Polynomial and rational inequalities

Lesson #12

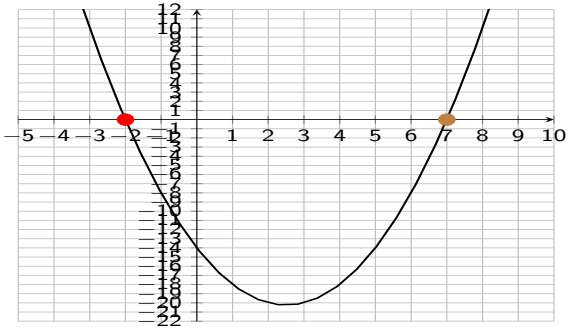
MAT 1375 Precalculus

New York City College of Technology CUNY

Polynomial inequalities

1. Solve for x: $x^2 - 5x - 14 \geq 0$

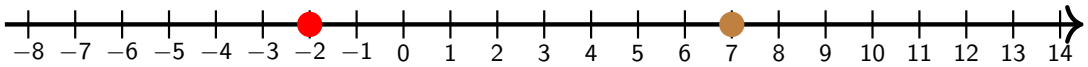
$f(x) = x^2 - 5x - 14$



Step 1: Solve the equality!

$x^2 - 5x - 14 = 0 \Rightarrow (x-7)(x+2) = 0 \Rightarrow x = 7, x = -2$

Step 2: Check a point in each interval!



$f(-3) = (-3)^2 - 5 \cdot (-3) - 14$	$f(0) = 0^2 - 5 \cdot 0 - 14$	$f(8) = 8^2 - 5 \cdot 8 - 14$
$= 9 + 15 - 14 = 10 \geq 0$ ✓	$= 0 - 0 - 14 = -14 \not\geq 0$ ✗	$= 64 - 40 - 14 = 10 \geq 0$ ✓
TRUE	FALSE	TRUE

$\Rightarrow \quad S = (-\infty, -2] \cup [7, \infty)$

Solving polynomial inequalities

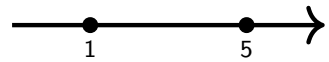
Step 1: Solve the equality! That is, find the roots!
Step 2: Check a point in each interval!

2. Solve: $x^2 - 6x + 5 < 0$

Step 1: $(x-1)(x-5) = 0$

$\Rightarrow x = 1, x = 5$

Step 2:

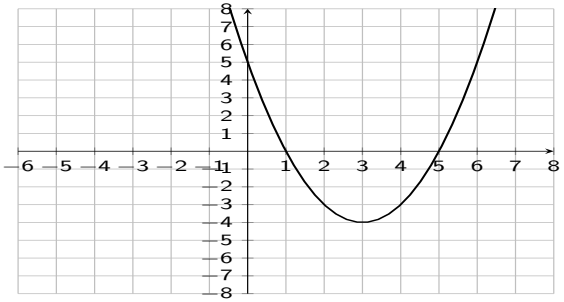


$f(0) = 0 - 0 + 5 = 5 \not< 0$ ✗

$f(3) = 9 - 18 + 5 = -4 < 0$ ✓

$f(6) = 36 - 36 + 5 = 5 \not< 0$ ✗

$\Rightarrow S = (1, 5)$

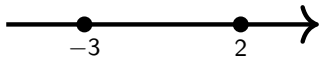


Polynomial inequalities - exercises

3. Solve:
$-2x^2 - 2x + 12 < 0$

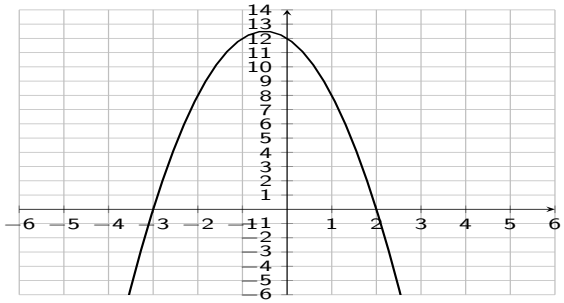
Step 1:
$-2(x^2 + x - 6) = 0$
$\Rightarrow -2(x-2)(x+3) = 0$
$\Rightarrow x = 2, x = -3$

