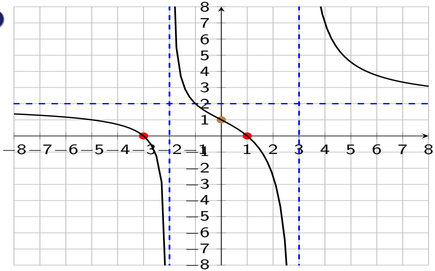
Exploring discontinuities and asymptotes

Lesson #11

MAT 1375 Precalculus

New York City College of Technology CUNY

Find the intercepts and the asymptotes of the rational function displayed below. What is its domain? What is its formula?

1. 

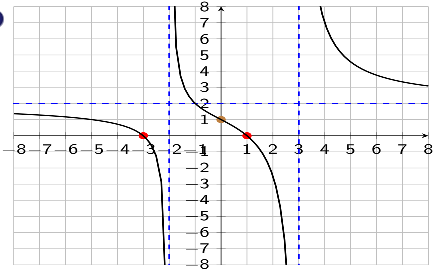
x-int.: y-int.:

VA: HA:

Domain:

$f(x) =$

Find the intercepts and the asymptotes of the rational function displayed below. What is its domain? What is its formula?

1. 

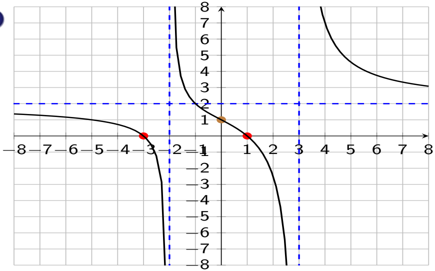
x-int.: $x=-3$, $x=1$ y-int.:

VA: HA:

Domain:

$f(x)=$

Find the intercepts and the asymptotes of the rational function displayed below. What is its domain? What is its formula?

1. 

x-int.: $x = -3$, $x = 1$ y-int.: $y = 1$

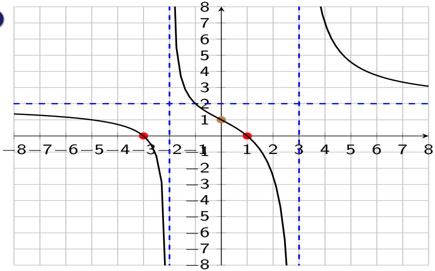
VA: HA:

Domain:

$f(x) =$

Find the intercepts and the asymptotes of the rational function displayed below. What is its domain? What is its formula?

1



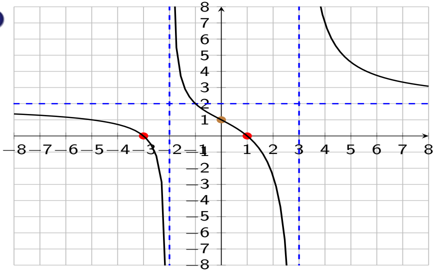
x-int.: $x = -3$, $x = 1$ y-int.: $y = 1$

VA: $x = -2, x = 3$ HA:

Domain:

$f(x) =$

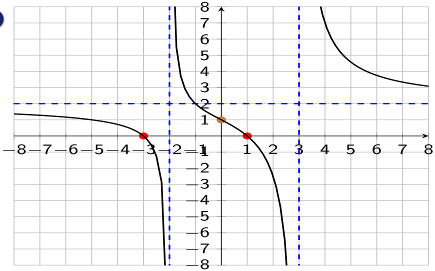
Find the intercepts and the asymptotes of the rational function displayed below. What is its domain? What is its formula?

1. 

x-int.: $x = -3$, $x = 1$ y-int.: $y = 1$
VA: $x = -2, x = 3$ HA: $y = 2$
Domain:

$f(x) =$

Find the intercepts and the asymptotes of the rational function displayed below. What is its domain? What is its formula?

