Polynomial and rational inequalities

Lesson #12

MAT 1375 Precalculus

New York City College of Technology CUNY

Polynomial inequalities

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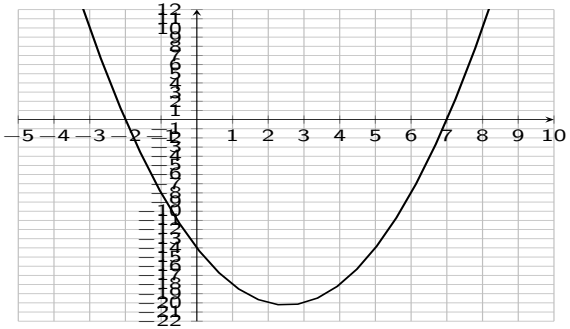

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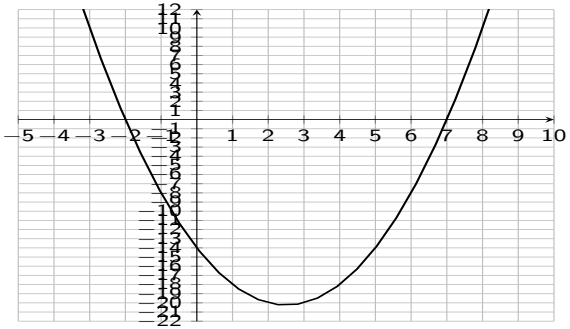

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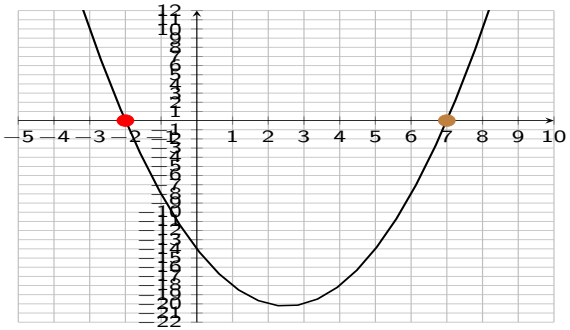

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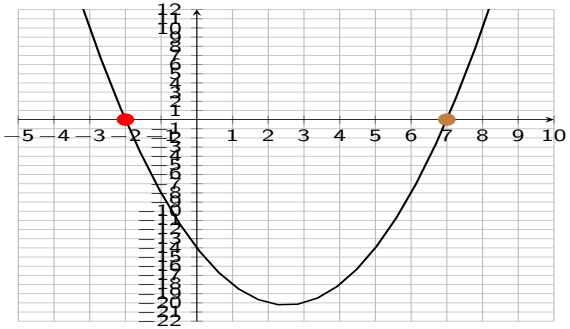

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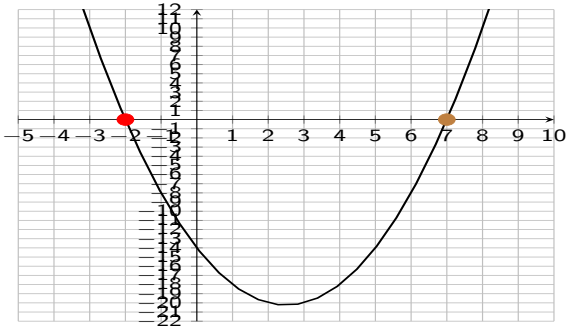

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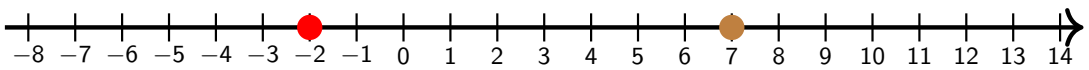

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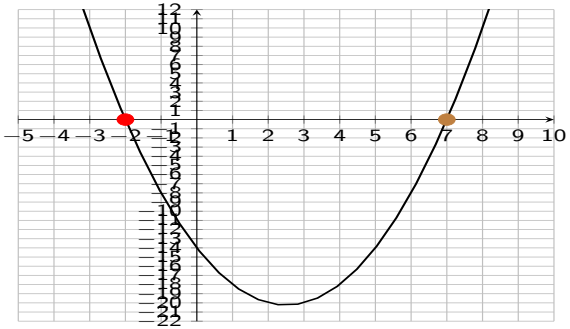

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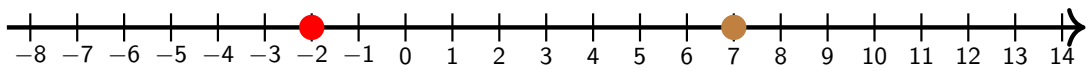

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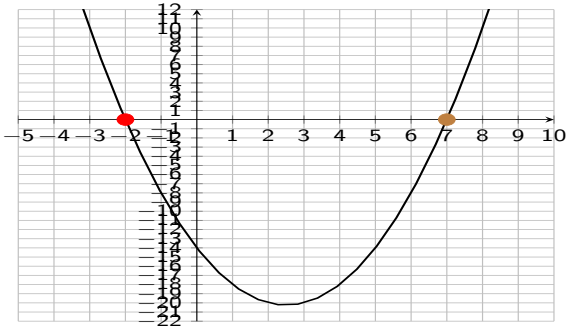

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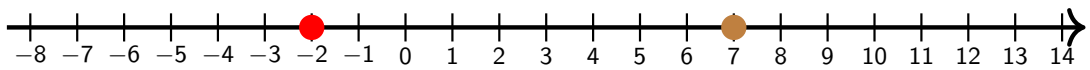

