Exponential and logarithmic functions

Lesson #13

MAT 1375 Precalculus

New York City College of Technology CUNY

Graphs of exponential functions

Example

Graph $f(x) = 2^x$.

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x	-2	-1	0	1	2	3
y						

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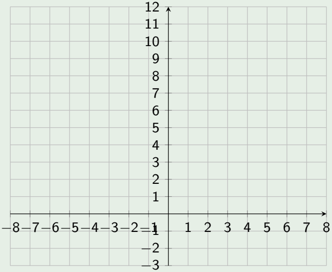

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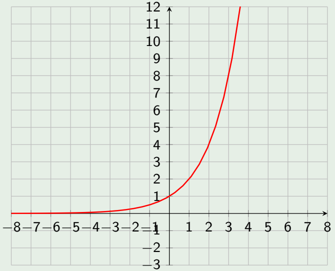

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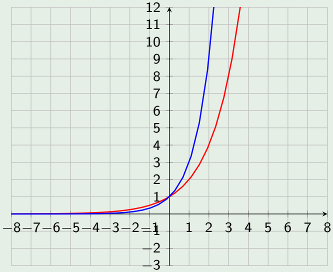

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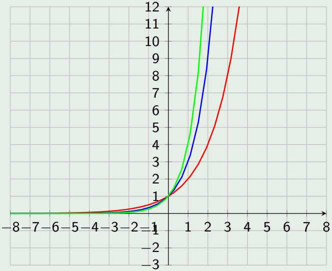

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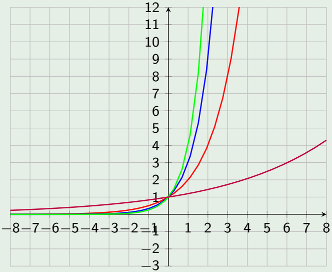

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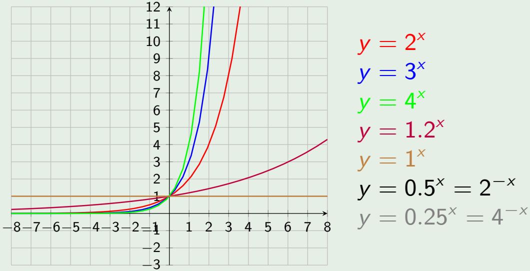

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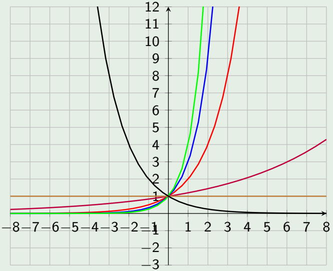

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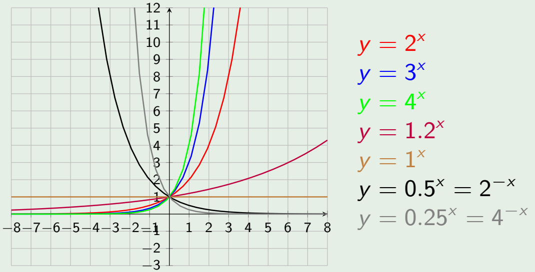

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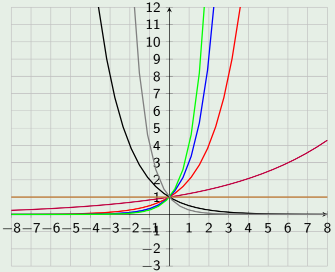

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Definition (Exponential function)

A function $f(x) = A \cdot B^x$ is called an **exponential function**.

We require $B > 0$. B is called the **base**.

Important bases: $B = 10$, $B = e \approx 2.71828$ (Euler's number)

For $f(x) = B^x$: domain $D = \mathbb{R}$ range $R = (0, \infty)$ (when $B \neq 1$)

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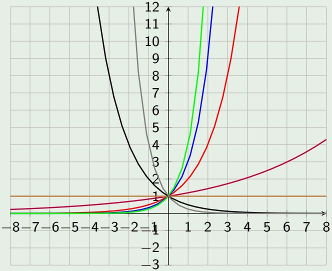

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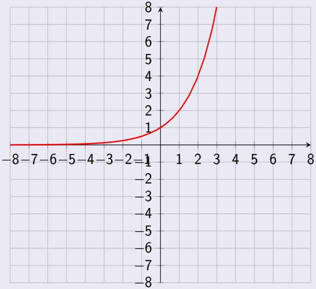
Varying the coefficient A

Graph $y = A \cdot 2^x$.