1. 

x-int.: $x = -3$, $x = 1$ y-int.: $y = 1$

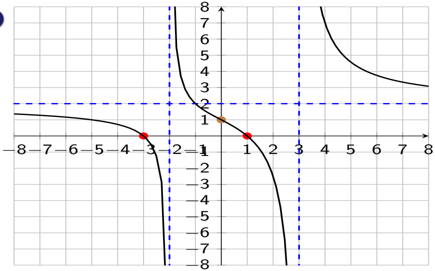
VA: $x = -2, x = 3$ HA: $y = 2$

Domain: $D = (-\infty, -2) \cup (-2, 3) \cup (3, \infty)$

$f(x) =$

Find the intercepts and the asymptotes of the rational function displayed below. What is its domain? What is its formula?

1



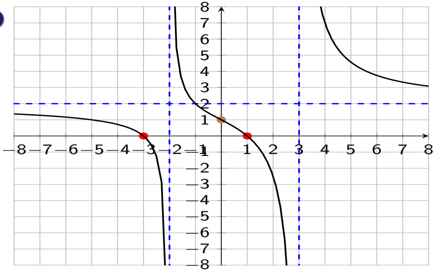
x-int.:	$x = -3$, $x = 1$	y-int.:	$y = 1$
VA:	$x = -2, x = 3$	HA:	$y = 2$

Domain: $D = (-\infty, -2) \cup (-2, 3) \cup (3, \infty)$

$$f(x) = \frac{a\cdot(x-1)\cdot(x+3)}{(x+2)\cdot(x-3)}$$

Find the intercepts and the asymptotes of the rational function displayed below. What is its domain? What is its formula?

1



x-int.: $x = -3$, $x = 1$ y-int.: $y = 1$

VA: $x = -2, x = 3$ HA: $y = 2$

Domain: $D = (-\infty, -2) \cup (-2, 3) \cup (3, \infty)$

$$f(x) = \frac{a\cdot(x-1)\cdot(x+3)}{(x+2)\cdot(x-3)}$$

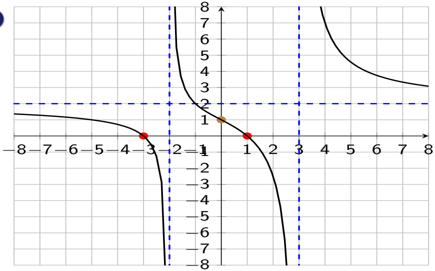
$$f(0) = \frac{a\cdot(-1)\cdot 3}{2\cdot(-3)} = \frac{a}{2} \overset{!}{=} 1 \implies a = 2$$

$$\implies f(x) = \frac{2\cdot(x-1)\cdot(x+3)}{(x+2)\cdot(x-3)}$$

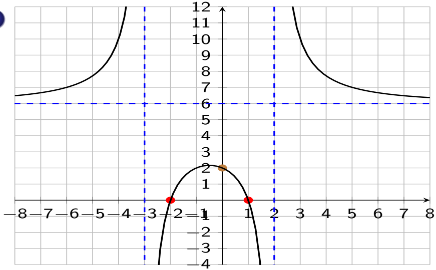
Rational function via its graph - exercises

Find the intercepts and the asymptotes of the rational function displayed below. What is its domain? What is its formula?

1



2



1

x-int.: $x = -3$, $x = 1$ y-int.: $y = 1$

VA: $x = -2, x = 3$ HA: $y = 2$

Domain: $D = (-\infty, -2) \cup (-2, 3) \cup (3, \infty)$

$$f(x) = \frac{a\cdot(x-1)\cdot(x+3)}{(x+2)\cdot(x-3)}$$

$$f(0) = \frac{a\cdot(-1)\cdot 3}{2\cdot(-3)} = \frac{a}{2} \overset{!}{=} 1 \implies a = 2$$

$$\implies f(x) = \frac{2\cdot(x-1)\cdot(x+3)}{(x+2)\cdot(x-3)}$$

2

x-int.: y-int.:

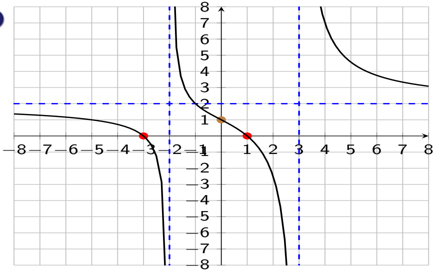
VA: HA:

Domain:

$f(x) =$

Find the intercepts and the asymptotes of the rational function displayed below. What is its domain? What is its formula?