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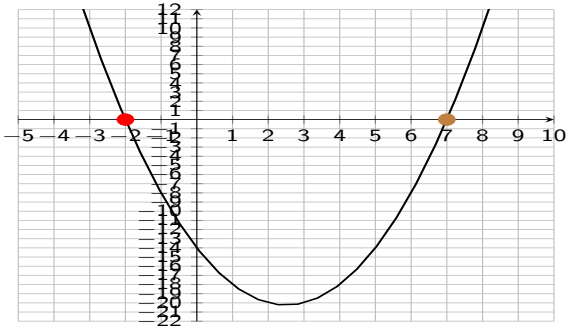

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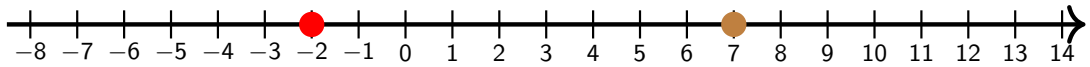

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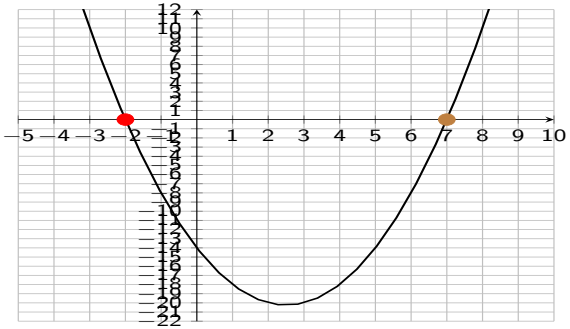

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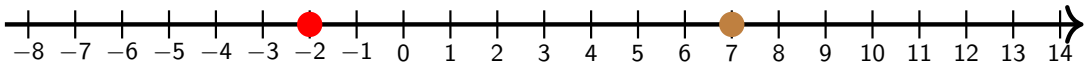

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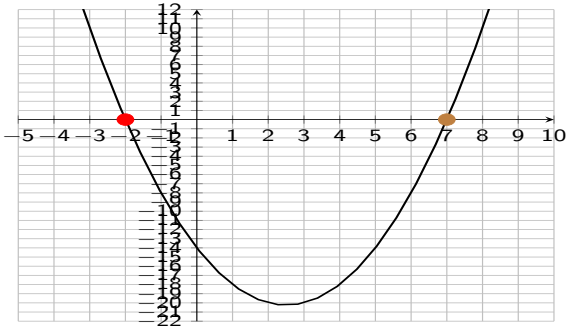

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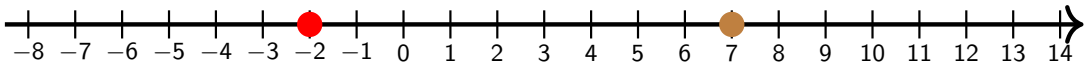

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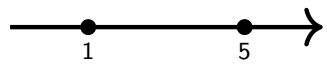

$f(0) = 0 - 0 + 5 = 5 \not< 0$ ✗

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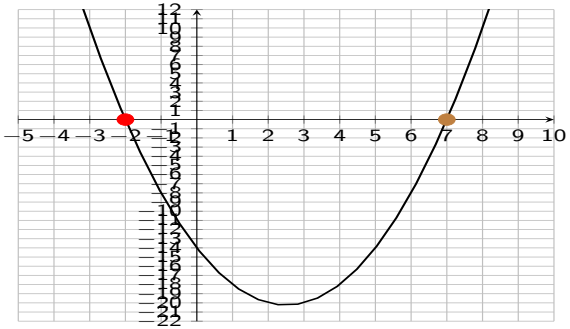

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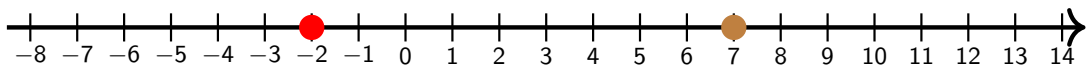

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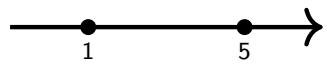

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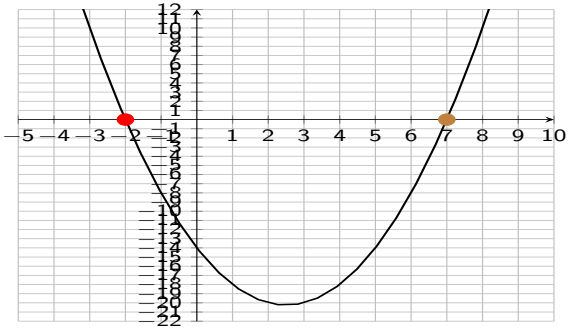

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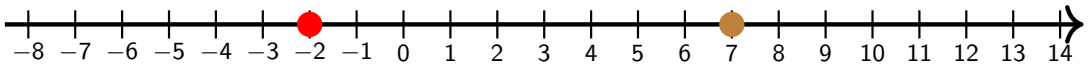

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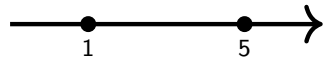

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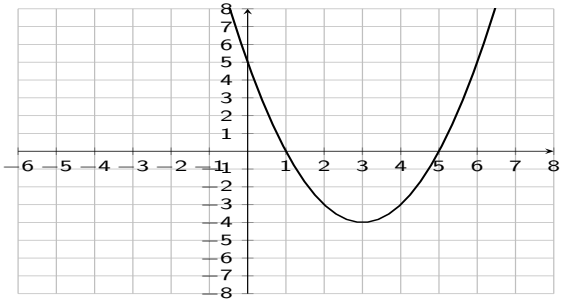

Polynomial inequalities - exercises

3. Solve:
$-2x^2 - 2x + 12 < 0$

4. Solve:
$(x - 3)(7x - 2)(x + 4) > 0$

5. Solve:
$(x + 2)(x - 4)(3x + 5) \leq 0$

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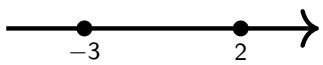
Step 2: 

