

Exponential and logarithmic functions

Lesson #13

MAT 1375 Precalculus

New York City College of Technology CUNY



Graphs of exponential functions

Example

Graph $f(x) = 2^x$.

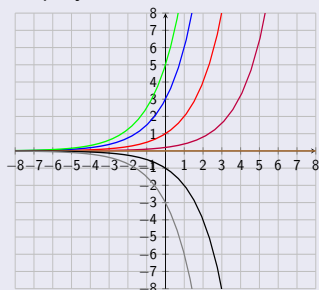
x	-2	-1	0	1	2	3
y	$2^{-2} = 0.25$	$2^{-1} = 0.5$	$2^0 = 1$	$2^1 = 2$	$2^2 = 4$	$2^3 = 8$



$$\begin{aligned}y &= 2^x \\y &= 3^x \\y &= 4^x \\y &= 1.2^x \\y &= 1^x \\y &= 0.5^x = 2^{-x} \\y &= 0.25^x = 4^{-x}\end{aligned}$$

Varying the coefficient A

Graph $y = A \cdot 2^x$.



$$\begin{aligned}y &= 1 \cdot 2^x \\y &= 3 \cdot 2^x \\y &= 5 \cdot 2^x \\y &= 0.2 \cdot 2^x \\y &= 0 \cdot 2^x \\y &= (-1) \cdot 2^x \\y &= (-3) \cdot 2^x\end{aligned}$$

Thus: A is the y -intercept!

Definition (Exponential function)

A function $f(x) = A \cdot B^x$ is called an **exponential function**.

We require $B > 0$. B is called the **base**.

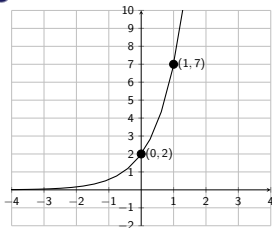
Important bases: $B = 10$, $B = e \approx 2.71828$ (Euler's number)

For $f(x) = B^x$: domain $D = \mathbb{R}$ range $R = (0, \infty)$
(when $B \neq 1$)

Exponential functions - exercises

Find the coefficient A and the base B of the displayed exponential function $y = A \cdot B^x$.

1



y-intercept: $A = 2$

plug $(x, y) = (1, 7)$

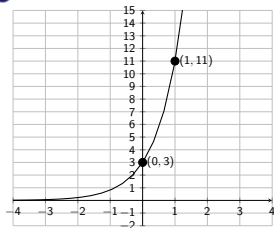
into $y = 2 \cdot B^x$:

$$\Rightarrow 7 = 2 \cdot B^1$$

$$\Rightarrow \frac{7}{2} = B$$

$$\Rightarrow f(x) = 2 \cdot \left(\frac{7}{2}\right)^x$$

2



y-intercept: $A = 3$

plug $(x, y) = (1, 11)$

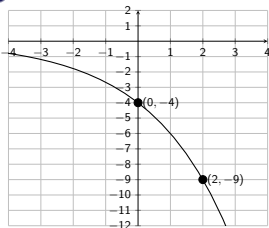
into $y = 3 \cdot B^x$:

$$\Rightarrow 11 = 3 \cdot B^1$$

$$\Rightarrow \frac{11}{3} = B$$

$$\Rightarrow f(x) = 3 \cdot \left(\frac{11}{3}\right)^x$$

3



y-intercept: $A = -4$

plug $(x, y) = (2, -9)$

into $y = (-4) \cdot B^x$:

$$\Rightarrow -9 = (-4) \cdot B^2$$

$$\Rightarrow \frac{9}{4} = B^2$$

$$\Rightarrow B = \sqrt{\frac{9}{4}} = \frac{3}{2}$$

$$\Rightarrow f(x) = (-4) \cdot \left(\frac{3}{2}\right)^x$$

Definition of the logarithm

Definition (Logarithm)

The function $y = B^x$ (for $B \neq 1$) is one-to-one. Let $y = \log_B(x)$ be its inverse function:

$$y = \log_B(x) \Leftrightarrow x = B^y$$

Important cases: $B = 10$: $\log_{10}(x) = \log(x)$
 $B = e$: $\log_e(x) = \ln(x)$ "natural logarithm"

- 1 Find $\log_2(8)$
 $y = \log_2(8) \Leftrightarrow 8 = 2^y = 2^3$
 $\Rightarrow y = 3$
 $\Rightarrow \log_2(8) = 3$
- 2 Find $\log_3(9)$
 $y = \log_3(9) \Leftrightarrow 9 = 3^y = 3^2$
 $\Rightarrow \log_3(9) = 2$
- 3 Find $\log(10,000)$
 $y = \log_{10}(10000)$
 $\Leftrightarrow 10000 = 10^y = 10^4$
 $\Rightarrow \log(10000) = 4$

