

# Properties of log and logarithmic equations

## Lesson #14

### MAT 1375 Precalculus

New York City College of Technology CUNY



## Algebraic properties of exp and log - review

Recall the following exponential identities:

$$B^r \cdot B^s = B^{r+s}$$

$$\frac{B^r}{B^s} = B^{r-s}$$

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These translate into logarithmic identities:

## Logarithmic identities

$$\log_B(r \cdot s) = \log_B(r) + \log_B(s)$$

$$\log_B\left(\frac{r}{s}\right) = \log_B(r) - \log_B(s)$$

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Combine to one logarithm:

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$$4 \log_5(x) + 2 \log_5(y) + 6 \log_5(z)$$

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$$5 \log(x) - 7 \log(y) - 4 \log(z)$$

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## Combining and expanding logarithms - exercises

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Recall:

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$$\begin{aligned}9 \quad & \log_2\left(\frac{\sqrt{x} \cdot y^7}{z^{\frac{1}{3}}}\right) = \log_2\left(\frac{x^{\frac{1}{2}} \cdot y^7}{z^{\frac{1}{3}}}\right) \\ &= \log_2(x^{\frac{1}{2}}) + \log_2(y^7) - \log_2(z^{\frac{1}{3}}) \\ &= \frac{1}{2} \cdot \log_2(x) + 7 \cdot \log_2(y) - \frac{1}{3} \cdot \log_2(z)\end{aligned}$$

$$\begin{aligned}10 \quad & \log\left(\sqrt{\frac{x^9}{y^7 \cdot z^5}}\right) = \log\left(\frac{x^9}{y^7 \cdot z^5}\right)^{\frac{1}{2}} = \frac{1}{2} \cdot \log\left(\frac{x^9}{y^7 \cdot z^5}\right) \\ &= \frac{1}{2} \cdot (9 \log(x) - 7 \log(y) - 5 \cdot \log(z)) \\ &= \frac{9}{2} \log(x) - \frac{7}{2} \log(y) - \frac{5}{2} \log(z)\end{aligned}$$

## Solving logarithmic equations - exercises

Solve the logarithmic equation.

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$$\log_4(2x + 3) = \log_4(7x - 5)$$

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$$\log_4(2x + 3) = \log_4(7x - 5)$$

$$\Rightarrow 2x + 3 = 7x - 5$$

$$\Rightarrow -5x = -8$$

$$\Rightarrow x = \frac{-8}{-5} = \frac{8}{5}$$

The logarithm is one-to-one

$$\log_B(r) = \log_B(s) \Leftrightarrow r = s$$

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$$\ln(2x - 7) = \ln(x - 4)$$

$$\Rightarrow 2x - 7 = x - 4 \quad \Rightarrow x = 3 \quad \mathbf{X}$$

Note that  $x = 3$  is **NOT** a solution,  
because  $\ln(2 \cdot 3 - 7) = \ln(-1) \quad \mathbf{X}$   
is undefined!



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More precisely:

The logarithm is one-to-one

When  $r > 0$  and  $s > 0$ :

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$$\log(2x + 11) = \log(4x + 17)$$

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$$\log_3(3x - 8) = \log_3(2x - 6)$$

$$\Rightarrow 3x - 8 = 2x - 6$$

$$\Rightarrow x = 2 \quad \times \quad \text{not a solution:}$$

$$\log_3(3 \cdot 2 - 8) = \log_3(-2) \text{ is undefined}$$

$$\log_3(2 \cdot 2 - 6) = \log_3(-2) \text{ is undefined}$$

$\Rightarrow$  There is no solution!

5

$$\log(2x + 11) = \log(4x + 17)$$

# Solving logarithmic equations - exercises

Solve the logarithmic equation.

