{5\pi}{4}$

② $y = 3 \sin(5x - \pi)$

$$A = 3, P = \frac{2\pi}{5}, S = \frac{-(-\pi)}{5} = \frac{\pi}{5}$$



Roots:

- $S = \frac{\pi}{5}$
- $S + P = \frac{\pi}{5} + \frac{2\pi}{5} = \frac{3\pi}{5}$
- $(\frac{\pi}{5} + \frac{3\pi}{5}) \div 2 = \frac{4\pi}{5} \cdot \frac{1}{2} = \frac{2\pi}{5}$

Maximum:

- $(\frac{\pi}{5} + \frac{2\pi}{5}) \div 2 = \frac{3\pi}{5} \cdot \frac{1}{2} = \frac{3\pi}{10}$

Minimum:

- $(\frac{2\pi}{5} + \frac{3\pi}{5}) \div 2 = \frac{5\pi}{5} \cdot \frac{1}{2} = \frac{\pi}{2}$

Graphing sin and cos - exercises

Find amplitude, period, and phase shift. Graph the function over one period. Label all maxima, minima, and roots.

$$3 \quad y = 5 \cos\left(2x - \frac{\pi}{3}\right)$$

$$4 \quad y = 2 \cos\left(x - \frac{\pi}{2}\right)$$

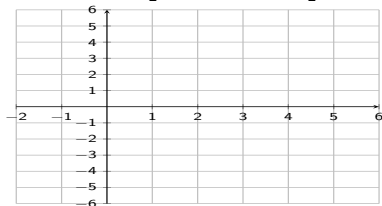
Graphing sin and cos - exercises

Find amplitude, period, and phase shift. Graph the function over one period. Label all maxima, minima, and roots.

8 $y = 5 \cos(2x - \frac{\pi}{3})$

9 $y = 2 \cos(x - \frac{\pi}{2})$

$$A = 5, P = \frac{2\pi}{2} = \pi, S = \frac{-(-\frac{\pi}{3})}{2} = \frac{\pi}{6}$$



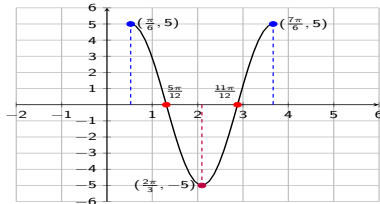
Graphing sin and cos - exercises

Find amplitude, period, and phase shift. Graph the function over one period. Label all maxima, minima, and roots.

③ $y = 5 \cos(2x - \frac{\pi}{3})$

④ $y = 2 \cos(x - \frac{\pi}{2})$

$$A = 5, P = \frac{2\pi}{2} = \pi, S = \frac{-(-\frac{\pi}{3})}{2} = \frac{\pi}{6}$$



Maxima:

- $S = \frac{\pi}{6}$
- $S + P = \frac{\pi}{6} + \pi = \frac{7\pi}{6}$

Minimum:

- $(\frac{\pi}{6} + \frac{7\pi}{6}) \div 2 = \frac{8\pi}{6} \cdot \frac{1}{2} = \frac{8\pi}{12} = \frac{2\pi}{3}$

Roots:

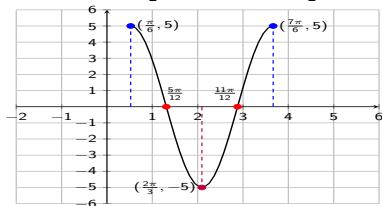
- $(\frac{\pi}{6} + \frac{2\pi}{3}) \div 2 = \frac{5\pi}{6} \cdot \frac{1}{2} = \frac{5\pi}{12}$
- $(\frac{2\pi}{3} + \frac{7\pi}{6}) \div 2 = \frac{11\pi}{6} \cdot \frac{1}{2} = \frac{11\pi}{12}$

Graphing sin and cos - exercises

Find amplitude, period, and phase shift. Graph the function over one period. Label all maxima, minima, and roots.