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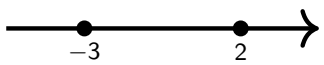
Step 2: 

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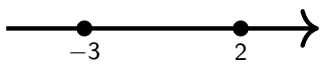
Step 2: 

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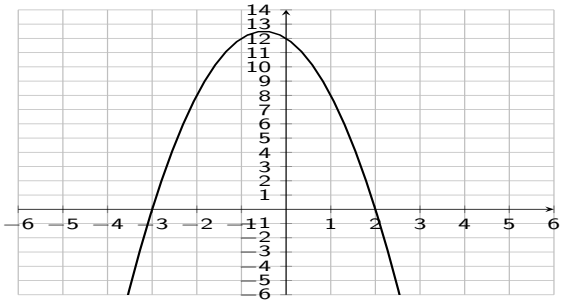

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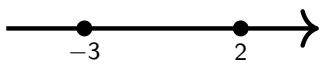
Step 2: 

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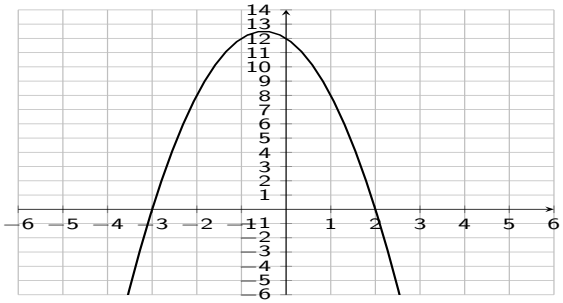

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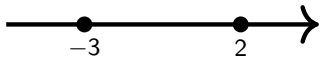
Step 2: 

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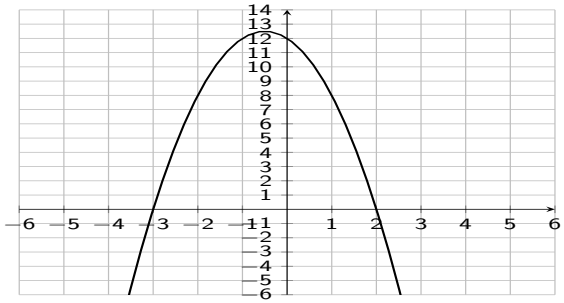

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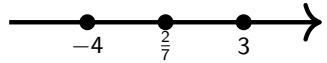
Step 2: 

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Polynomial inequalities - exercises

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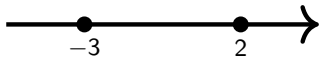

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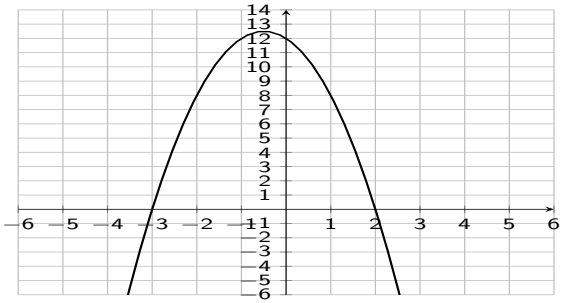

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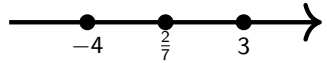

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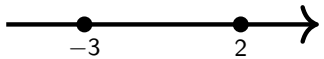

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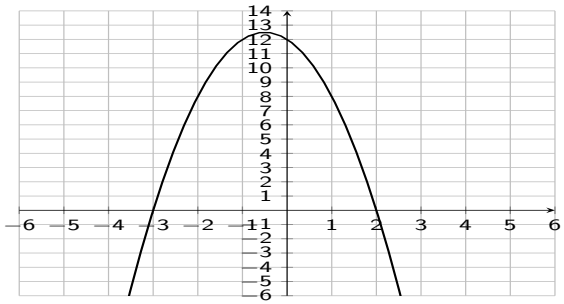

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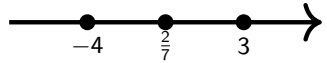

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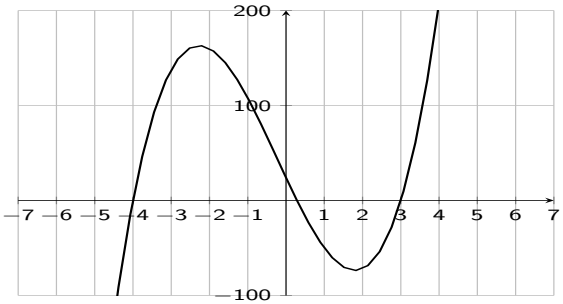

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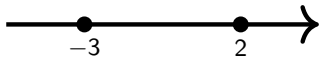

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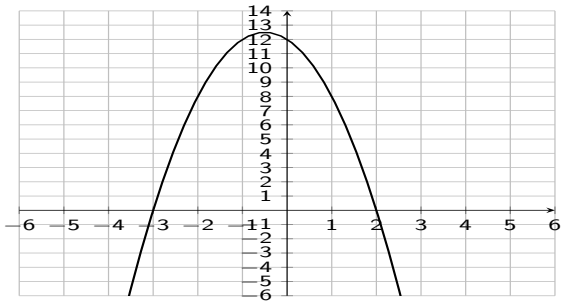

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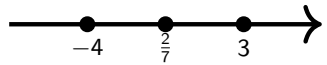

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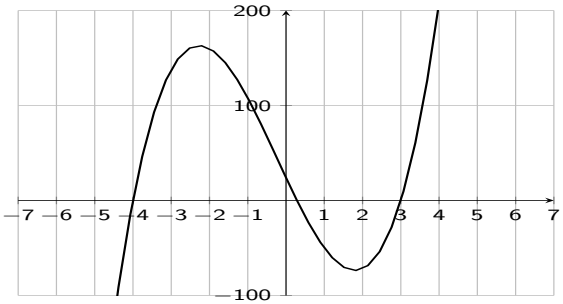