1



x-int.: $x = -3$, $x = 1$ y-int.: $y = 1$

VA: $x = -2, x = 3$ HA: $y = 2$

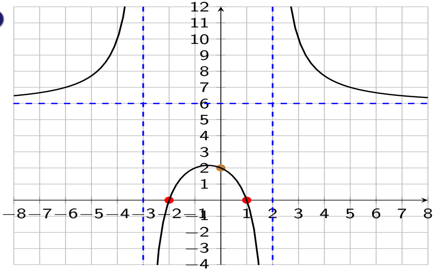
Domain: $D = (-\infty, -2) \cup (-2, 3) \cup (3, \infty)$

$$f(x) = \frac{a\cdot(x-1)\cdot(x+3)}{(x+2)\cdot(x-3)}$$

$$f(0) = \frac{a\cdot(-1)\cdot 3}{2\cdot(-3)} = \frac{a}{2} \overset{!}{=} 1 \implies a = 2$$

$$\implies f(x) = \frac{2\cdot(x-1)\cdot(x+3)}{(x+2)\cdot(x-3)}$$

2



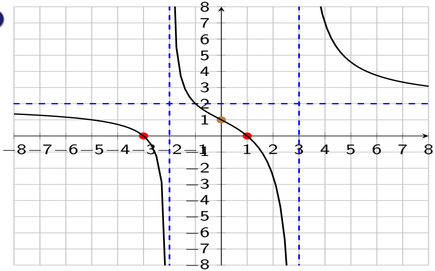
x-int.: $x = -2$, $x = 1$ y-int.: $y = 2$

VA: $x = -3, x = 2$ HA: $y = 6$

Domain: $D = (-\infty, -3) \cup (-3, 2) \cup (2, \infty)$

$f(x) =$

Find the intercepts and the asymptotes of the rational function displayed below. What is its domain? What is its formula?

1. 

x-int.: $x = -3$, $x = 1$ y-int.: $y = 1$

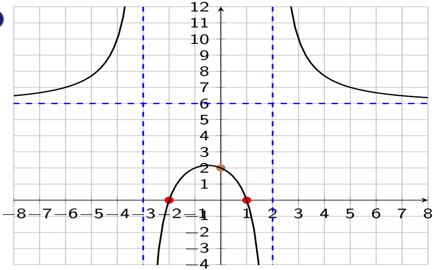
VA: $x = -2, x = 3$ HA: $y = 2$

Domain: $D = (-\infty, -2) \cup (-2, 3) \cup (3, \infty)$

$$f(x) = \frac{a \cdot (x-1) \cdot (x+3)}{(x+2) \cdot (x-3)}$$

$$f(0) = \frac{a \cdot (-1) \cdot 3}{2 \cdot (-3)} = \frac{a}{2} \stackrel{!}{=} 1 \implies a = 2$$

$$\implies f(x) = \frac{2 \cdot (x-1) \cdot (x+3)}{(x+2) \cdot (x-3)}$$

2. 

x-int.: $x = -2$, $x = 1$ y-int.: $y = 2$

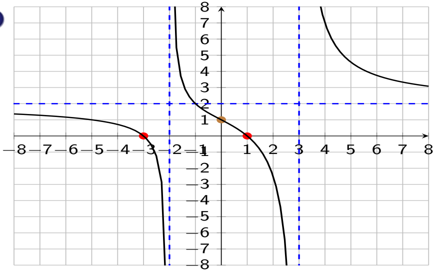
VA: $x = -3, x = 2$ HA: $y = 6$

Domain: $D = (-\infty, -3) \cup (-3, 2) \cup (2, \infty)$

$$f(x) = \frac{a \cdot (x+2) \cdot (x-1)}{(x-2) \cdot (x+3)}$$

Find the intercepts and the asymptotes of the rational function displayed below. What is its domain? What is its formula?