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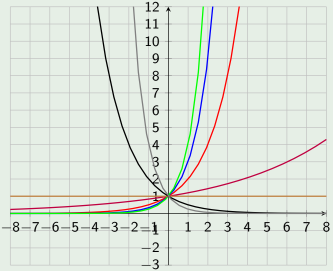

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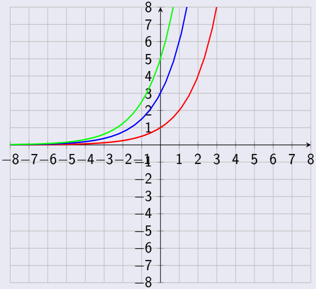

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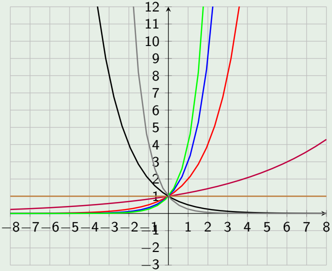

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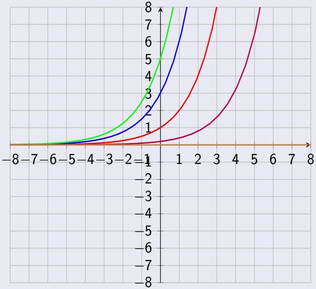

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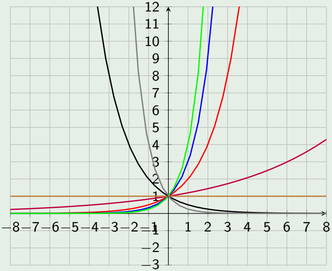

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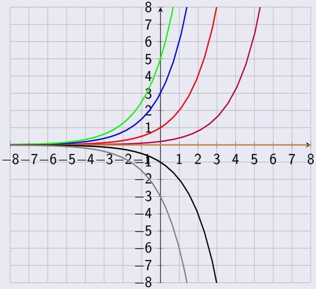

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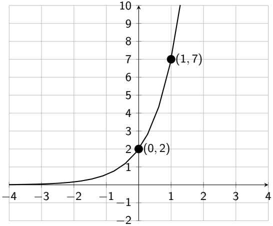
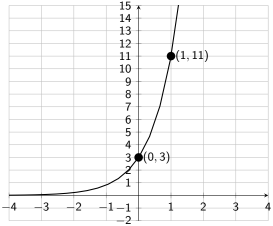
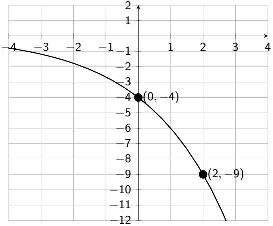
Thus: A is the y-intercept!

Find the coefficient A and the base B of the displayed exponential function $y = A \cdot B^x$.

1. 

2. 

3. 

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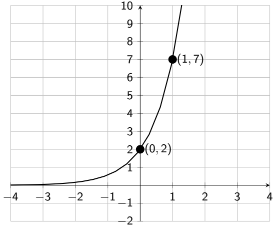
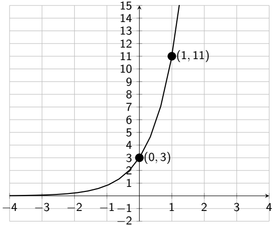
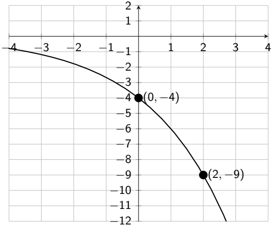
1. 

y-intercept: $A = 2$
plug $(x, y) = (1, 7)$
into $y = 2 \cdot B^x$:
$\Rightarrow 7 = 2 \cdot B^1$
$\Rightarrow \frac{7}{2} = B$
$\Rightarrow f(x) = 2 \cdot \left(\frac{7}{2}\right)^x$

2. 

3. 

Exponential functions - exercises

Find the coefficient A and the base B of the displayed exponential function $y = A \cdot B^x$.