⑧ $y = 5 \cos(2x - \frac{\pi}{3})$

$$A = 5, P = \frac{2\pi}{2} = \pi, S = \frac{-(-\frac{\pi}{3})}{2} = \frac{\pi}{6}$$



Maxima:

- $S = \frac{\pi}{6}$
- $S + P = \frac{\pi}{6} + \pi = \frac{7\pi}{6}$

Minimum:

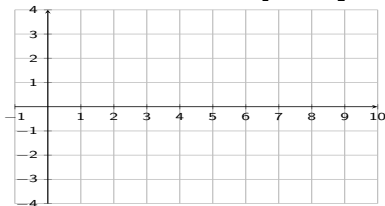
- $(\frac{\pi}{6} + \frac{7\pi}{6}) \div 2 = \frac{8\pi}{6} \cdot \frac{1}{2} = \frac{8\pi}{12} = \frac{2\pi}{3}$

Roots:

- $(\frac{\pi}{6} + \frac{2\pi}{3}) \div 2 = \frac{5\pi}{6} \cdot \frac{1}{2} = \frac{5\pi}{12}$
- $(\frac{2\pi}{3} + \frac{7\pi}{6}) \div 2 = \frac{11\pi}{6} \cdot \frac{1}{2} = \frac{11\pi}{12}$

⑨ $y = 2 \cos(x - \frac{\pi}{2})$

$$A = 2, P = 2\pi, S = \frac{-(-\frac{\pi}{2})}{1} = \frac{\pi}{2}$$

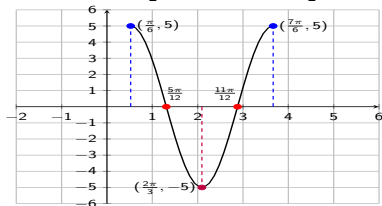


Graphing sin and cos - exercises

Find amplitude, period, and phase shift. Graph the function over one period. Label all maxima, minima, and roots.

⑧ $y = 5 \cos(2x - \frac{\pi}{3})$

$$A = 5, P = \frac{2\pi}{2} = \pi, S = \frac{-(-\frac{\pi}{3})}{2} = \frac{\pi}{6}$$



Maxima:

- $S = \frac{\pi}{6}$
- $S + P = \frac{\pi}{6} + \pi = \frac{7\pi}{6}$

Minimum:

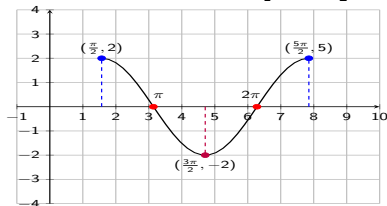
- $(\frac{\pi}{6} + \frac{7\pi}{6}) \div 2 = \frac{8\pi}{6} \cdot \frac{1}{2} = \frac{8\pi}{12} = \frac{2\pi}{3}$

Roots:

- $(\frac{\pi}{6} + \frac{2\pi}{3}) \div 2 = \frac{5\pi}{6} \cdot \frac{1}{2} = \frac{5\pi}{12}$
- $(\frac{2\pi}{3} + \frac{7\pi}{6}) \div 2 = \frac{11\pi}{6} \cdot \frac{1}{2} = \frac{11\pi}{12}$

⑨ $y = 2 \cos(x - \frac{\pi}{2})$

$$A = 2, P = 2\pi, S = \frac{-(-\frac{\pi}{2})}{1} = \frac{\pi}{2}$$



Maxima:

- $S = \frac{\pi}{2}$
- $S + P = \frac{\pi}{2} + 2\pi = \frac{5\pi}{2}$

Minimum:

- $(\frac{\pi}{2} + \frac{5\pi}{2}) \div 2 = \frac{6\pi}{2} \cdot \frac{1}{2} = \frac{3\pi}{2}$

Roots:

- $(\frac{\pi}{2} + \frac{3\pi}{2}) \div 2 = \frac{4\pi}{2} \cdot \frac{1}{2} = \frac{4\pi}{4} = \pi$
- $(\frac{3\pi}{2} + \frac{5\pi}{2}) \div 2 = \frac{8\pi}{2} \cdot \frac{1}{2} = \frac{8\pi}{4} = 2\pi$

Graphing sin and cos - exercises

Find amplitude, period, and phase shift. Graph the function over one period. Label all maxima, minima, and roots.