Step 2:



$f(-4) = -32 + 8 + 12 = -12 < 0$ ✓
$f(0) = 0 - 0 + 12 = 12 \not< 0$ ✗
$f(4) = -32 - 8 + 12 = -28 < 0$ ✓

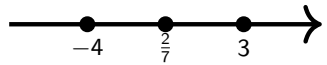
$\Rightarrow S = (-\infty, -3) \cup (2, \infty)$



4. Solve:
$(x-3)(7x-2)(x+4) > 0$

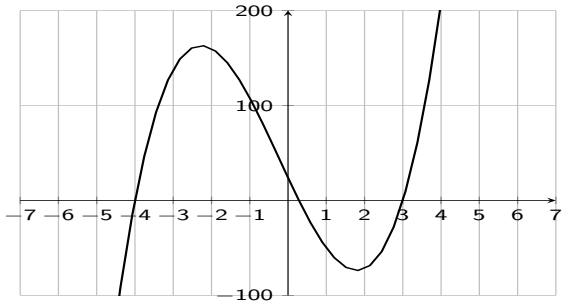
Step 1:
$(x-3)(7x-2)(x+4) = 0$
$\Rightarrow x = 3, x = \frac{2}{7}, x = -4$

Step 2:



From graphing calculator:
$f(-5) = -296 \not> 0$ ✗
$f(0) = 24 > 0$ ✓
$f(1) = -50 \not> 0$ ✗
$f(4) = 208 > 0$ ✓

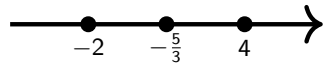
$\Rightarrow S = (-4, \frac{2}{7}) \cup (3, \infty)$



5. Solve:
$(x+2)(x-4)(3x+5) \leq 0$

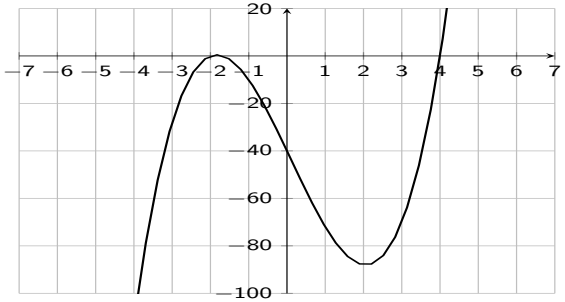
Step 1:
$(x+2)(x-4)(3x+5) = 0$
$\Rightarrow x = -2, x = 4, x = -\frac{5}{3}$

Step 2:



$f(-3) = -28 \leq 0$ ✓
$f(-1.7) = 0.171 \not\leq 0$ ✗
$f(0) = -40 \leq 0$ ✓
$f(5) = 140 \not\leq 0$ ✗

$\Rightarrow S = (-\infty, -2] \cup [-\frac{5}{3}, 4]$



Polynomial and absolute value inequalities - exercises

6. Solve: $2x^3 + 3x^2 - 11x - 6 \geq 0$

Step 1: $2x^3 + 3x^2 - 11x - 6 = 0$

$\Rightarrow x = 2$ (for example)

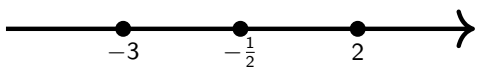
Long division gives:

$f(x) = (x-2)(2x^2 + 7x + 3)$

$= (x-2)(2x+1)(x+3)$

$\Rightarrow$ roots: $x = 2, x = -\frac{1}{2}, x = -3$

Step 2:



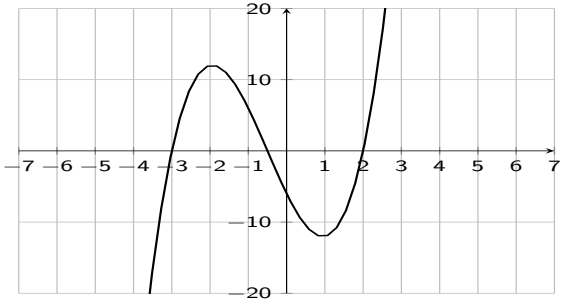
$f(-4) = -42 \not\geq 0$ ✗

$f(-2) = 12 \geq 0$ ✓

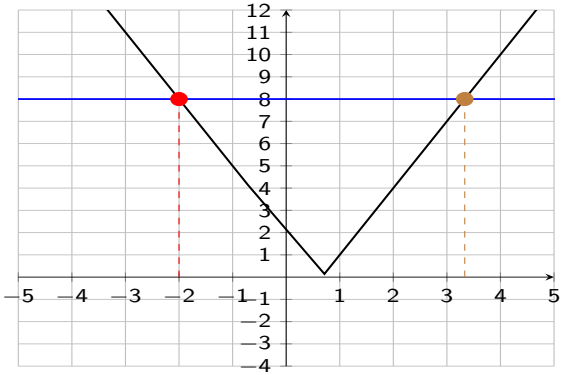
$f(0) = -6 \not\geq 0$ ✗

$f(3) = 42 \geq 0$ ✓

$\Rightarrow S = [-3, -\frac{1}{2}] \cup [2, \infty)$



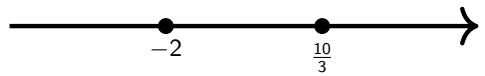
7. Solve: $|3x - 2| > 8$



Step 1: $|3x - 2| = 8$

$3x - 2 = 8$	$3x - 2 = -8$
$3x = 10$	$3x = -6$
$x = \frac{10}{3}$	$x = -2$

Step 2:



$|3(-4) - 2| = |-12 - 2| = |-14| = 14 > 8$ ✓

$|3 \cdot 0 - 2| = |-2| = 2 \not> 8$ ✗

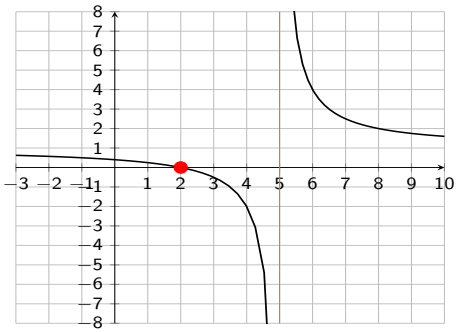
$|3 \cdot 4 - 2| = |12 - 2| = |10| = 10 > 8$ ✓

$\Rightarrow S = (-\infty, -2) \cup (\frac{10}{3}, \infty)$

Rational inequalities

1. Solve for x: $\frac{x-2}{x-5} \geq 0$

$$f(x) = \frac{x-2}{x-5}$$



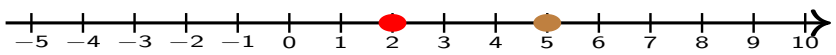
Step 1: Find the **roots** and **vertical asymptotes**!

roots:

$f(x) = 0 \quad \Rightarrow x - 2 = 0 \quad \Rightarrow x = 2$

VA: $\quad x - 5 = 0 \quad \Rightarrow x = 5$

Step 2: Check a point in each interval!



$f(0) = \frac{0-2}{0-5}$	$f(3) = \frac{3-2}{3-5}$	$f(6) = \frac{6-2}{6-5}$
$= \frac{2}{5} \geq 0$ ✓	$= -\frac{1}{2} \not\geq 0$ ✗	$= \frac{4}{1} = 4 \geq 0$ ✓
TRUE	FALSE	TRUE

$\Rightarrow \quad S = (-\infty, 2] \cup (5, \infty)$

Note: $x = 5$ is **not** a solution, since $f(5)$ is undefined!

Solving rational inequalities

Step 1: Find the **roots** and **vertical asymptotes**!

Step 2: Check a point in each interval!