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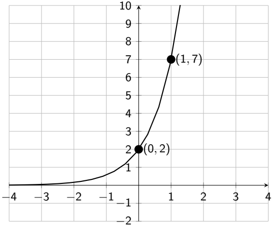

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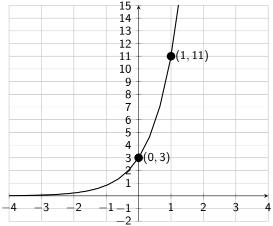


y-intercept: $A = 3$

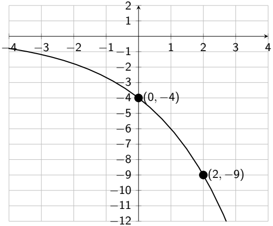
plug $(x, y) = (1, 11)$ into $y = 3 \cdot B^x$:

$\Rightarrow 11 = 3 \cdot B^1$

$\Rightarrow \frac{11}{3} = B$

$\Rightarrow f(x) = 3 \cdot \left(\frac{11}{3}\right)^x$

3.



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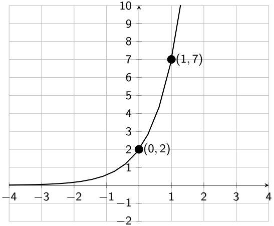

y-intercept: $A = 2$
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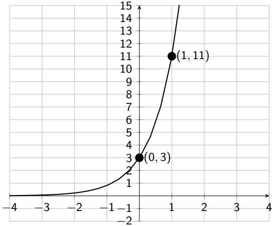

y-intercept: $A = 3$
plug $(x, y) = (1, 11)$
into $y = 3 \cdot B^x$:
$\Rightarrow 11 = 3 \cdot B^1$
$\Rightarrow \frac{11}{3} = B$
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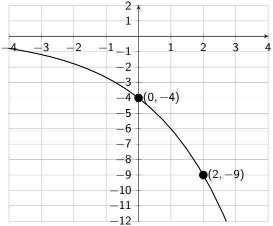

y-intercept: $A = -4$
plug $(x, y) = (2, -9)$
into $y = (-4) \cdot B^x$:
$\Rightarrow -9 = (-4) \cdot B^2$
$\Rightarrow \frac{9}{4} = B^2$
$\Rightarrow B = \sqrt{\frac{9}{4}} = \frac{3}{2}$

$\Rightarrow f(x) = (-4) \cdot \left(\frac{3}{2}\right)^x$

Definition of the logarithm

Definition (Logarithm)

The function $y = B^x$ (for $B \neq 1$) is one-to-one. Let $y = \log_B(x)$ be its inverse function:

$$y = \log_B(x) \quad \Leftrightarrow \quad x = B^y$$

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Important cases: $B = 10$: $\log_{10}(x) = \log(x)$

$B = e$: $\log_e(x) = \ln(x)$ "natural logarithm"

1. Find $\log_2(8)$

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   $y = \log_2(8) \Leftrightarrow 8 = 2^y = 2^3$
   $\Rightarrow y = 3$
   $\Rightarrow \log_2(8) = 3$

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1. Find $\log_2(8)$
   $y = \log_2(8) \Leftrightarrow 8 = 2^y = 2^3$
   $\Rightarrow y = 3$
   $\Rightarrow \log_2(8) = 3$
2. Find $\log_3(9)$
3. Find $\log(10,000)$
4. Find $\log_8(8)$
5. Find $\log_5(1)$
6. Find $\log_7(7^3)$

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$B = 10$: $\log_{10}(x) = \log(x)$
$B = e$: $\log_e(x) = \ln(x)$ "natural logarithm"

1. Find $\log_2(8)$
   $y = \log_2(8) \Leftrightarrow 8 = 2^y = 2^3$
   $\Rightarrow y = 3$
   $\Rightarrow \log_2(8) = 3$
2. Find $\log_3(9)$
   $y = \log_3(9) \Leftrightarrow 9 = 3^y = 3^2$
   $\Rightarrow \log_3(9) = 2$
3. Find $\log(10,000)$
4. Find $\log_8(8)$
5. Find $\log_5(1)$
6. Find $\log_7(7^3)$

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1. Find $\log_2(8)$
   $y = \log_2(8) \Leftrightarrow 8 = 2^y = 2^3$
   $\Rightarrow y = 3$
   $\Rightarrow \log_2(8) = 3$
2. Find $\log_3(9)$
   $y = \log_3(9) \Leftrightarrow 9 = 3^y = 3^2$
   $\Rightarrow \log_3(9) = 2$
3. Find $\log(10,000)$
   $y = \log_{10}(10000)$
   $\Leftrightarrow 10000 = 10^y = 10^4$
   $\Rightarrow \log(10000) = 4$
4. Find $\log_8(8)$
5. Find $\log_5(1)$
6. Find $\log_7(7^3)$