1

$$\begin{aligned}\log_4(2x + 3) &= \log_4(7x - 5) \\ \Rightarrow 2x + 3 &= 7x - 5 \\ \Rightarrow -5x &= -8 \\ \Rightarrow x &= \frac{-8}{-5} = \frac{8}{5}\end{aligned}$$

The logarithm is one-to-one

$$\log_B(r) = \log_B(s) \Leftrightarrow r = s$$

2

$$\begin{aligned}\log_7(2x + 4) &= \log_7(6x - 3) \\ \Rightarrow 2x + 4 &= 6x - 3 \\ \Rightarrow -4x &= -7 \\ \Rightarrow x &= \frac{7}{4}\end{aligned}$$

3

$$\begin{aligned}\ln(2x - 7) &= \ln(x - 4) \\ \Rightarrow 2x - 7 &= x - 4 \quad \Rightarrow x = 3 \quad \times \\ \text{Note that } x = 3 &\text{ is } \mathbf{NOT} \text{ a solution,} \\ \text{because } \ln(2 \cdot 3 - 7) &= \ln(-1) \quad \times \\ &\text{is undefined!}\end{aligned}$$

More precisely:

The logarithm is one-to-one

When  $r > 0$  and  $s > 0$ :

$$\log_B(r) = \log_B(s) \Leftrightarrow r = s$$

4

$$\begin{aligned}\log_3(3x - 8) &= \log_3(2x - 6) \\ \Rightarrow 3x - 8 &= 2x - 6 \\ \Rightarrow x &= 2 \quad \times \quad \text{not a solution:} \\ \log_3(3 \cdot 2 - 8) &= \log_3(-2) \text{ is undefined} \\ \log_3(2 \cdot 2 - 6) &= \log_3(-2) \text{ is undefined} \\ \Rightarrow &\text{There is no solution!}\end{aligned}$$

5

$$\begin{aligned}\log(2x + 11) &= \log(4x + 17) \\ \Rightarrow 2x + 11 &= 4x + 17 \\ \Rightarrow -6 &= 2x \\ \Rightarrow x &= -3 \quad \checkmark \quad \text{is a solution:} \\ \log(2 \cdot (-3) + 11) &= \log(5) \text{ is defined} \\ \log(4 \cdot (-3) + 17) &= \log(5) \text{ is defined}\end{aligned}$$

## Solving logarithmic equations - exercises

Solve the logarithmic equation.

1

$$\log_3(x) = 2$$

## Solving logarithmic equations - exercises

Solve the logarithmic equation.

4

$$\log_3(x) = 2$$

$$\Rightarrow x = 3^2 = 9 \quad \checkmark$$

Recall

$$\log_B(x) = y \quad \Leftrightarrow \quad x = B^y$$

## Solving logarithmic equations - exercises

Solve the logarithmic equation.

1

$$\begin{aligned}\log_3(x) &= 2 \\ \Rightarrow x &= 3^2 = 9 \quad \checkmark\end{aligned}$$

Recall

$$\log_B(x) = y \quad \Leftrightarrow \quad x = B^y$$

2

$$\log_5(x) = 7$$

## Solving logarithmic equations - exercises

Solve the logarithmic equation.

1

$$\begin{aligned}\log_3(x) &= 2 \\ \Rightarrow x &= 3^2 = 9 \quad \checkmark\end{aligned}$$

Recall

$$\log_B(x) = y \quad \Leftrightarrow \quad x = B^y$$

2

$$\begin{aligned}\log_5(x) &= 7 \\ \Rightarrow x &= 5^7 = 78125 \quad \checkmark\end{aligned}$$



## Solving logarithmic equations - exercises

Solve the logarithmic equation.

1

$$\begin{aligned}\log_3(x) &= 2 \\ \Rightarrow x &= 3^2 = 9 \quad \checkmark\end{aligned}$$

Recall

$$\log_B(x) = y \quad \Leftrightarrow \quad x = B^y$$

2

$$\begin{aligned}\log_5(x) &= 7 \\ \Rightarrow x &= 5^7 = 78125 \quad \checkmark\end{aligned}$$

3

$$\log_2(x + 5) = 4$$

4

$$\log_7(5x + 8) = 3$$

## Solving logarithmic equations - exercises

Solve the logarithmic equation.

1

$$\begin{aligned}\log_3(x) &= 2 \\ \Rightarrow x &= 3^2 = 9 \quad \checkmark\end{aligned}$$

### Recall

$$\log_B(x) = y \quad \Leftrightarrow \quad x = B^y$$

2

$$\begin{aligned}\log_5(x) &= 7 \\ \Rightarrow x &= 5^7 = 78125 \quad \checkmark\end{aligned}$$

3

$$\begin{aligned}\log_2(x + 5) &= 4 \\ \Rightarrow x + 5 &= 2^4 = 16 \\ \Rightarrow x &= 16 - 5 = 11 \quad \checkmark\end{aligned}$$

4

$$\log_7(5x + 8) = 3$$

# Solving logarithmic equations - exercises

Solve the logarithmic equation.