1



x-int.: $x = -3,\ x = 1$ y-int.: $y = 1$

VA: $x = -2, x = 3$ HA: $y = 2$

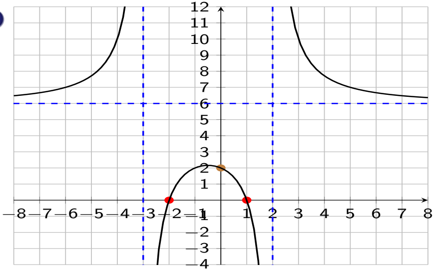
Domain: $D = (-\infty, -2) \cup (-2, 3) \cup (3, \infty)$

$$f(x) = \frac{a\cdot(x-1)\cdot(x+3)}{(x+2)\cdot(x-3)}$$

$$f(0) = \frac{a\cdot(-1)\cdot 3}{2\cdot(-3)} = \frac{a}{2} \stackrel{!}{=} 1 \implies a = 2$$

$$\implies f(x) = \frac{2\cdot(x-1)\cdot(x+3)}{(x+2)\cdot(x-3)}$$

2



x-int.: $x = -2,\ x = 1$ y-int.: $y = 2$

VA: $x = -3, x = 2$ HA: $y = 6$

Domain: $D = (-\infty, -3) \cup (-3, 2) \cup (2, \infty)$

$$f(x) = \frac{a\cdot(x+2)\cdot(x-1)}{(x-2)\cdot(x+3)}$$

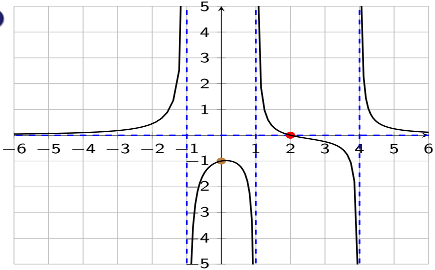
$$f(0) = \frac{a\cdot 2\cdot(-1)}{(-2)\cdot 3} = \frac{a}{3} \stackrel{!}{=} 2 \implies a = 6$$

$$\implies f(x) = \frac{6\cdot(x+2)\cdot(x-1)}{(x-2)\cdot(x+3)}$$

Rational function via its graph - exercises

Find the intercepts and the asymptotes of the rational function displayed below. What is its domain? What is its formula?

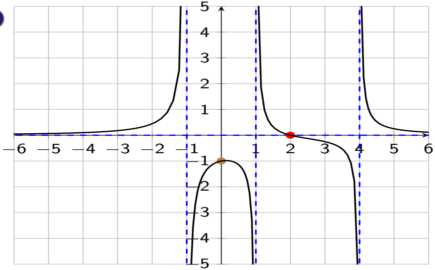
3.



x-int.: *y*-int.:
VA:
HA:
Domain:

$f(x) =$

Find the intercepts and the asymptotes of the rational function displayed below. What is its domain? What is its formula?

3. 

x-int.: $x = 2$ y-int.: $y = -1$

VA: $x = -1, x = 1, x = 4$

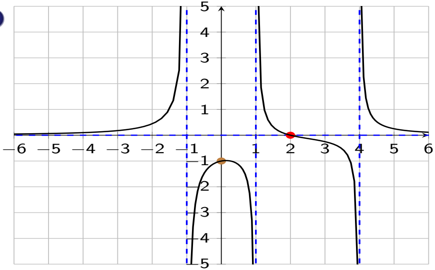
HA: $y = 0$

Domain: $D = (-\infty, -1) \cup (-1, 1) \cup (1, 4) \cup (4, \infty)$

$f(x) =$

Rational function via its graph - exercises

Find the intercepts and the asymptotes of the rational function displayed below. What is its domain? What is its formula?