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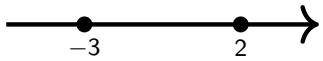

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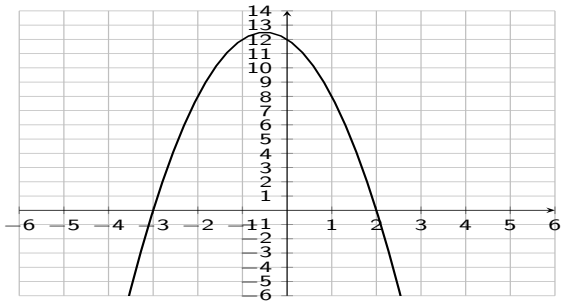

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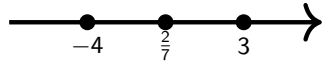

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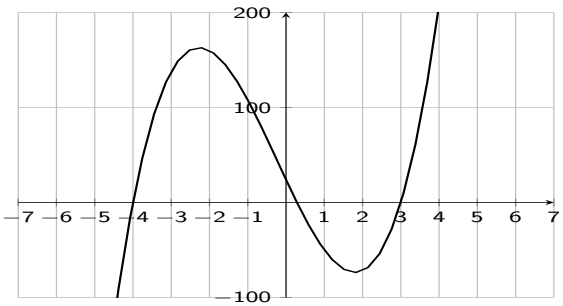

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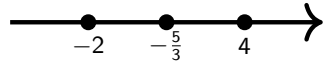

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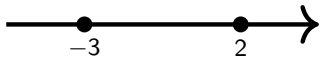

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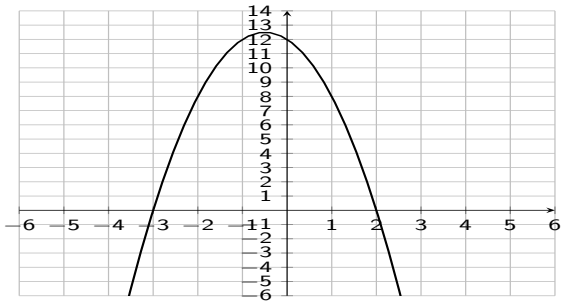

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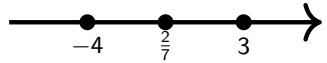

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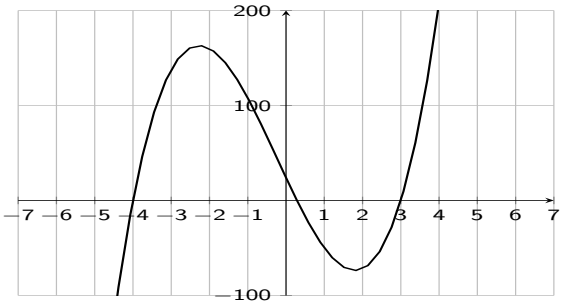

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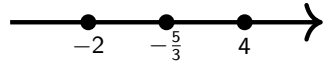

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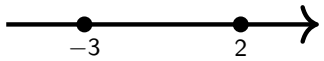

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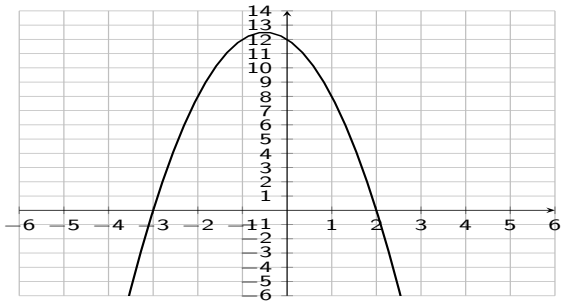

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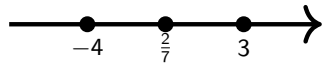

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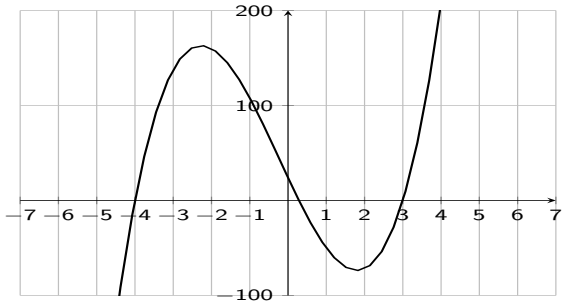

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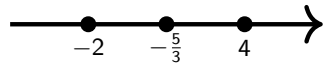

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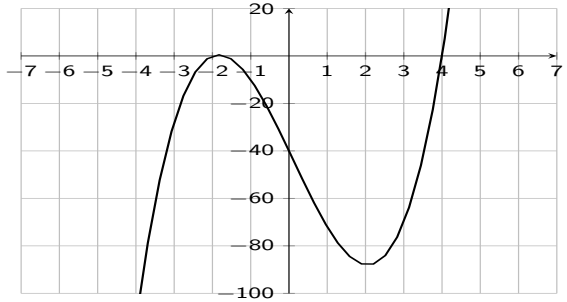

Polynomial and absolute value inequalities - exercises

6. Solve: $2x^3 + 3x^2 - 11x - 6 \geq 0$

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   Long division gives:
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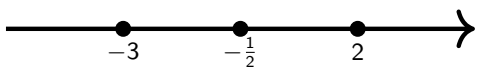

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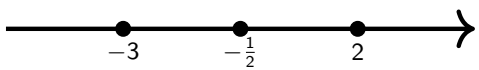

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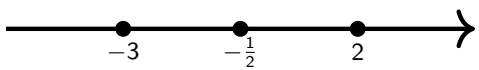