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$$y = \log_B(x) \quad \Leftrightarrow \quad x = B^y$$

Important cases: $B = 10$: $\log_{10}(x) = \log(x)$
$B = e$: $\log_e(x) = \ln(x)$ "natural logarithm"

1. Find $\log_2(8)$
   $y = \log_2(8) \Leftrightarrow 8 = 2^y = 2^3$
   $\Rightarrow y = 3$
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2. Find $\log_3(9)$
   $y = \log_3(9) \Leftrightarrow 9 = 3^y = 3^2$
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3. Find $\log(10,000)$
   $y = \log_{10}(10000)$
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4. Find $\log_8(8)$
   $y = \log_8(8) \Leftrightarrow 8 = 8^y = 8^1$
   $\Rightarrow \log_8(8) = 1$
5. Find $\log_5(1)$
6. Find $\log_7(7^3)$

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Definition of the logarithm

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Definition of the logarithm

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$$\log_B(B^n) = n$$

$$\log_B(1) = 0, \quad \log_B(B) = 1$$

Change of base formula

$$\log_B(x) = \frac{\log(x)}{\log(B)} = \frac{\ln(x)}{\ln(B)}$$

7. $\log_2(5) =$

Definition (Logarithm)

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Change of base formula

$$\log_B(x) = \frac{\log(x)}{\log(B)} = \frac{\ln(x)}{\ln(B)}$$

7. $\log_2(5) = \frac{\log(5)}{\log(2)} \approx 2.322$

Example

Graph $y = \log_2(x)$.

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x	-2	-1	0	1	2	3
$y = 2^x$	0.25	0.5	1	2	4	8

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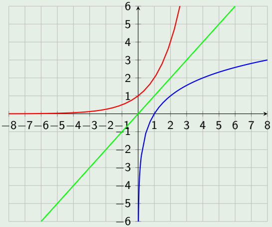

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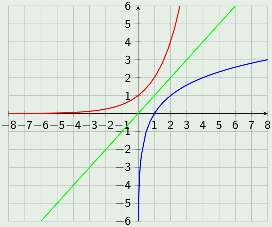
Range: $R = \mathbb{R}$ HA: no HA

x-intercept: $x = 1$



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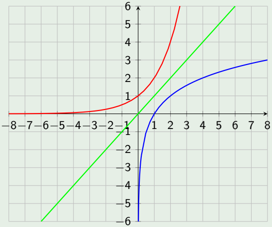

Graphs of logarithmic functions

Graph $y = \log_B(x)$.

Example

Graph $y = \log_2(x)$.

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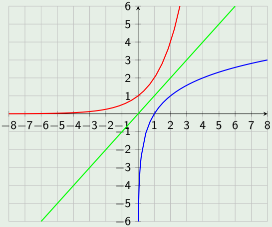

Graphs of logarithmic functions

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Graphs of logarithmic functions

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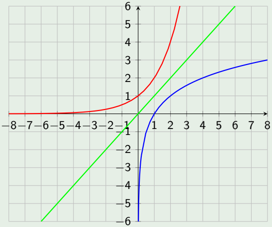

Graphs of logarithmic functions

Graph $y = \log_B(x)$.

Domain: $D = (0, \infty)$ VA: $x = 0$

Range: $R = \mathbb{R}$ HA: no HA

x-intercept: $y = 0$

$\Rightarrow \log_B(x) = 0$

$\Rightarrow x = B^0 = 1$

y-intercept: no y-intercept

Graphing logarithmic functions - exercises

Find the domain, asymptotes, and x-intercept of the logarithmic function, and sketch a graph.

1. $y = \log_3(2x - 5)$
2. $y = \log_2(4x + 7)$

Find the domain, asymptotes, and x-intercept of the logarithmic function, and sketch a graph.