1

$$\begin{aligned}\log_3(x) &= 2 \\ \Rightarrow x &= 3^2 = 9 \quad \checkmark\end{aligned}$$

## Recall

$$\log_B(x) = y \quad \Leftrightarrow \quad x = B^y$$

2

$$\begin{aligned}\log_5(x) &= 7 \\ \Rightarrow x &= 5^7 = 78125 \quad \checkmark\end{aligned}$$

3

$$\begin{aligned}\log_2(x + 5) &= 4 \\ \Rightarrow x + 5 &= 2^4 = 16 \\ \Rightarrow x &= 16 - 5 = 11 \quad \checkmark\end{aligned}$$

4

$$\begin{aligned}\log_7(5x + 8) &= 3 \\ \Rightarrow 5x + 8 &= 7^3 = 343 \\ \Rightarrow 5x &= 335 \\ \Rightarrow x &= 67 \quad \checkmark\end{aligned}$$

## Solving logarithmic equations - exercises

Solve the logarithmic equation.

1

$$\begin{aligned}\log_3(x) &= 2 \\ \Rightarrow x &= 3^2 = 9 \quad \checkmark\end{aligned}$$

5

$$\log_2(x + 1) + \log_2(x - 3) = 5$$

### Recall

$$\log_B(x) = y \quad \Leftrightarrow \quad x = B^y$$

2

$$\begin{aligned}\log_5(x) &= 7 \\ \Rightarrow x &= 5^7 = 78125 \quad \checkmark\end{aligned}$$

3

$$\begin{aligned}\log_2(x + 5) &= 4 \\ \Rightarrow x + 5 &= 2^4 = 16 \\ \Rightarrow x &= 16 - 5 = 11 \quad \checkmark\end{aligned}$$

4

$$\begin{aligned}\log_7(5x + 8) &= 3 \\ \Rightarrow 5x + 8 &= 7^3 = 343 \\ \Rightarrow 5x &= 335 \\ \Rightarrow x &= 67 \quad \checkmark\end{aligned}$$

# Solving logarithmic equations - exercises

Solve the logarithmic equation.

1

$$\begin{aligned}\log_3(x) &= 2 \\ \Rightarrow x &= 3^2 = 9 \quad \checkmark\end{aligned}$$

Recall

$$\log_B(x) = y \quad \Leftrightarrow \quad x = B^y$$

2

$$\begin{aligned}\log_5(x) &= 7 \\ \Rightarrow x &= 5^7 = 78125 \quad \checkmark\end{aligned}$$

3

$$\begin{aligned}\log_2(x + 5) &= 4 \\ \Rightarrow x + 5 &= 2^4 = 16 \\ \Rightarrow x &= 16 - 5 = 11 \quad \checkmark\end{aligned}$$

4

$$\begin{aligned}\log_7(5x + 8) &= 3 \\ \Rightarrow 5x + 8 &= 7^3 = 343 \\ \Rightarrow 5x &= 335 \\ \Rightarrow x &= 67 \quad \checkmark\end{aligned}$$

5

$$\begin{aligned}\log_2(x + 1) + \log_2(x - 3) &= 5 \\ \Rightarrow \log_2((x + 1) \cdot (x - 3)) &= 5 \\ \Rightarrow (x + 1) \cdot (x - 3) &= 2^5 \\ \Rightarrow x^2 - 2x - 3 &= 32 \\ \Rightarrow x^2 - 2x - 35 &= 0 \\ \Rightarrow (x + 5)(x - 7) &= 0 \\ \Rightarrow x = -5 \quad \times \quad \text{or} \quad x = 7 \quad \checkmark \\ \Rightarrow x &= 7\end{aligned}$$

# Solving logarithmic equations - exercises

Solve the logarithmic equation.