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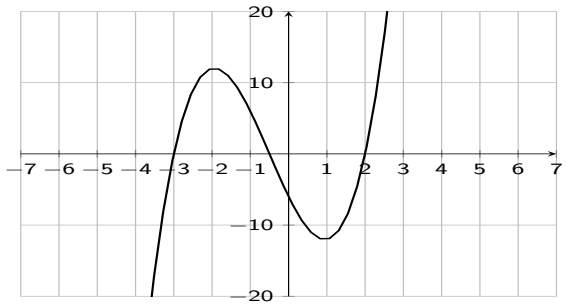

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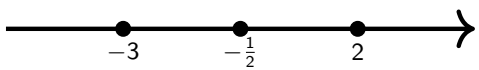

$f(-4) = -42 \not\geq 0$ ✗

$f(-2) = 12 \geq 0$ ✓

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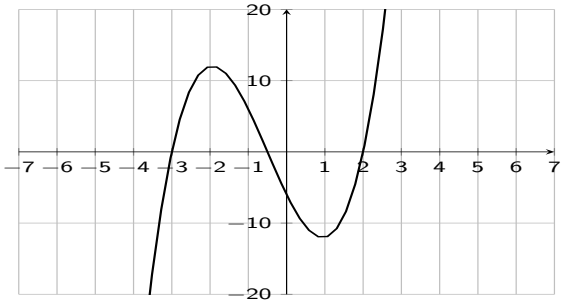
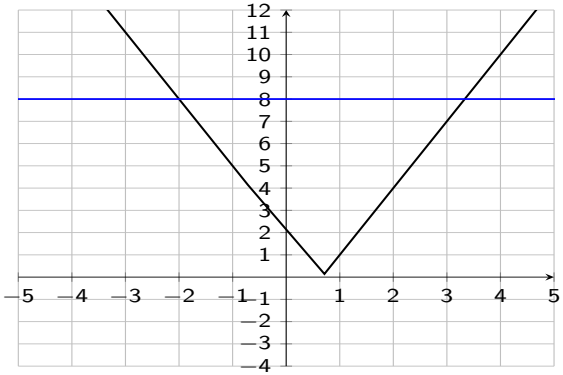
$\Rightarrow S = [-3, -\frac{1}{2}] \cup [2, \infty)$



7. Solve: $|3x - 2| > 8$



6. Solve: $2x^3 + 3x^2 - 11x - 6 \geq 0$

Step 1: $2x^3 + 3x^2 - 11x - 6 = 0$

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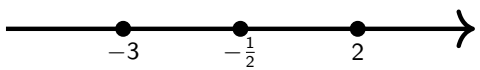

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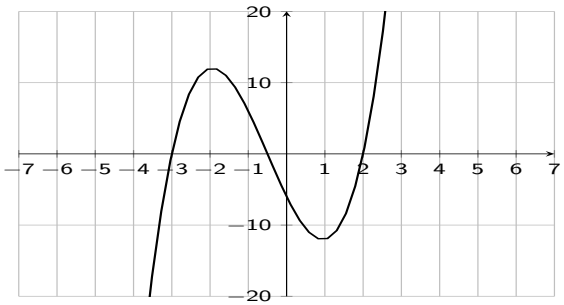

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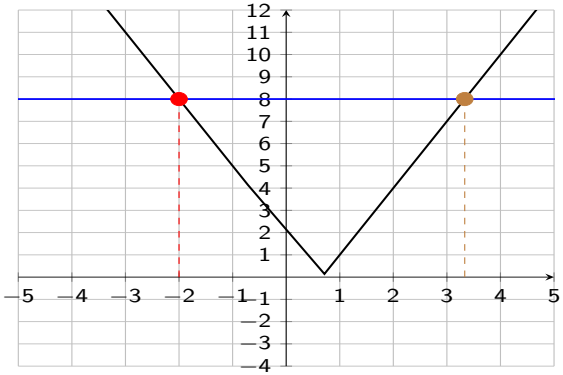

Step 1: $|3x - 2| = 8$

$3x - 2 = 8$	$3x - 2 = -8$
$3x = 10$	$3x = -6$
$x = \frac{10}{3}$	$x = -2$

Polynomial and absolute value inequalities - exercises

6. Solve: $2x^3 + 3x^2 - 11x - 6 \geq 0$

Step 1: $2x^3 + 3x^2 - 11x - 6 = 0$

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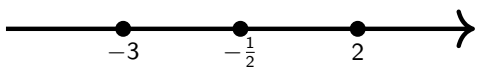

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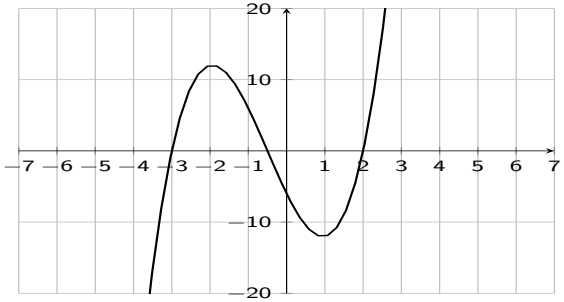

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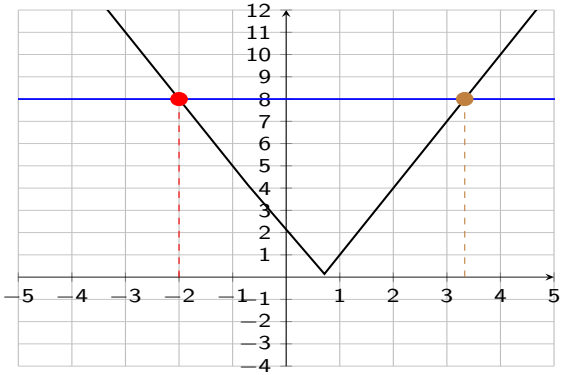

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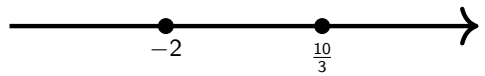