5 $y = 7 \sin(3x + \pi)$

6 $y = -3 \cos(2x - 2\pi)$

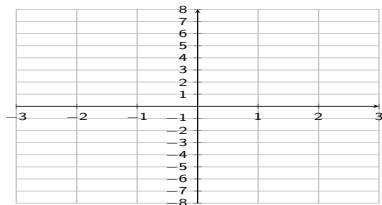
Graphing sin and cos - exercises

Find amplitude, period, and phase shift. Graph the function over one period. Label all maxima, minima, and roots.

5 $y = 7 \sin(3x + \pi)$

$$A = 7, P = \frac{2\pi}{3}, S = \frac{-\pi}{3}$$

6 $y = -3 \cos(2x - 2\pi)$

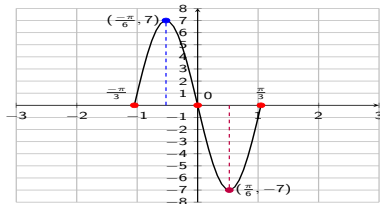


Graphing sin and cos - exercises

Find amplitude, period, and phase shift. Graph the function over one period. Label all maxima, minima, and roots.

5 $y = 7 \sin(3x + \pi)$

$$A = 7, P = \frac{2\pi}{3}, S = \frac{-\pi}{3}$$



6 $y = -3 \cos(2x - 2\pi)$

Roots:

- $S = \frac{-\pi}{3}$
- $S + P = \frac{-\pi}{3} + \frac{2\pi}{3} = \frac{\pi}{3}$
- $(\frac{-\pi}{3} + \frac{\pi}{3}) \div 2 = 0 \cdot \frac{1}{2} = 0$

Maximum:

- $(\frac{-\pi}{3} + 0) \div 2 = \frac{-\pi}{3} \cdot \frac{1}{2} = \frac{-\pi}{6}$

Minimum:

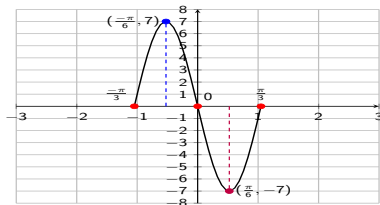
- $(0 + \frac{\pi}{3}) \div 2 = \frac{\pi}{3} \cdot \frac{1}{2} = \frac{\pi}{6}$

Graphing sin and cos - exercises

Find amplitude, period, and phase shift. Graph the function over one period. Label all maxima, minima, and roots.

5 $y = 7 \sin(3x + \pi)$

$$A = 7, P = \frac{2\pi}{3}, S = \frac{-\pi}{3}$$



Roots:

- $S = \frac{-\pi}{3}$
- $S + P = \frac{-\pi}{3} + \frac{2\pi}{3} = \frac{\pi}{3}$
- $(\frac{-\pi}{3} + \frac{\pi}{3}) \div 2 = 0 \cdot \frac{1}{2} = 0$

Maximum:

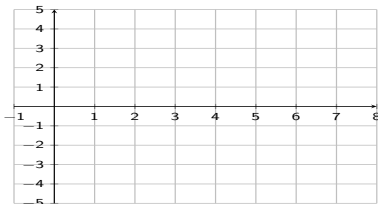
- $(\frac{-\pi}{3} + 0) \div 2 = \frac{-\pi}{3} \cdot \frac{1}{2} = \frac{-\pi}{6}$

Minimum:

- $(0 + \frac{\pi}{3}) \div 2 = \frac{\pi}{3} \cdot \frac{1}{2} = \frac{\pi}{6}$

6 $y = -3 \cos(2x - 2\pi)$

$$A = |-3| = 3, P = \frac{2\pi}{2} = \pi, S = \frac{-(-2\pi)}{2} = \pi$$

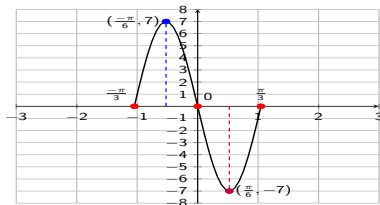


Graphing sin and cos - exercises

Find amplitude, period, and phase shift. Graph the function over one period. Label all maxima, minima, and roots.