1

$$\log_3(x) = 2$$
$$\Rightarrow x = 3^2 = 9 \checkmark$$

Recall

$$\log_B(x) = y \Leftrightarrow x = B^y$$

2

$$\log_5(x) = 7$$
$$\Rightarrow x = 5^7 = 78125 \checkmark$$

3

$$\log_2(x + 5) = 4$$
$$\Rightarrow x + 5 = 2^4 = 16$$
$$\Rightarrow x = 16 - 5 = 11 \checkmark$$

4

$$\log_7(5x + 8) = 3$$
$$\Rightarrow 5x + 8 = 7^3 = 343$$
$$\Rightarrow 5x = 335$$
$$\Rightarrow x = 67 \checkmark$$

5

$$\log_2(x + 1) + \log_2(x - 3) = 5$$
$$\Rightarrow \log_2((x + 1) \cdot (x - 3)) = 5$$
$$\Rightarrow (x + 1) \cdot (x - 3) = 2^5$$
$$\Rightarrow x^2 - 2x - 3 = 32$$
$$\Rightarrow x^2 - 2x - 35 = 0$$
$$\Rightarrow (x + 5)(x - 7) = 0$$
$$\Rightarrow x = -5 \text{ X or } x = 7 \checkmark$$
$$\Rightarrow x = 7$$

6

$$\log_3(x - 2) + \log_3(x + 6) = 2$$

# Solving logarithmic equations - exercises

Solve the logarithmic equation.

1

$$\begin{aligned}\log_3(x) &= 2 \\ \Rightarrow x &= 3^2 = 9 \quad \checkmark\end{aligned}$$

Recall

$$\log_B(x) = y \quad \Leftrightarrow \quad x = B^y$$

2

$$\begin{aligned}\log_5(x) &= 7 \\ \Rightarrow x &= 5^7 = 78125 \quad \checkmark\end{aligned}$$

3

$$\begin{aligned}\log_2(x + 5) &= 4 \\ \Rightarrow x + 5 &= 2^4 = 16 \\ \Rightarrow x &= 16 - 5 = 11 \quad \checkmark\end{aligned}$$

4

$$\begin{aligned}\log_7(5x + 8) &= 3 \\ \Rightarrow 5x + 8 &= 7^3 = 343 \\ \Rightarrow 5x &= 335 \\ \Rightarrow x &= 67 \quad \checkmark\end{aligned}$$

5

$$\begin{aligned}\log_2(x + 1) + \log_2(x - 3) &= 5 \\ \Rightarrow \log_2((x + 1) \cdot (x - 3)) &= 5 \\ \Rightarrow (x + 1) \cdot (x - 3) &= 2^5 \\ \Rightarrow x^2 - 2x - 3 &= 32 \\ \Rightarrow x^2 - 2x - 35 &= 0 \\ \Rightarrow (x + 5)(x - 7) &= 0 \\ \Rightarrow x = -5 \quad \times \quad \text{or} \quad x = 7 \quad \checkmark \\ \Rightarrow x &= 7\end{aligned}$$

6

$$\begin{aligned}\log_3(x - 2) + \log_3(x + 6) &= 2 \\ \Rightarrow (x - 2)(x + 6) &= 3^2 \\ \Rightarrow x^2 + 4x - 12 &= 9 \\ \Rightarrow x^2 + 4x - 21 &= 0 \\ \Rightarrow (x + 7)(x - 3) &= 0 \\ \Rightarrow x = -7 \quad \times \quad \text{or} \quad x = 3 \quad \checkmark \\ \Rightarrow x &= 3\end{aligned}$$

## Solving logarithmic equations - exercises

Solve the logarithmic equation.

7

$$\log_5(x + 3) + \log_5(x + 4) = \log_5(2)$$



## Solving logarithmic equations - exercises

Solve the logarithmic equation.

7

$$\begin{aligned}\log_5(x+3) + \log_5(x+4) &= \log_5(2) \\ \Rightarrow \log_5((x+3) \cdot (x+4)) &= \log_5(2) \\ \Rightarrow (x+3) \cdot (x+4) &= 2 \\ \Rightarrow x^2 + 7x + 12 &= 2 \\ \Rightarrow x^2 + 7x + 10 &= 0 \\ \Rightarrow (x+2)(x+5) &= 0 \\ \Rightarrow x = -2 \checkmark \text{ or } x = -5 \times \\ \Rightarrow x &= -2\end{aligned}$$

## Solving logarithmic equations - exercises

Solve the logarithmic equation.