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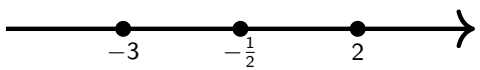

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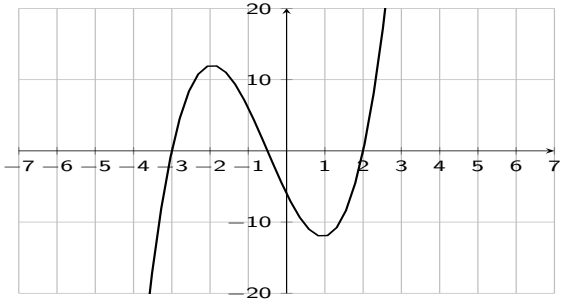

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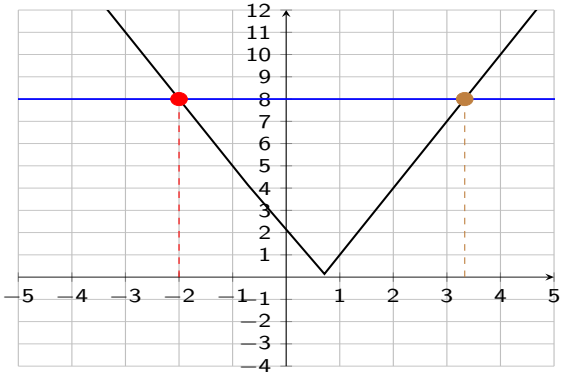

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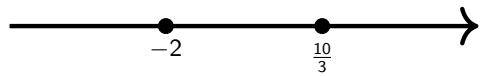

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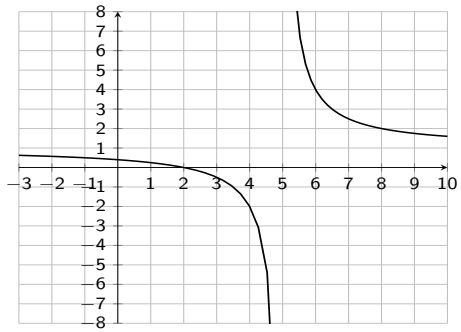

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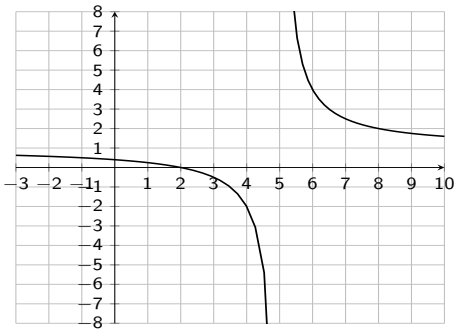

Step 1: Find the **roots** and **vertical asymptotes**!

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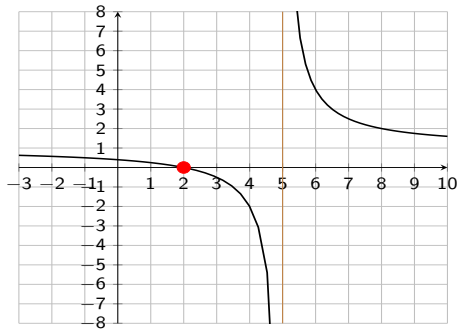

Step 1: Find the **roots** and **vertical asymptotes**!

roots:

$f(x) = 0 \quad \Rightarrow x - 2 = 0 \quad \Rightarrow x = 2$

VA: $\quad x - 5 = 0 \quad \Rightarrow x = 5$

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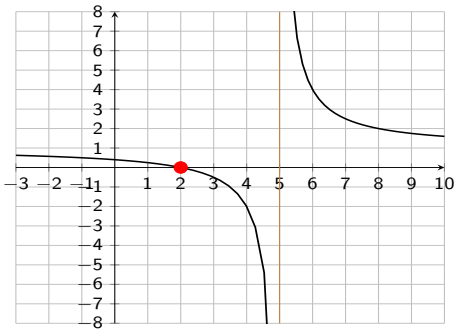

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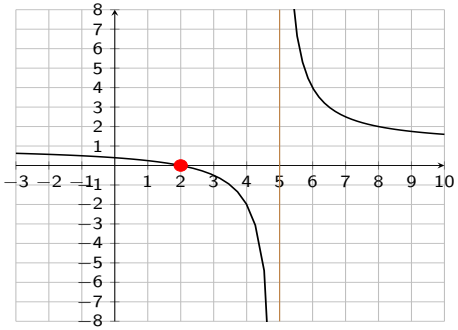

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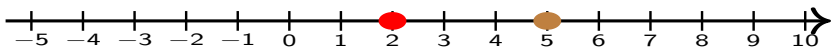

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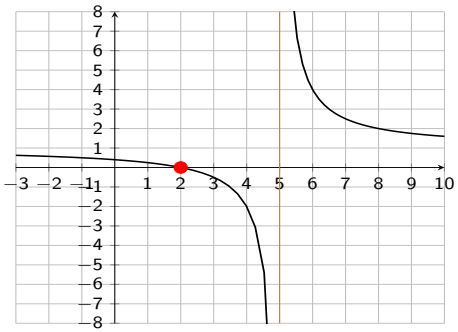

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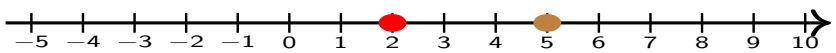

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Note: $x = 5$ is **not** a solution, since $f(5)$ is undefined!