3. 

x-int.: $x = 2$ y-int.: $y = -1$

VA: $x = -1, x = 1, x = 4$

HA: $y = 0$

Domain: $D = (-\infty, -1) \cup (-1, 1) \cup (1, 4) \cup (4, \infty)$

$$f(x) = \frac{a \cdot (x-2)}{(x+1)\cdot(x-1)\cdot(x-4)}$$

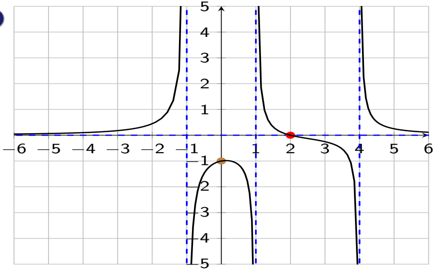
$$f(0) = \frac{-2 \cdot a}{4} = \frac{-a}{2} \overset{!}{=} -1 \implies a = 2$$

$$\implies f(x) = \frac{2\cdot(x-2)}{(x+1)\cdot(x-1)\cdot(x-4)}$$

Rational function via its graph - exercises

Find the intercepts and the asymptotes of the rational function displayed below. What is its domain? What is its formula?

3



x-int.: $x = 2$ y-int.: $y = -1$

VA: $x = -1, x = 1, x = 4$

HA: $y = 0$

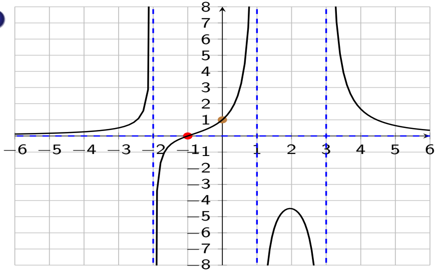
Domain: $D = (-\infty, -1) \cup (-1, 1) \cup (1, 4) \cup (4, \infty)$

$$f(x) = \frac{a\cdot(x-2)}{(x+1)\cdot(x-1)\cdot(x-4)}$$

$$f(0) = \frac{-2\cdot a}{4} = \frac{-a}{2} \overset{!}{=} -1 \implies a = 2$$

$$\implies f(x) = \frac{2\cdot(x-2)}{(x+1)\cdot(x-1)\cdot(x-4)}$$

4



x-int.: y-int.:

VA:

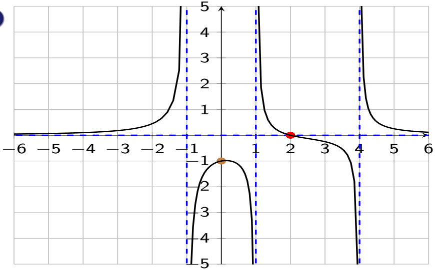
HA:

Domain:

$f(x) =$

Find the intercepts and the asymptotes of the rational function displayed below. What is its domain? What is its formula?

3



x-int.: $x = 2$ y-int.: $y = -1$

VA: $x = -1, x = 1, x = 4$

HA: $y = 0$

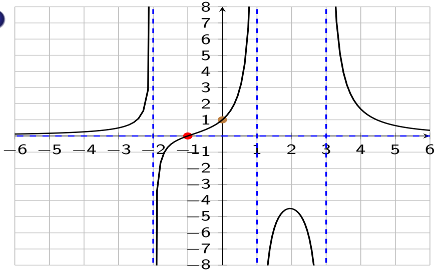
Domain: $D = (-\infty, -1) \cup (-1, 1) \cup (1, 4) \cup (4, \infty)$

$$f(x) = \frac{a\cdot(x-2)}{(x+1)\cdot(x-1)\cdot(x-4)}$$

$$f(0) = \frac{-2\cdot a}{4} = \frac{-a}{2} \overset{!}{=} -1 \implies a = 2$$

$$\implies f(x) = \frac{2\cdot(x-2)}{(x+1)\cdot(x-1)\cdot(x-4)}$$

4



x-int.: $x = -1$ y-int.: $y = 1$

VA: $x = -2, x = 1, x = 3$

HA: $y = 0$

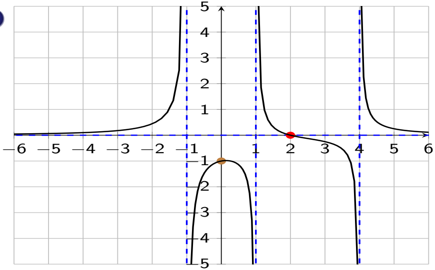
Domain: $D = (-\infty, -2) \cup (-2, 1) \cup (1, 3) \cup (3, \infty)$

$f(x) =$

Rational function via its graph - exercises

Find the intercepts and the asymptotes of the rational function displayed below. What is its domain? What is its formula?