7

$$\begin{aligned}\log_5(x+3) + \log_5(x+4) &= \log_5(2) \\ \Rightarrow \log_5((x+3) \cdot (x+4)) &= \log_5(2) \\ \Rightarrow (x+3) \cdot (x+4) &= 2 \\ \Rightarrow x^2 + 7x + 12 &= 2 \\ \Rightarrow x^2 + 7x + 10 &= 0 \\ \Rightarrow (x+2)(x+5) &= 0 \\ \Rightarrow x = -2 \checkmark \text{ or } x = -5 \times \\ \Rightarrow x &= -2\end{aligned}$$

8

$$\log_3(x-7) + \log_3(2-x) = \log_3(4)$$

9

$$\ln(x+1) + \ln(7-x) = \ln(12)$$

10

$$\log_4(x+2) + \log_4(x+5) = 3$$

## Solving logarithmic equations - exercises

Solve the logarithmic equation.

7

$$\begin{aligned}\log_5(x+3) + \log_5(x+4) &= \log_5(2) \\ \Rightarrow \log_5((x+3) \cdot (x+4)) &= \log_5(2) \\ \Rightarrow (x+3) \cdot (x+4) &= 2 \\ \Rightarrow x^2 + 7x + 12 &= 2 \\ \Rightarrow x^2 + 7x + 10 &= 0 \\ \Rightarrow (x+2)(x+5) &= 0 \\ \Rightarrow x = -2 \checkmark \text{ or } x = -5 \times \\ \Rightarrow x &= -2\end{aligned}$$

8

$$\begin{aligned}\log_3(x-7) + \log_3(2-x) &= \log_3(4) \\ \Rightarrow \log_3((x-7) \cdot (2-x)) &= \log_3(4) \\ \Rightarrow (x-7) \cdot (2-x) &= 4 \\ \Rightarrow 2x - x^2 - 14 + 7x &= 4 \\ \Rightarrow -x^2 + 9x - 18 &= 0 \\ \xrightarrow{\cdot(-1)} \Rightarrow x^2 - 9x + 18 &= 0 \\ \Rightarrow (x-3) \cdot (x-6) &= 0 \\ \Rightarrow x = 3 \times \text{ or } x = 6 \times \\ \Rightarrow \text{There is no solution!}\end{aligned}$$

9

$$\ln(x+1) + \ln(7-x) = \ln(12)$$

10

$$\log_4(x+2) + \log_4(x+5) = 3$$

## Solving logarithmic equations - exercises

Solve the logarithmic equation.

7

$$\begin{aligned}\log_5(x+3) + \log_5(x+4) &= \log_5(2) \\ \Rightarrow \log_5((x+3) \cdot (x+4)) &= \log_5(2) \\ \Rightarrow (x+3) \cdot (x+4) &= 2 \\ \Rightarrow x^2 + 7x + 12 &= 2 \\ \Rightarrow x^2 + 7x + 10 &= 0 \\ \Rightarrow (x+2)(x+5) &= 0 \\ \Rightarrow x = -2 \checkmark \text{ or } x = -5 \times \\ \Rightarrow x = -2\end{aligned}$$

8

$$\begin{aligned}\log_3(x-7) + \log_3(2-x) &= \log_3(4) \\ \Rightarrow \log_3((x-7) \cdot (2-x)) &= \log_3(4) \\ \Rightarrow (x-7) \cdot (2-x) &= 4 \\ \Rightarrow 2x - x^2 - 14 + 7x &= 4 \\ \Rightarrow -x^2 + 9x - 18 &= 0 \\ \xrightarrow{\cdot(-1)} \Rightarrow x^2 - 9x + 18 &= 0 \\ \Rightarrow (x-3) \cdot (x-6) &= 0 \\ \Rightarrow x = 3 \times \text{ or } x = 6 \times \\ \Rightarrow \text{There is no solution!}\end{aligned}$$