1. $y = \log_3(2x-5)$
   Domain: $2x-5>0$
   $\Rightarrow 2x>5 \quad \Rightarrow x>\frac{5}{2}$
   $\Rightarrow D=(\frac{5}{2},\infty)$
   VA: $x=\frac{5}{2}$
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   x-intercept: $y = 0$

   $\Rightarrow 0 = \log_3(2x - 5)$

   $\Rightarrow 2x - 5 = 1$

   $\Rightarrow 2x = 6$

   $\Rightarrow x = 3$

2. $y = \log_2(4x + 7)$

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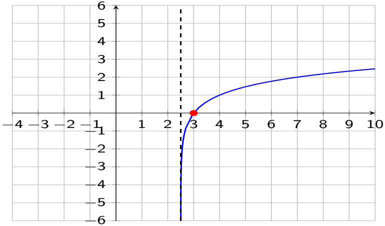

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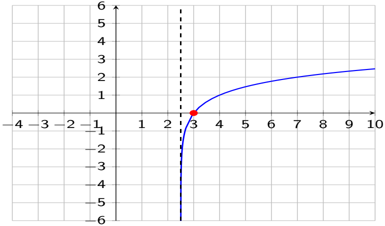

2. $y = \log_2(4x + 7)$
   Domain: $4x + 7 > 0$
   $\Rightarrow 4x > -7 \quad \Rightarrow x > -\frac{7}{4}$
   $\Rightarrow D = (-\frac{7}{4}, \infty)$
   VA: $x = -\frac{7}{4}$
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Find the domain, asymptotes, and x-intercept of the logarithmic function, and sketch a graph.

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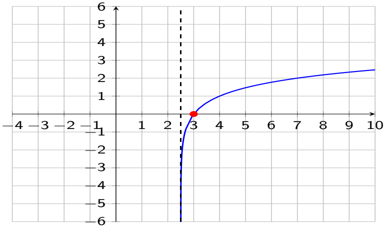

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   x-intercept: $y = 0$

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   $\Rightarrow 4x = -6$

   $\Rightarrow x = -\frac{6}{4} = -\frac{3}{2}$

Find the domain, asymptotes, and x-intercept of the logarithmic function, and sketch a graph.

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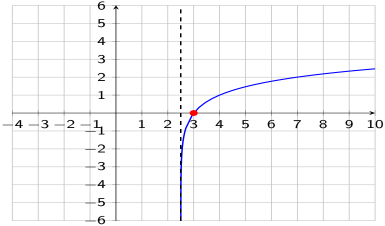

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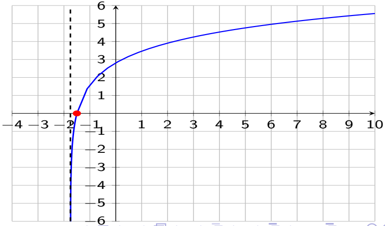

Graphing logarithmic functions - exercises

Find the domain, asymptotes, and x-intercept of the logarithmic function, and sketch a graph.

3. $y = \log(17 - 7x)$
4. $y = \log_6(-x - 4)$

Graphing logarithmic functions - exercises

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   x-intercept: $y = 0$

   $\Rightarrow 0 = \log(17 - 7x)$

   $\Rightarrow 17 - 7x = 1$

   $\Rightarrow -7x = -16$

   $\Rightarrow x = \frac{16}{7}$

4. $y = \log_6(-x - 4)$

Find the domain, asymptotes, and x-intercept of the logarithmic function, and sketch a graph.

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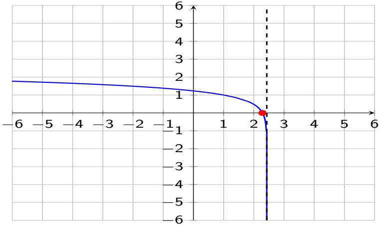

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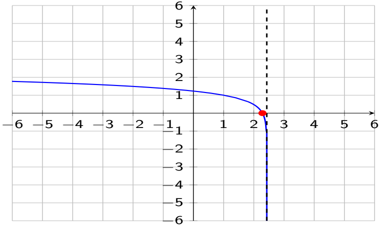

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   Domain: $-x - 4 > 0$

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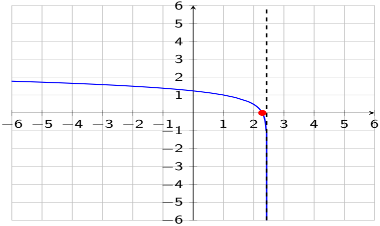

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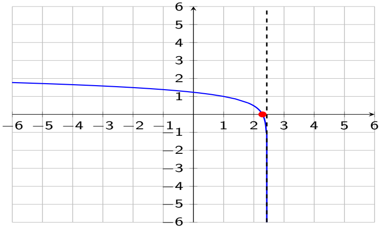

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