

Exponential equations and applications

Lesson #15

MAT 1375 Precalculus

New York City College of Technology CUNY



Solving exponential equations - exercises

Solve the exponential equation.



$$2^{x+3} = 16$$

Solving exponential equations - exercises

Solve the exponential equation.

1

$$2^{x+3} = 16$$

$$\Rightarrow 2^{x+3} = 2^4$$

$$\Rightarrow x + 3 = 4$$

$$\Rightarrow x = 1$$

Solving exponential equations - exercises

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Exponential function is one-to-one

$$B^r = B^s \Leftrightarrow r = s$$

2

$$5^{2x-6} = 25$$

Solving exponential equations - exercises

Solve the exponential equation.

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$$\Rightarrow 2x - 6 = 2$$

$$\Rightarrow 2x = 8$$

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Solving exponential equations - exercises

Solve the exponential equation.

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$$\begin{aligned}5^{2x-6} &= 25 \\ \Rightarrow 5^{2x-6} &= 5^2 \\ \Rightarrow 2x - 6 &= 2 \\ \Rightarrow 2x &= 8 \\ \Rightarrow x &= 4\end{aligned}$$

3

$$6^{x+7} = 36^x$$

4

$$8^{x-7} = 4^{3x+5}$$

5

$$2^x = 7$$

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$$\begin{aligned}8^{x-7} &= 4^{3x+5} \\ \Rightarrow 2^{3(x-7)} &= 2^{2(3x+5)} \\ \Rightarrow 3(x-7) &= 2(3x+5) \\ \Rightarrow 3x - 21 &= 6x + 10 \\ \Rightarrow -3x &= 31 \\ \Rightarrow x &= -\frac{31}{3}\end{aligned}$$

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$$2^x = 7$$

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5

$$\begin{aligned}2^x &= 7 \\ \Rightarrow \log(2^x) &= \log(7) \\ \Rightarrow x \cdot \log(2) &= \log(7) \\ \Rightarrow x &= \frac{\log(7)}{\log(2)}\end{aligned}$$

Solving exponential equations - exercises

Solve the exponential equation.

6

$$3^{x+5} = 8$$

Solving exponential equations - exercises

Solve the exponential equation.

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$$3^{x+5} = 8$$

$$\Rightarrow \log(3^{x+5}) = \log(8)$$

$$\Rightarrow (x + 5) \cdot \log 3 = \log 8$$

$$\Rightarrow x \cdot \log 3 + 5 \cdot \log 3 = \log 8$$

$$\Rightarrow x \cdot \log 3 = \log 8 - 5 \cdot \log 3$$

$$\Rightarrow x = \frac{\log 8 - 5 \cdot \log 3}{\log 3}$$

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$$5^{2x+6} = 2^{x+9}$$

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$$\Rightarrow \log(5^{2x+6}) = \log(2^{x+9})$$

$$\Rightarrow (2x + 6) \cdot \log 5 = (x + 9) \cdot \log 2$$

$$\Rightarrow 2x \cdot \log 5 + 6 \cdot \log 5 = x \cdot \log 2 + 9 \cdot \log 2$$

$$\Rightarrow 2x \cdot \log 5 - x \cdot \log 2 = 9 \cdot \log 2 - 6 \cdot \log 5$$

$$\Rightarrow x \cdot (2 \log 5 - \log 2) = 9 \cdot \log 2 - 6 \cdot \log 5$$

$$\Rightarrow x = \frac{9 \cdot \log 2 - 6 \cdot \log 5}{2 \log 5 - \log 2}$$

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$$3^{2x+4} = 6^{5-x}$$

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$$\Rightarrow 2x \cdot \log 5 + 6 \cdot \log 5 = x \cdot \log 2 + 9 \cdot \log 2$$