9

$$\begin{aligned}\ln(x+1) + \ln(7-x) &= \ln(12) \\ \Rightarrow \ln((x+1) \cdot (7-x)) &= \ln(12) \\ \Rightarrow (x+1) \cdot (7-x) &= 12 \\ \Rightarrow 7x - x^2 + 7 - x &= 12 \\ \Rightarrow -x^2 + 6x - 5 &= 0 \\ \Rightarrow x^2 - 6x + 5 &= 0 \\ \Rightarrow (x-1) \cdot (x-5) &= 0 \\ \Rightarrow x = 1 \checkmark \text{ or } x = 5 \checkmark \\ \Rightarrow \text{There are two solutions!}\end{aligned}$$

10

$$\log_4(x+2) + \log_4(x+5) = 3$$

# Solving logarithmic equations - exercises

Solve the logarithmic equation.

7

$$\begin{aligned}\log_5(x+3) + \log_5(x+4) &= \log_5(2) \\ \Rightarrow \log_5((x+3) \cdot (x+4)) &= \log_5(2) \\ \Rightarrow (x+3) \cdot (x+4) &= 2 \\ \Rightarrow x^2 + 7x + 12 &= 2 \\ \Rightarrow x^2 + 7x + 10 &= 0 \\ \Rightarrow (x+2)(x+5) &= 0 \\ \Rightarrow x = -2 \checkmark \text{ or } x = -5 \times \\ \Rightarrow x &= -2\end{aligned}$$

8

$$\begin{aligned}\log_3(x-7) + \log_3(2-x) &= \log_3(4) \\ \Rightarrow \log_3((x-7) \cdot (2-x)) &= \log_3(4) \\ \Rightarrow (x-7) \cdot (2-x) &= 4 \\ \Rightarrow 2x - x^2 - 14 + 7x &= 4 \\ \Rightarrow -x^2 + 9x - 18 &= 0 \\ \stackrel{(-1)}{\Rightarrow} x^2 - 9x + 18 &= 0 \\ \Rightarrow (x-3) \cdot (x-6) &= 0 \\ \Rightarrow x = 3 \times \text{ or } x = 6 \times \\ \Rightarrow \text{There is no solution!}\end{aligned}$$

9

$$\begin{aligned}\ln(x+1) + \ln(7-x) &= \ln(12) \\ \Rightarrow \ln((x+1) \cdot (7-x)) &= \ln(21) \\ \Rightarrow (x+1) \cdot (7-x) &= 12 \\ \Rightarrow 7x - x^2 + 7 - x &= 12 \\ \Rightarrow -x^2 + 6x - 5 &= 0 \\ \Rightarrow x^2 - 6x + 5 &= 0 \\ \Rightarrow (x-1) \cdot (x-5) &= 0 \\ \Rightarrow x = 1 \checkmark \text{ or } x = 5 \checkmark \\ \Rightarrow \text{There are two solutions!}\end{aligned}$$

10

$$\begin{aligned}\log_4(x+2) + \log_4(x+5) &= 3 \\ \Rightarrow (x+2)(x+5) &= 4^3 \\ \Rightarrow x^2 + 7x + 10 &= 64 \\ \Rightarrow x^2 + 7x - 54 &= 0 \\ \Rightarrow x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-7 \pm \sqrt{7^2 - 4 \cdot 1 \cdot (-54)}}{2} \\ \Rightarrow x &= \frac{-7 \pm \sqrt{265}}{2} \\ \Rightarrow x &= \frac{-7 + \sqrt{265}}{2} \text{ or } x = \frac{-7 - \sqrt{265}}{2}\end{aligned}$$

# Solving logarithmic equations - exercises

Solve the logarithmic equation.

7

$$\begin{aligned}\log_5(x+3) + \log_5(x+4) &= \log_5(2) \\ \Rightarrow \log_5((x+3) \cdot (x+4)) &= \log_5(2) \\ \Rightarrow (x+3) \cdot (x+4) &= 2 \\ \Rightarrow x^2 + 7x + 12 &= 2 \\ \Rightarrow x^2 + 7x + 10 &= 0 \\ \Rightarrow (x+2)(x+5) &= 0 \\ \Rightarrow x = -2 \checkmark \text{ or } x = -5 \times \\ \Rightarrow x &= -2\end{aligned}$$