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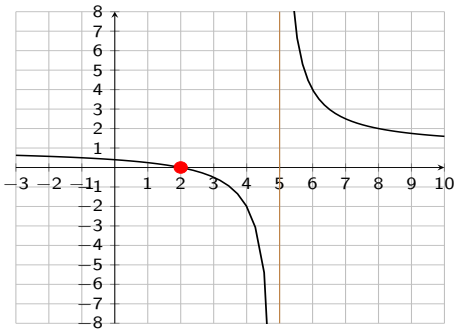

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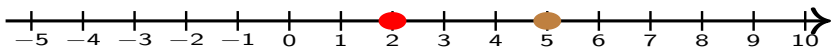

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Solving rational inequalities

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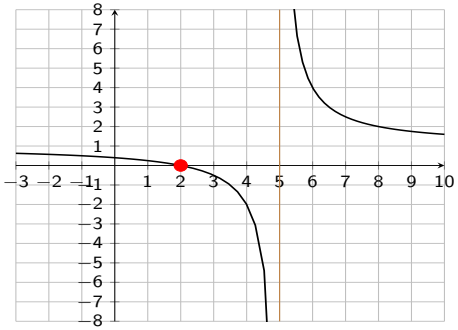

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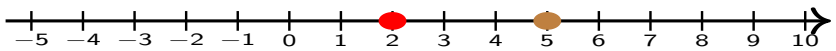

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Solving rational inequalities

Step 1: Find the **roots** and **vertical asymptotes**!

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Rational inequalities

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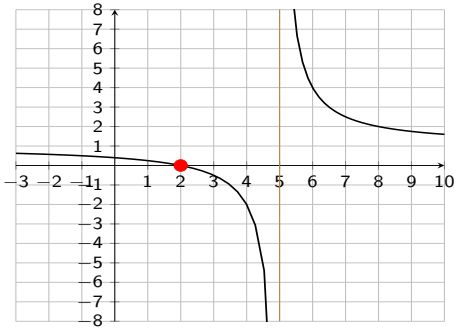

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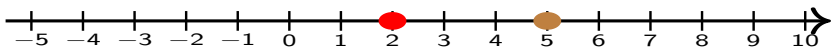

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roots: $3x + 5 = 0 \implies x = -\frac{5}{3}$

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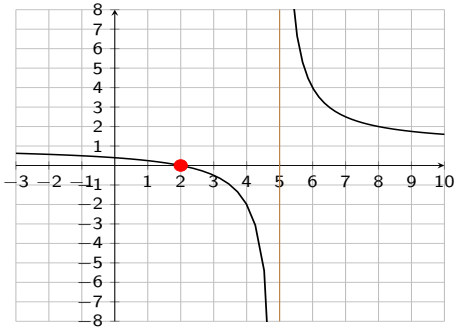

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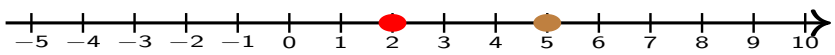

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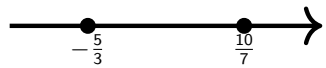

$f(-2) = \frac{3\cdot(-2)+5}{7\cdot(-2)-10} = \frac{-1}{-24} = \frac{1}{24} \not\leq 0$ ✗

$f(0) = \frac{3\cdot 0+5}{7\cdot 0-10} = \frac{5}{-10} = -\frac{1}{2} \leq 0$ ✓

$f(2) = \frac{3\cdot 2+5}{7\cdot 2-10} = \frac{11}{4} \not\leq 0$ ✗

Rational inequalities

1. Solve for x: $\frac{x-2}{x-5} \geq 0$

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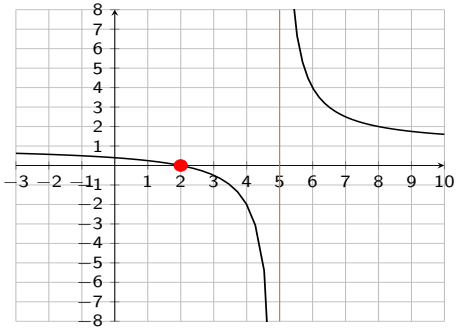


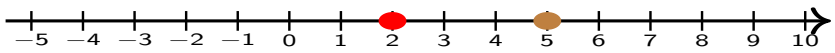
Step 1: Find the **roots** and **vertical asymptotes**!

roots:

$f(x) = 0 \quad \Rightarrow x - 2 = 0 \quad \Rightarrow x = 2$

VA: $\quad x - 5 = 0 \quad \Rightarrow x = 5$

Step 2: Check a point in each interval!



$f(0) = \frac{0-2}{0-5}$	$f(3) = \frac{3-2}{3-5}$	$f(6) = \frac{6-2}{6-5}$
$= \frac{2}{5} \geq 0$ ✓	$= -\frac{1}{2} \not\geq 0$ ✗	$= \frac{4}{1} = 4 \geq 0$ ✓
TRUE	FALSE	TRUE

$\Rightarrow \quad S = (-\infty, 2] \cup (5, \infty)$

Note: $x = 5$ is **not** a solution, since $f(5)$ is undefined!

Solving rational inequalities

Step 1: Find the **roots** and **vertical asymptotes**!

Step 2: Check a point in each interval!

2. Solve: $\frac{3x+5}{7x-10} \leq 0$

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roots: $3x + 5 = 0 \implies x = -\frac{5}{3}$

VA: $\quad 7x - 10 = 0 \implies x = \frac{10}{7}$

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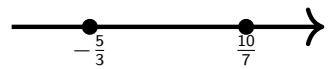

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Rational inequalities

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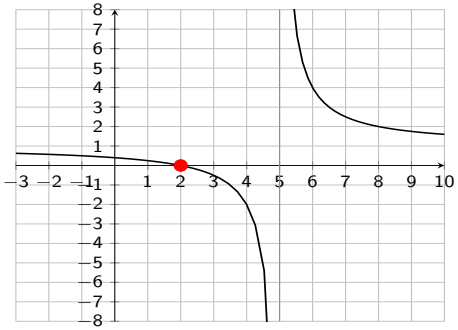
$$f(x) = \frac{x-2}{x-5}$$