5 $y = 7 \sin(3x + \pi)$

$$A = 7, P = \frac{2\pi}{3}, S = \frac{-\pi}{3}$$



Roots:

- $S = \frac{-\pi}{3}$
- $S + P = \frac{-\pi}{3} + \frac{2\pi}{3} = \frac{\pi}{3}$
- $(\frac{-\pi}{3} + \frac{\pi}{3}) \div 2 = 0 \cdot \frac{1}{2} = 0$

Maximum:

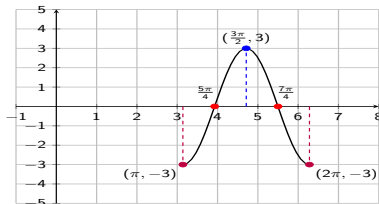
- $(\frac{-\pi}{3} + 0) \div 2 = \frac{-\pi}{3} \cdot \frac{1}{2} = \frac{-\pi}{6}$

Minimum:

- $(0 + \frac{\pi}{3}) \div 2 = \frac{\pi}{3} \cdot \frac{1}{2} = \frac{\pi}{6}$

6 $y = -3 \cos(2x - 2\pi)$

$$A = |-3| = 3, P = \frac{2\pi}{2} = \pi, S = \frac{-(-2\pi)}{2} = \pi$$



Minima:

- $S = \pi$
- $S + P = \pi + \pi = 2\pi$

Maximum:

- $(\pi + 2\pi) \div 2 = 3\pi \cdot \frac{1}{2} = \frac{3\pi}{2}$

Roots:

- $(\pi + \frac{3\pi}{2}) \div 2 = \frac{5\pi}{2} \cdot \frac{1}{2} = \frac{5\pi}{4}$
- $(\frac{3\pi}{2} + 2\pi) \div 2 = \frac{7\pi}{2} \cdot \frac{1}{2} = \frac{7\pi}{4}$

Graphing sin and cos - exercises

Find amplitude, period, and phase shift. Graph the function over one period. Label all maxima, minima, and roots.

7 $y = -5 \sin(6x)$

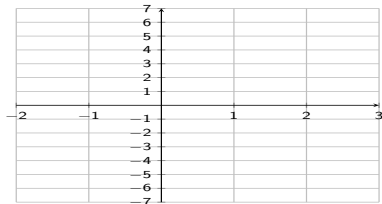
8 $y = 4 \cos\left(2x - \frac{\pi}{4}\right)$

Graphing sin and cos - exercises

Find amplitude, period, and phase shift. Graph the function over one period. Label all maxima, minima, and roots.

7 $y = -5 \sin(6x)$

$$A = 5, P = \frac{2\pi}{6} = \frac{\pi}{3}, S = \frac{-0}{6} = 0$$



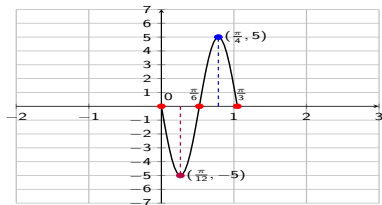
8 $y = 4 \cos(2x - \frac{\pi}{4})$

Graphing sin and cos - exercises

Find amplitude, period, and phase shift. Graph the function over one period. Label all maxima, minima, and roots.

7 $y = -5 \sin(6x)$

$$A = 5, P = \frac{2\pi}{6} = \frac{\pi}{3}, S = \frac{-0}{6} = 0$$



8 $y = 4 \cos(2x - \frac{\pi}{4})$

Roots:

- $S = 0$
- $S + P = 0 + \frac{\pi}{3} = \frac{\pi}{3}$
- $(0 + \frac{\pi}{3}) \div 2 = \frac{\pi}{3} \cdot \frac{1}{2} = \frac{\pi}{6}$

Minimum:

- $(0 + \frac{\pi}{6}) \div 2 = \frac{\pi}{6} \cdot \frac{1}{2} = \frac{\pi}{12}$

Maximum:

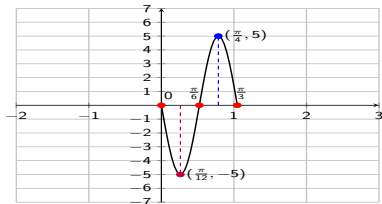
- $(\frac{\pi}{6} + \frac{\pi}{3}) \div 2 = \frac{3\pi}{6} \cdot \frac{1}{2} = \frac{3\pi}{12} = \frac{\pi}{4}$

Graphing sin and cos - exercises

Find amplitude, period, and phase shift. Graph the function over one period. Label all maxima, minima, and roots.