8

$$\begin{aligned}\log_3(x-7) + \log_3(2-x) &= \log_3(4) \\ \Rightarrow \log_3((x-7) \cdot (2-x)) &= \log_3(4) \\ \Rightarrow (x-7) \cdot (2-x) &= 4 \\ \Rightarrow 2x - x^2 - 14 + 7x &= 4 \\ \Rightarrow -x^2 + 9x - 18 &= 0 \\ \stackrel{\cdot(-1)}{\Rightarrow} x^2 - 9x + 18 &= 0 \\ \Rightarrow (x-3) \cdot (x-6) &= 0 \\ \Rightarrow x = 3 \times \text{ or } x = 6 \times \\ \Rightarrow \text{There is no solution!}\end{aligned}$$

9

$$\begin{aligned}\ln(x+1) + \ln(7-x) &= \ln(12) \\ \Rightarrow \ln((x+1) \cdot (7-x)) &= \ln(21) \\ \Rightarrow (x+1) \cdot (7-x) &= 12 \\ \Rightarrow 7x - x^2 + 7 - x &= 12 \\ \Rightarrow -x^2 + 6x - 5 &= 0 \\ \Rightarrow x^2 - 6x + 5 &= 0 \\ \Rightarrow (x-1) \cdot (x-5) &= 0 \\ \Rightarrow x = 1 \checkmark \text{ or } x = 5 \checkmark \\ \Rightarrow \text{There are two solutions!}\end{aligned}$$

10

$$\begin{aligned}\log_4(x+2) + \log_4(x+5) &= 3 \\ \Rightarrow (x+2)(x+5) &= 4^3 \\ \Rightarrow x^2 + 7x + 10 &= 64 \\ \Rightarrow x^2 + 7x - 54 &= 0 \\ \Rightarrow x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-7 \pm \sqrt{7^2 - 4 \cdot 1 \cdot (-54)}}{2} \\ \Rightarrow x &= \frac{-7 \pm \sqrt{265}}{2} \\ \Rightarrow x &= \frac{-7 + \sqrt{265}}{2} \text{ or } x = \frac{-7 - \sqrt{265}}{2} \\ &\approx 4.639 \checkmark \quad \approx -11.639 \times \\ \Rightarrow x &= \frac{-7 + \sqrt{265}}{2}\end{aligned}$$

## Solving logarithmic equations - exercises

Solve the logarithmic equation.

12

$$\log_2(x + 7) - \log_2(x + 3) = 5$$

## Solving logarithmic equations - exercises

Solve the logarithmic equation.

12

$$\log_2(x + 7) - \log_2(x + 3) = 5$$

$$\Rightarrow \log_2\left(\frac{x+7}{x+3}\right) = 5$$

$$\Rightarrow \frac{x+7}{x+3} = 2^5$$

$$\Rightarrow \frac{x+7}{x+3} = 32$$

$$\Rightarrow x + 7 = 32 \cdot (x + 3)$$

$$\Rightarrow x + 7 = 32x + 96$$

$$\Rightarrow -31x = 89$$

$$\Rightarrow x = \frac{89}{-31} = -\frac{89}{31} \checkmark$$

Note that this is a solution, since both  $\log_2(x + 7)$  and  $\log_2(x + 3)$  are defined for  $x = -\frac{89}{31}$ , because:

$$x + 7 = -\frac{89}{31} + 7 \approx 4.129 > 0,$$

$$x + 3 = -\frac{89}{31} + 3 \approx 0.129 > 0.$$



## Solving logarithmic equations - exercises

Solve the logarithmic equation.

12

$$\log_2(x + 7) - \log_2(x + 3) = 5$$

$$\Rightarrow \log_2\left(\frac{x+7}{x+3}\right) = 5$$

$$\Rightarrow \frac{x+7}{x+3} = 2^5$$

$$\Rightarrow \frac{x+7}{x+3} = 32$$

$$\Rightarrow x + 7 = 32 \cdot (x + 3)$$

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$$\Rightarrow -31x = 89$$

$$\Rightarrow x = \frac{89}{-31} = -\frac{89}{31} \checkmark$$

Note that this is a solution, since both  $\log_2(x + 7)$  and  $\log_2(x + 3)$  are defined for  $x = -\frac{89}{31}$ , because:

$$x + 7 = -\frac{89}{31} + 7 \approx 4.129 > 0,$$

$$x + 3 = -\frac{89}{31} + 3 \approx 0.129 > 0.$$

13

$$\log_3(x + 5) - \log_3(x + 2) = 4$$

14

$$\log_5(x + 8) - \log_5(3 - x) = 2$$

## Solving logarithmic equations - exercises

Solve the logarithmic equation.