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$$2^{3-4x} = 7^{6x-5}$$

Solving exponential equations - exercises

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8

$$3^{2x+4} = 6^{5-x}$$

$$\Rightarrow (2x+4) \log 3 = (5-x) \log 6$$

$$\Rightarrow 2x \log 3 + 4 \log 3 = 5 \log 6 - x \log 6$$

$$\Rightarrow 2x \log 3 + x \log 6 = 5 \log 6 - 4 \log 3$$

$$\Rightarrow x \cdot (2 \log 3 + \log 6) = 5 \log 6 - 4 \log 3$$

$$\Rightarrow x = \frac{5 \log 6 - 4 \log 3}{2 \log 3 + \log 6}$$

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$$3^{2x+4} = 6^{5-x}$$

$$\Rightarrow (2x+4) \log 3 = (5-x) \log 6$$

$$\Rightarrow 2x \log 3 + 4 \log 3 = 5 \log 6 - x \log 6$$

$$\Rightarrow 2x \log 3 + x \log 6 = 5 \log 6 - 4 \log 3$$

$$\Rightarrow x \cdot (2 \log 3 + \log 6) = 5 \log 6 - 4 \log 3$$

$$\Rightarrow x = \frac{5 \log 6 - 4 \log 3}{2 \log 3 + \log 6}$$

9

$$2^{3-4x} = 7^{6x-5}$$

$$\Rightarrow (3-4x) \log 2 = (6x-5) \log 7$$

$$\Rightarrow 3 \log 2 - 4x \log 2 = 6x \log 7 - 5 \log 7$$

$$\Rightarrow 3 \log 2 + 5 \log 7 = 6x \log 7 + 4x \log 2$$

$$\Rightarrow 3 \log 2 + 5 \log 7 = x \cdot (6 \log 7 + 4 \log 2)$$

$$\Rightarrow x = \frac{3 \log 2 + 5 \log 7}{6 \log 7 + 4 \log 2}$$

Solving exponential equations - exercises

Solve the exponential equation.

10

$$2e^x = 5$$

Solving exponential equations - exercises

Solve the exponential equation.

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$$2e^x = 5$$

$$\Rightarrow e^x = \frac{5}{2}$$

$$\Rightarrow \ln(e^x) = \ln\left(\frac{5}{2}\right)$$

$$\Rightarrow x \cdot \ln(e) = \ln\left(\frac{5}{2}\right)$$

Recall: $\ln(e) = 1$

$$\Rightarrow x = \ln\left(\frac{5}{2}\right)$$

Solving exponential equations - exercises

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$$4e^{5x-12} = 9$$

Solving exponential equations - exercises

Solve the exponential equation.

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$$4e^{5x-12} = 9$$

$$\Rightarrow e^{5x-12} = \frac{9}{4}$$

$$\Rightarrow \ln(e^{5x-12}) = \ln\left(\frac{9}{4}\right)$$

$$\Rightarrow (5x - 12) \cdot \ln(e) = \ln\left(\frac{9}{4}\right)$$

$$\Rightarrow 5x - 12 = \ln\left(\frac{9}{4}\right)$$

$$\Rightarrow 5x = \ln\left(\frac{9}{4}\right) + 12$$

$$\Rightarrow x = \frac{\ln\left(\frac{9}{4}\right) + 12}{5}$$

Solving exponential equations - exercises

Solve the exponential equation.

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Solve the exponential equation.

12

$$\frac{4e^x + 6}{7e^x - 3} = 12$$

Solving exponential equations - exercises

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Solve the exponential equation.

12

$$\frac{4e^x + 6}{7e^x - 3} = 12$$

Multiply both sides by the denominator $7e^x - 3$:

$$\Rightarrow 4e^x + 6 = 12 \cdot (7e^x - 3)$$

$$\Rightarrow 4e^x + 6 = 84e^x - 36$$

Separate e^x to one side:

$$\Rightarrow 4e^x - 84e^x = -36 - 6$$

$$\Rightarrow -80e^x = -42$$

$$\Rightarrow e^x = \frac{-42}{-80} = \frac{21}{40}$$

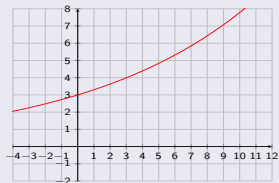
Solve for x :

$$\Rightarrow \ln(e^x) = \ln\left(\frac{21}{40}\right)$$

$$\Rightarrow x = \ln\left(\frac{21}{40}\right)$$

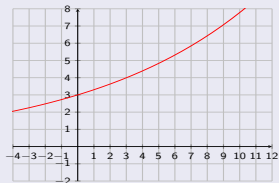
Exponential models

Exponential growth



Exponential models

Exponential growth



Examples:

-Population growth

Exponential models

Exponential growth



Examples:

- Population growth
- Investment at a fixed rate

Exponential models

Exponential growth



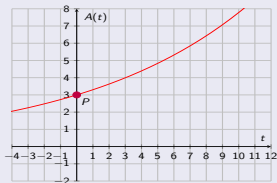
Examples:

- Population growth
- Investment at a fixed rate

Model: $A(t) = P \cdot e^{r \cdot t}$ or $A(t) = P \cdot (1 + r)^t$

Exponential models

Exponential growth



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- Population growth
- Investment at a fixed rate

Model: $A(t) = P \cdot e^{r \cdot t}$ or $A(t) = P \cdot (1 + r)^t$

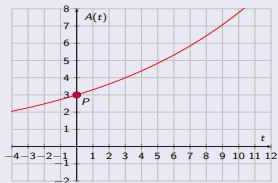
t = time

P = initial amount

$A(t)$ = amount at time t

Exponential models

Exponential growth



Examples:

- Population growth
- Investment at a fixed rate

Model: $A(t) = P \cdot e^{r \cdot t}$ or $A(t) = P \cdot (1 + r)^t$
 $t =$ time $r =$ rate of growth

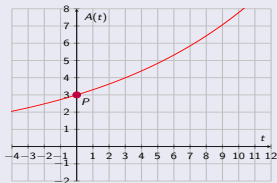
$P =$ initial amount

$A(t) =$ amount at time t

Exponential models in applications

Exponential models

Exponential growth



Exponential decline



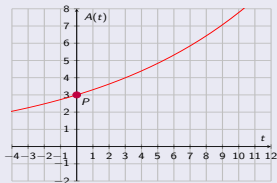
Examples:

- Population growth
- Investment at a fixed rate

Model: $A(t) = P \cdot e^{r \cdot t}$ or $A(t) = P \cdot (1 + r)^t$
 t = time
 P = initial amount
 $A(t)$ = amount at time t
 $r > 0$: exponential growth
 $r < 0$: exponential decline

Exponential models

Exponential growth



Exponential decline



Examples:

- Population growth
- Investment at a fixed rate

Model: $A(t) = P \cdot e^{r \cdot t}$ or

t = time

P = initial amount

$A(t)$ = amount at time t

Examples:

- Population decline

$A(t) = P \cdot (1 + r)^t$

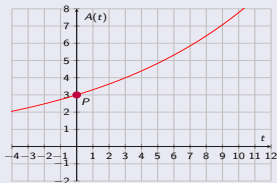
r = rate of growth or decay

$r > 0$: exponential growth

$r < 0$: exponential decline

Exponential models

Exponential growth



Exponential decline



Examples:

- Population growth
- Investment at a fixed rate

Model: $A(t) = P \cdot e^{r \cdot t}$ or

t = time

P = initial amount

$A(t)$ = amount at time t

Examples:

- Population decline
- Radioactive decay

$A(t) = P \cdot (1 + r)^t$

r = rate of growth or decay

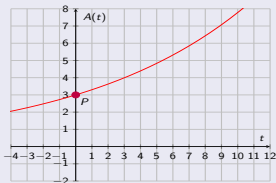
$r > 0$: exponential growth

$r < 0$: exponential decline

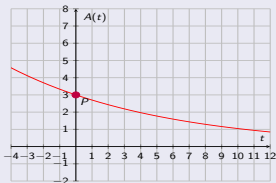
Exponential models in applications

Exponential models

Exponential growth



Exponential decline



Examples:

- Population growth
- Investment at a fixed rate

Model: $A(t) = P \cdot e^{r \cdot t}$ or

t = time

P = initial amount

$A(t)$ = amount at time t

Examples:

- Population decline
- Radioactive decay

$A(t) = P \cdot (1 + r)^t$

r = rate of growth or decay

$r > 0$: exponential growth

$r < 0$: exponential decline

Note:

Both

$$A(t) = P \cdot e^{r \cdot t}$$

and

$$A(t) = P \cdot (1 + r)^t$$

could be used to model a situation.

For small r , these are close:

$$e^r \approx 1 + r$$

Use the model as stated.

If no particular model is stated use $A = P \cdot e^{r \cdot t}$.

Exponential models - exercises

- 1 A city has a population size of 56,000 people. The city grows exponentially at a rate of 6% per year.
- (a) What is the population size after 15 years?
- (b) How long will it take for the population to reach 200,000?

Exponential models - exercises

- 1 A city has a population size of 56,000 people. The city grows exponentially at a rate of 6% per year.
- (a) What is the population size after 15 years?
- (b) How long will it take for the population to reach 200,000?

Solution:

- (a) $P = 56,000$, $r = 6\% = 0.06$
 $\Rightarrow A(t) = 56000 \cdot e^{0.06 \cdot t}$
 $t = 15$
 $\Rightarrow A(15) = 56000 \cdot e^{0.06 \cdot 15} \approx 138,000$
There will be approximately 138,000 people in the city.

Exponential models - exercises

- 1 A city has a population size of 56,000 people. The city grows exponentially at a rate of 6% per year.
- (a) What is the population size after 15 years?
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Solution:

(a) $P = 56,000$, $r = 6\% = 0.06$

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$$t = 15$$

$$\Rightarrow A(15) = 56000 \cdot e^{0.06 \cdot 15} \approx 138,000$$

There will be approximately 138,000 people in the city.

(b) $A(t) = 200,000$

$$\Rightarrow 200,000 = 56,000 \cdot e^{0.06 \cdot t}$$

$$\Rightarrow \frac{200,000}{56,000} = e^{0.06 \cdot t}$$

$$\Rightarrow \ln\left(\frac{200,000}{56,000}\right) = 0.06 \cdot t$$

$$\Rightarrow t = \frac{\ln\left(\frac{200,000}{56,000}\right)}{0.06} \approx 21.2$$

It will take approximately 21.2 years.

Exponential models - exercises

① A city has a population size of 56,000 people. The city grows exponentially at a rate of 6% per year.

- (a) What is the population size after 15 years?
(b) How long will it take for the population to reach 200,000?

Solution:

(a) $P = 56,000$, $r = 6\% = 0.06$

$$\Rightarrow A(t) = 56000 \cdot e^{0.06 \cdot t}$$

$$t = 15$$

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$$\Rightarrow 200,000 = 56,000 \cdot e^{0.06 \cdot t}$$

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It will take approximately 21.2 years.

② The world population in 2020 consists of about 7.8 billion people and grows at a rate of 1.1% per year.

- (a) Assuming constant growth, what will the world population be in 2075?
(b) How long does it take for the world population to triple?

Exponential models - exercises

① A city has a population size of 56,000 people. The city grows exponentially at a rate of 6% per year.

- (a) What is the population size after 15 years?
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$$\Rightarrow 200,000 = 56,000 \cdot e^{0.06 \cdot t}$$

$$\Rightarrow \frac{200,000}{56,000} = e^{0.06 \cdot t}$$

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It will take approximately 21.2 years.

② The world population in 2020 consists of about 7.8 billion people and grows at a rate of 1.1% per year.

- (a) Assuming constant growth, what will the world population be in 2075?
(b) How long does it take for the world population to triple?