Step 1: Find the **roots** and **vertical asymptotes**!

roots:

$f(x) = 0 \quad \Rightarrow x - 2 = 0 \quad \Rightarrow x = 2$

VA: $\quad x - 5 = 0 \quad \Rightarrow x = 5$

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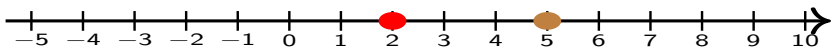

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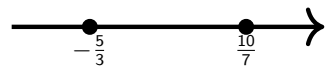

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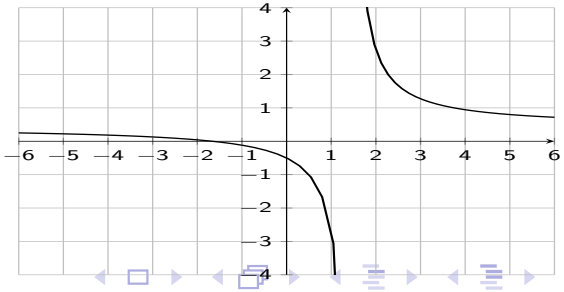

Rational inequalities - exercises

3. Solve: $\frac{x-1}{x^2-4} > 0$

4. Solve: $\frac{x^2-4x-21}{2x+2} \leq 0$

Rational inequalities - exercises

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Step 1:

roots: $x = 1$

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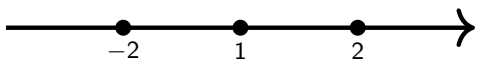

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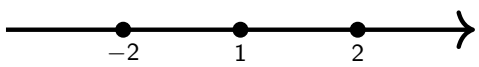

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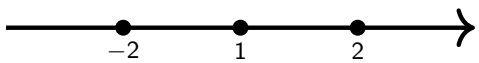

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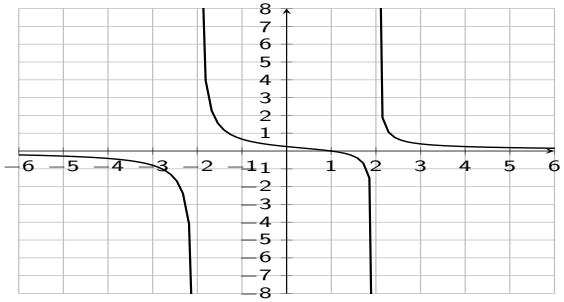

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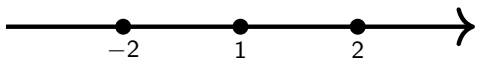

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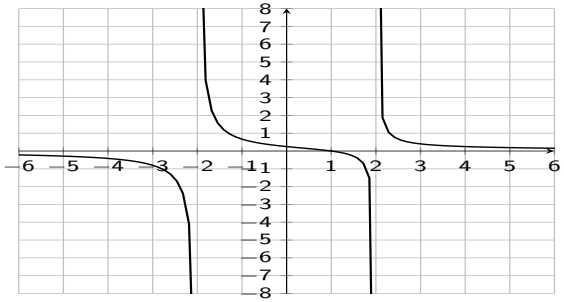

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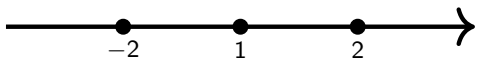

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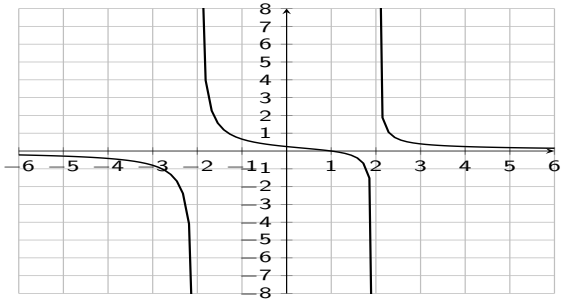

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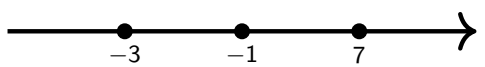

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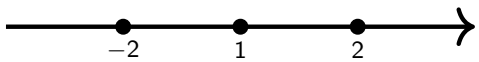

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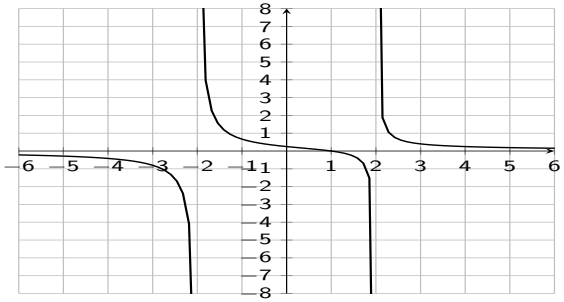

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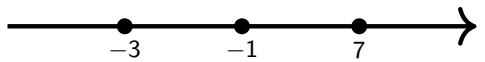

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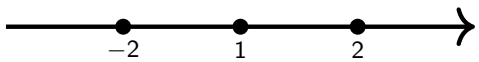

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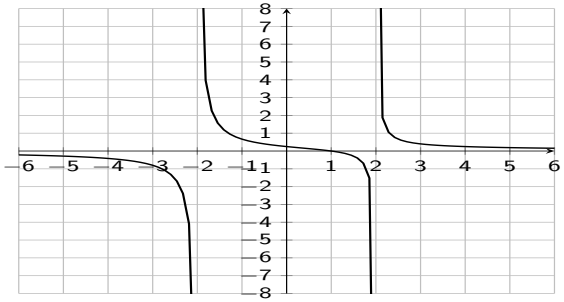

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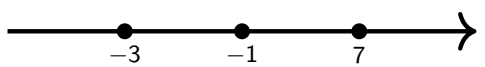

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