12

$$\log_2(x + 7) - \log_2(x + 3) = 5$$

$$\Rightarrow \log_2\left(\frac{x+7}{x+3}\right) = 5$$

$$\Rightarrow \frac{x+7}{x+3} = 2^5$$

$$\Rightarrow \frac{x+7}{x+3} = 32$$

$$\Rightarrow x + 7 = 32 \cdot (x + 3)$$

$$\Rightarrow x + 7 = 32x + 96$$

$$\Rightarrow -31x = 89$$

$$\Rightarrow x = \frac{89}{-31} = -\frac{89}{31} \checkmark$$

Note that this is a solution, since both  $\log_2(x + 7)$  and  $\log_2(x + 3)$  are defined for  $x = -\frac{89}{31}$ , because:

$$x + 7 = -\frac{89}{31} + 7 \approx 4.129 > 0,$$

$$x + 3 = -\frac{89}{31} + 3 \approx 0.129 > 0.$$

13

$$\log_3(x + 5) - \log_3(x + 2) = 4$$

$$\Rightarrow \log_3\left(\frac{x+5}{x+2}\right) = 4$$

$$\Rightarrow \frac{x+5}{x+2} = 3^4 = 81$$

$$\Rightarrow x + 5 = 81 \cdot (x + 2)$$

$$\Rightarrow x + 5 = 81x + 162$$

$$\Rightarrow -80x = 157$$

$$\Rightarrow x = -\frac{157}{80} \checkmark$$

This is a solution, since  $-\frac{157}{80} \approx -1.96$ .

14

$$\log_5(x + 8) - \log_5(3 - x) = 2$$

## Solving logarithmic equations - exercises

Solve the logarithmic equation.

12

$$\log_2(x + 7) - \log_2(x + 3) = 5$$

$$\Rightarrow \log_2\left(\frac{x+7}{x+3}\right) = 5$$

$$\Rightarrow \frac{x+7}{x+3} = 2^5$$

$$\Rightarrow \frac{x+7}{x+3} = 32$$

$$\Rightarrow x + 7 = 32 \cdot (x + 3)$$

$$\Rightarrow x + 7 = 32x + 96$$

$$\Rightarrow -31x = 89$$

$$\Rightarrow x = \frac{89}{-31} = -\frac{89}{31} \checkmark$$

Note that this is a solution, since both  $\log_2(x + 7)$  and  $\log_2(x + 3)$  are defined for  $x = -\frac{89}{31}$ , because:

$$x + 7 = -\frac{89}{31} + 7 \approx 4.129 > 0,$$

$$x + 3 = -\frac{89}{31} + 3 \approx 0.129 > 0.$$

13

$$\log_3(x + 5) - \log_3(x + 2) = 4$$

$$\Rightarrow \log_3\left(\frac{x+5}{x+2}\right) = 4$$

$$\Rightarrow \frac{x+5}{x+2} = 3^4 = 81$$

$$\Rightarrow x + 5 = 81 \cdot (x + 2)$$

$$\Rightarrow x + 5 = 81x + 162$$

$$\Rightarrow -80x = 157$$

$$\Rightarrow x = -\frac{157}{80} \checkmark$$

This is a solution, since  $-\frac{157}{80} \approx -1.96$ .

14

$$\log_5(x + 8) - \log_5(3 - x) = 2$$

$$\Rightarrow \log_5\left(\frac{x+8}{3-x}\right) = 2$$

$$\Rightarrow \frac{x+8}{3-x} = 5^2 = 25$$

$$\Rightarrow x + 8 = 25 \cdot (3 - x)$$

$$\Rightarrow x + 8 = 75 - 25x$$

$$\Rightarrow 26x = 67$$

$$\Rightarrow x = \frac{67}{26} \checkmark$$

This is a solution, since  $\frac{67}{26} \approx 2.58$ , and so  $x + 8 > 0$  and  $3 - x > 0$ .