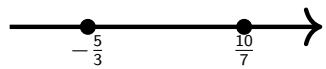
2. Solve: $\frac{3x+5}{7x-10} \leq 0$

Step 1:

roots: $3x + 5 = 0 \implies x = -\frac{5}{3}$

VA: $\quad 7x - 10 = 0 \implies x = \frac{10}{7}$

Step 2:

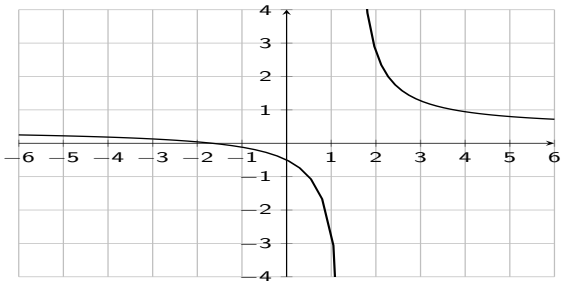


$f(-2) = \frac{3\cdot(-2)+5}{7\cdot(-2)-10} = \frac{-1}{-24} = \frac{1}{24} \not\leq 0$ ✗

$f(0) = \frac{3\cdot 0+5}{7\cdot 0-10} = \frac{5}{-10} = -\frac{1}{2} \leq 0$ ✓

$f(2) = \frac{3\cdot 2+5}{7\cdot 2-10} = \frac{11}{4} \not\leq 0$ ✗

$\Rightarrow S = [-\frac{5}{3}, \frac{10}{7})$



Rational inequalities - exercises

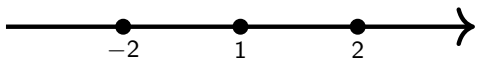
3. Solve: $\frac{x-1}{x^2-4} > 0$

Step 1:

roots: $x = 1$

VA: $x = 2, x = -2$

Step 2:



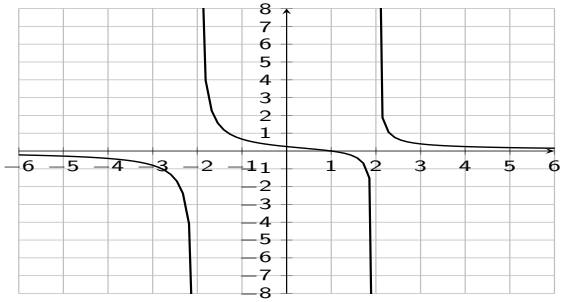
$f(-3) = \frac{-3-1}{(-3)^2-4} = \frac{-4}{5} \not> 0$ ✗

$f(0) = \frac{0-1}{0^2-4} = \frac{-1}{-4} = \frac{1}{4} > 0$ ✓

$f(1.5) = \frac{1.5-1}{1.5^2-4} = \frac{.5}{-1.75} \not> 0$ ✗

$f(3) = \frac{3-1}{3^2-4} = \frac{2}{5} > 0$ ✓

$\Rightarrow S = (-2, 1) \cup (2, \infty)$



4. Solve: $\frac{x^2-4x-21}{2x+2} \leq 0$

Step 1:

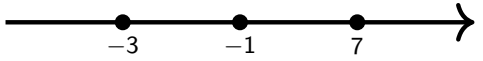
roots: $x^2 - 4x - 21 = 0$

$\Rightarrow (x-7)(x+3) = 0$

$\Rightarrow x = 7, x = -3$

VA: $2x + 2 = 0 \quad \Rightarrow x = -1$

Step 2:



$f(-4) = \frac{(-4)^2-4(-4)-21}{2(-4)+2} = \frac{11}{-6} \leq 0$ ✓

$f(-2) = \frac{(-2)^2-4(-2)-21}{2(-2)+2} = \frac{-9}{-2} = \frac{9}{2} \not\leq 0$ ✗

$f(0) = \frac{-21}{2} \leq 0$ ✓

$f(8) = \frac{8^2-4\cdot 8-21}{2\cdot 8+2} = \frac{11}{18} \not\leq 0$ ✗

$\Rightarrow S = (-\infty, -3] \cup (-1, 7]$