3



x-int.: $x = 2$ y-int.: $y = -1$

VA: $x = -1, x = 1, x = 4$

HA: $y = 0$

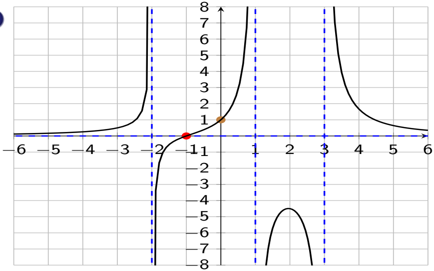
Domain: $D = (-\infty, -1) \cup (-1, 1) \cup (1, 4) \cup (4, \infty)$

$$f(x) = \frac{a\cdot(x-2)}{(x+1)\cdot(x-1)\cdot(x-4)}$$

$$f(0) = \frac{-2\cdot a}{4} = \frac{-a}{2} \overset{!}{=} -1 \implies a = 2$$

$$\implies f(x) = \frac{2\cdot(x-2)}{(x+1)\cdot(x-1)\cdot(x-4)}$$

4



x-int.: $x = -1$ y-int.: $y = 1$

VA: $x = -2, x = 1, x = 3$

HA: $y = 0$

Domain: $D = (-\infty, -2) \cup (-2, 1) \cup (1, 3) \cup (3, \infty)$

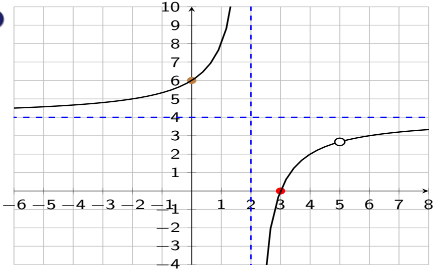
$$f(x) = \frac{a\cdot(x+1)}{(x-1)\cdot(x-3)\cdot(x+2)}$$

$$f(0) = \frac{a}{6} \overset{!}{=} 1 \implies a = 6$$

$$\implies f(x) = \frac{6\cdot(x+1)}{(x-1)\cdot(x-3)\cdot(x+2)}$$

Find the intercepts and the asymptotes of the rational function displayed below. What is its domain? What is its formula?

5

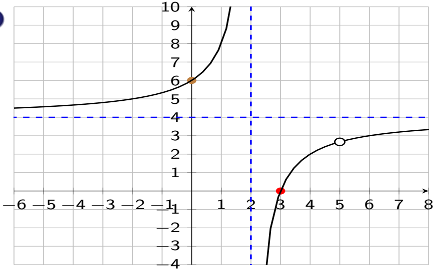


x-int.: *y*-int.:
VA: HA:
Hole:
Domain:

$f(x) =$

Find the intercepts and the asymptotes of the rational function displayed below. What is its domain? What is its formula?

5



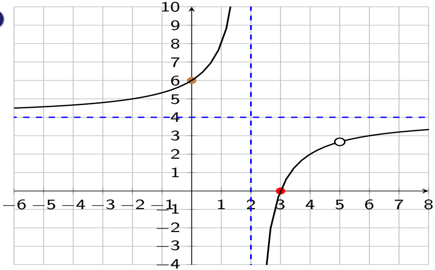
x-int.:	$x = 3$	y-int.:	$y = 6$
VA:	$x = 2$	HA:	$y = 4$
Hole:	$x = 5$		

Domain: $D = (-\infty, 2) \cup (2, 5) \cup (5, \infty)$

$f(x) =$

Find the intercepts and the asymptotes of the rational function displayed below. What is its domain? What is its formula?