7 $y = -5 \sin(6x)$

$$A = 5, P = \frac{2\pi}{6} = \frac{\pi}{3}, S = \frac{-0}{6} = 0$$



Roots:

- $S = 0$
- $S + P = 0 + \frac{\pi}{3} = \frac{\pi}{3}$
- $(0 + \frac{\pi}{3}) \div 2 = \frac{\pi}{3} \cdot \frac{1}{2} = \frac{\pi}{6}$

Minimum:

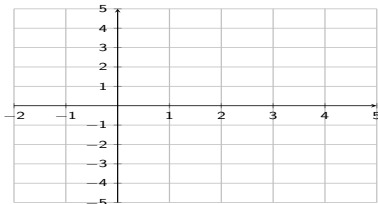
- $(0 + \frac{\pi}{6}) \div 2 = \frac{\pi}{6} \cdot \frac{1}{2} = \frac{\pi}{12}$

Maximum:

- $(\frac{\pi}{6} + \frac{\pi}{3}) \div 2 = \frac{3\pi}{6} \cdot \frac{1}{2} = \frac{3\pi}{12} = \frac{\pi}{4}$

8 $y = 4 \cos(2x - \frac{\pi}{4})$

$$A = 4, P = \frac{2\pi}{2} = \pi, S = \frac{-(-\frac{\pi}{4})}{2} = \frac{\pi}{8}$$

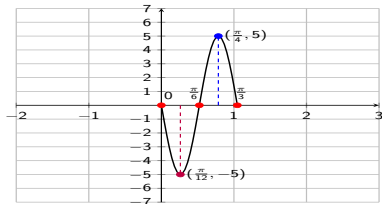


Graphing sin and cos - exercises

Find amplitude, period, and phase shift. Graph the function over one period. Label all maxima, minima, and roots.

7 $y = -5 \sin(6x)$

$$A = 5, P = \frac{2\pi}{6} = \frac{\pi}{3}, S = \frac{-0}{6} = 0$$



Roots:

- $S = 0$
- $S + P = 0 + \frac{\pi}{3} = \frac{\pi}{3}$
- $(0 + \frac{\pi}{3}) \div 2 = \frac{\pi}{3} \cdot \frac{1}{2} = \frac{\pi}{6}$

Minimum:

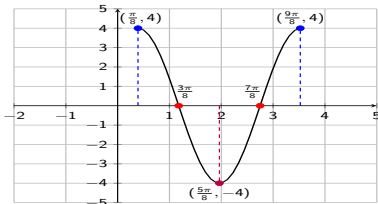
- $(0 + \frac{\pi}{6}) \div 2 = \frac{\pi}{6} \cdot \frac{1}{2} = \frac{\pi}{12}$

Maximum:

- $(\frac{\pi}{6} + \frac{\pi}{3}) \div 2 = \frac{3\pi}{6} \cdot \frac{1}{2} = \frac{3\pi}{12} = \frac{\pi}{4}$

8 $y = 4 \cos(2x - \frac{\pi}{4})$

$$A = 4, P = \frac{2\pi}{2} = \pi, S = \frac{-(-\frac{\pi}{4})}{2} = \frac{\pi}{8}$$



Maxima:

- $S = \frac{\pi}{8}$
- $S + P = \frac{\pi}{8} + \pi = \frac{9\pi}{8}$

Minimum:

- $(\frac{\pi}{8} + \frac{9\pi}{8}) \div 2 = \frac{10\pi}{8} \cdot \frac{1}{2} = \frac{10\pi}{16} = \frac{5\pi}{8}$

Roots:

- $(\frac{\pi}{8} + \frac{5\pi}{8}) \div 2 = \frac{6\pi}{8} \cdot \frac{1}{2} = \frac{3\pi}{8}$
- $(\frac{5\pi}{8} + \frac{9\pi}{8}) \div 2 = \frac{14\pi}{8} \cdot \frac{1}{2} = \frac{7\pi}{8}$