- 4 Find $\log_8(8)$
 $y = \log_8(8) \Leftrightarrow 8 = 8^y = 8^1$
 $\Rightarrow \log_8(8) = 1$
- 5 Find $\log_5(1)$
 $y = \log_5(1) \Leftrightarrow 1 = 5^y = 5^0$
 $\Rightarrow \log_5(1) = 0$
- 6 Find $\log_7(7^3)$
 $y = \log_7(7^3) \Leftrightarrow 7^3 = 7^y$
 $\Rightarrow y = 3$
 $\Rightarrow \log_7(7^3) = 3$

$$\log_B(B^n) = n$$

$$\log_B(1) = 0, \log_B(B) = 1$$

Change of base formula

$$\log_B(x) = \frac{\log(x)}{\log(B)} = \frac{\ln(x)}{\ln(B)}$$

$$\log_2(5) = \frac{\log(5)}{\log(2)} \approx 2.322$$

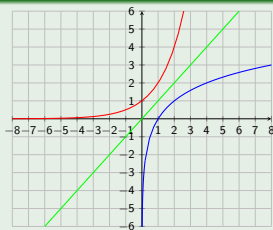
Graphs of logarithmic functions

Example

Graph $y = \log_2(x)$.

x	-2	-1	0	1	2	3	$y = \log_2(x)$
$y = 2^x$	0.25	0.5	1	2	4	8	x

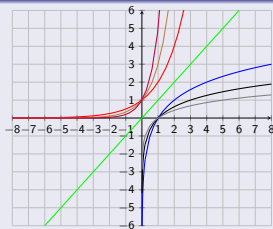
Domain: $D = (0, \infty)$ VA: $x = 0$
Range: $R = \mathbb{R}$ HA: no HA
x-intercept: $x = 1$



Graphs of logarithmic functions

Graph $y = \log_B(x)$.

Domain: $D = (0, \infty)$ VA: $x = 0$
Range: $R = \mathbb{R}$ HA: no HA
x-intercept: $y = 0$
 $\Rightarrow \log_B(x) = 0$
 $\Rightarrow x = B^0 = 1$
y-intercept: no y-intercept



Graphing logarithmic functions - exercises

Find the domain, asymptotes, and x -intercept of the logarithmic function, and sketch a graph.

① $y = \log_3(2x - 5)$

Domain: $2x - 5 > 0$

$$\Rightarrow 2x > 5 \Rightarrow x > \frac{5}{2}$$

$$\Rightarrow D = \left(\frac{5}{2}, \infty\right)$$

VA: $x = \frac{5}{2}$

HA: no HA

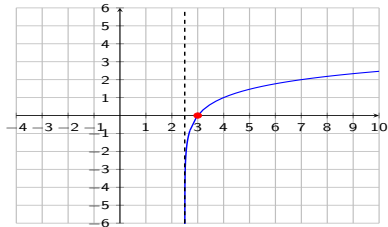
x -intercept: $y = 0$

$$\Rightarrow 0 = \log_3(2x - 5)$$

$$\Rightarrow 2x - 5 = 1$$

$$\Rightarrow 2x = 6$$

$$\Rightarrow x = 3$$



② $y = \log_2(4x + 7)$

Domain: $4x + 7 > 0$

$$\Rightarrow 4x > -7 \Rightarrow x > -\frac{7}{4}$$

$$\Rightarrow D = \left(-\frac{7}{4}, \infty\right)$$

VA: $x = -\frac{7}{4}$

HA: no HA

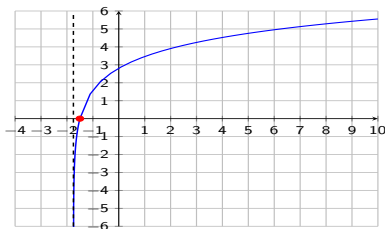
x -intercept: $y = 0$

$$\Rightarrow 0 = \log_2(4x + 7)$$

$$\Rightarrow 4x + 7 = 1$$

$$\Rightarrow 4x = -6$$

$$\Rightarrow x = -\frac{6}{4} = -\frac{3}{2}$$



Graphing logarithmic functions - exercises

Find the domain, asymptotes, and x -intercept of the logarithmic function, and sketch a graph.

3 $y = \log(17 - 7x)$

Domain: $17 - 7x > 0$

$$\Rightarrow -7x > -17 \Rightarrow x < \frac{(-17)}{-7} = \frac{17}{7}$$

$$\Rightarrow D = (-\infty, \frac{17}{7})$$

VA: $x = \frac{17}{7}$

HA: no HA

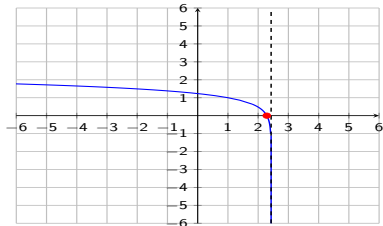
x -intercept: $y = 0$

$$\Rightarrow 0 = \log(17 - 7x)$$

$$\Rightarrow 17 - 7x = 1$$

$$\Rightarrow -7x = -16$$

$$\Rightarrow x = \frac{16}{7}$$



4 $y = \log_6(-x - 4)$

Domain: $-x - 4 > 0$

$$\Rightarrow -x > 4 \Rightarrow x < -4$$

$$\Rightarrow D = (-\infty, -4)$$

VA: $x = -4$

HA: no HA

x -intercept: $y = 0$

$$\Rightarrow 0 = \log_6(-x - 4)$$

$$\Rightarrow -x - 4 = 1$$

$$\Rightarrow -x = 5$$

$$\Rightarrow x = -5$$