Solution:

(a) $P = 7.8$ [billion], $r = 1.1\% = 0.011$

$$\Rightarrow A(t) = 7.8 \cdot e^{0.011 \cdot t}$$

$$t = 2075 - 2020 = 55$$

$$\Rightarrow A(55) = 7.8 \cdot e^{0.011 \cdot 55} \approx 14.3$$

There will be approximately 14.3 billion people.

Exponential models - exercises

① A city has a population size of 56,000 people. The city grows exponentially at a rate of 6% per year.

- (a) What is the population size after 15 years?
(b) How long will it take for the population to reach 200,000?

Solution:

(a) $P = 56,000$, $r = 6\% = 0.06$
 $\Rightarrow A(t) = 56000 \cdot e^{0.06 \cdot t}$
 $t = 15$
 $\Rightarrow A(15) = 56000 \cdot e^{0.06 \cdot 15} \approx 138,000$
There will be approximately 138,000 people in the city.

(b) $A(t) = 200,000$
 $\Rightarrow 200,000 = 56,000 \cdot e^{0.06 \cdot t}$
 $\Rightarrow \frac{200,000}{56,000} = e^{0.06 \cdot t}$
 $\Rightarrow \ln\left(\frac{200,000}{56,000}\right) = 0.06 \cdot t$
 $\Rightarrow t = \frac{\ln\left(\frac{200,000}{56,000}\right)}{0.06} \approx 21.2$
It will take approximately 21.2 years.

② The world population in 2020 consists of about 7.8 billion people and grows at a rate of 1.1% per year.

- (a) Assuming constant growth, what will the world population be in 2075?
(b) How long does it take for the world population to triple?

Solution:

(a) $P = 7.8$ [billion], $r = 1.1\% = 0.011$
 $\Rightarrow A(t) = 7.8 \cdot e^{0.011 \cdot t}$
 $t = 2075 - 2020 = 55$
 $\Rightarrow A(55) = 7.8 \cdot e^{0.011 \cdot 55} \approx 14.3$
There will be approximately 14.3 billion people.

(b) $A(t) = 3 \cdot 7.8$
 $\Rightarrow 3 \cdot 7.8 = 7.8 \cdot e^{0.011 \cdot t}$
 $\Rightarrow 3 = e^{0.011 \cdot t}$
 $\Rightarrow \ln(3) = 0.011 \cdot t$
 $\Rightarrow t = \frac{\ln(3)}{0.011} \approx 99.87$
It takes about 99.87 years for the world population to triple in size.

Exponential models - exercises

- 9 An ant colony of 4900 ants decreases exponentially at a rate of 3.5% per month.
- (a) How many ants will be left after 2 years?
- (b) When will only half of the ants be left?

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Solution:

- (a) $P = 4900$, $r = -3.5\% = -0.035$
 $\Rightarrow A(t) = 4900 \cdot e^{-0.035 \cdot t}$
 $t = 2 \cdot 12 = 24$ [months]
 $\Rightarrow A(24) = 4900 \cdot e^{-0.035 \cdot 24} \approx 2100$
After 2 years, there will be approximately 2100 ants left.

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(b) $A(t) = \frac{1}{2} \cdot 4900 = 2450$

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$$\Rightarrow \frac{1}{2} = e^{-0.035 \cdot t}$$

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It will take approximately 19.8 months until half of the ants will be left.

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Solution:

$P = ?$ (unknown)
 $r = 12.7\% = 0.127$
 $\Rightarrow A(t) = P \cdot (1 + 0.127)^t = P \cdot 1.127^t$
 $A(t) = 2 \cdot P$
 $\Rightarrow 2 \cdot P = P \cdot 1.127^t$
 $\Rightarrow 2 = 1.127^t$
 $\Rightarrow \ln(2) = t \cdot \ln(1.127)$
 $\Rightarrow t = \frac{\ln(2)}{\ln(1.127)} \approx 5.8$

It takes approximately 5.8 days for the bacteria colony to double in size.

Note: It takes the same amount of time for the culture to double no matter what the initial number P of bacterias was.