5



x-int.:	$x = 3$	y-int.:	$y = 6$
VA:	$x = 2$	HA:	$y = 4$
Hole:	$x = 5$		

Domain: $D = (-\infty, 2) \cup (2, 5) \cup (5, \infty)$

$$f(x) = \frac{a \cdot (x-3) \cdot (x-5)}{(x-2) \cdot (x-5)}$$

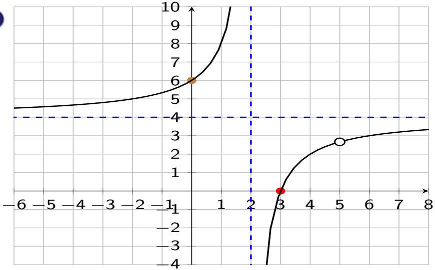
$$f(0) = \frac{3a}{2} \overset{!}{=} 6 \implies a = \frac{2}{3} \cdot 6 = 4$$

$$\implies f(x) = \frac{4 \cdot (x-3) \cdot (x-5)}{(x-2) \cdot (x-5)}$$

Rational function via its graph - exercises

Find the intercepts and the asymptotes of the rational function displayed below. What is its domain? What is its formula?

5.



x-int.:	$x = 3$	y-int.:	$y = 6$
VA:	$x = 2$	HA:	$y = 4$
Hole:	$x = 5$		

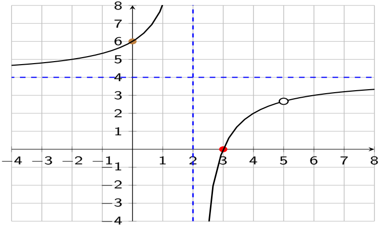
Domain: $D = (-\infty, 2) \cup (2, 5) \cup (5, \infty)$

$$f(x) = \frac{a\cdot(x-3)\cdot(x-5)}{(x-2)\cdot(x-5)}$$

$$f(0) = \frac{3a}{2} \stackrel{!}{=} 6 \implies a = \frac{2}{3}\cdot 6 = 4$$

$$\implies f(x) = \frac{4\cdot(x-3)\cdot(x-5)}{(x-2)\cdot(x-5)}$$

6. From previous exercise:

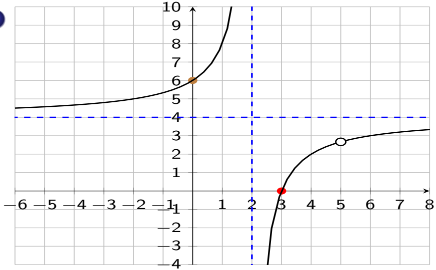


$f(x) = \frac{4\cdot(x-3)\cdot(x-5)}{(x-2)\cdot(x-5)}$ has hole at $x = 5$.

Q: What is the y-coordinate of the removable singularity?

Find the intercepts and the asymptotes of the rational function displayed below. What is its domain? What is its formula?

5



x-int.:	$x = 3$	y-int.:	$y = 6$
VA:	$x = 2$	HA:	$y = 4$
Hole:	$x = 5$		

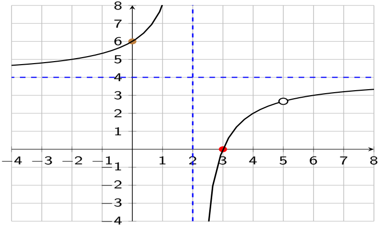
Domain: $D = (-\infty, 2) \cup (2, 5) \cup (5, \infty)$

$$f(x) = \frac{a\cdot(x-3)\cdot(x-5)}{(x-2)\cdot(x-5)}$$

$$f(0) = \frac{3a}{2} \overset{!}{=} 6 \implies a = \frac{2}{3}\cdot 6 = 4$$

$$\implies f(x) = \frac{4\cdot(x-3)\cdot(x-5)}{(x-2)\cdot(x-5)}$$

6 From previous exercise:



$f(x) = \frac{4\cdot(x-3)\cdot(x-5)}{(x-2)\cdot(x-5)}$ has hole at $x = 5$.

Q: What is the y-coordinate of the removable singularity?

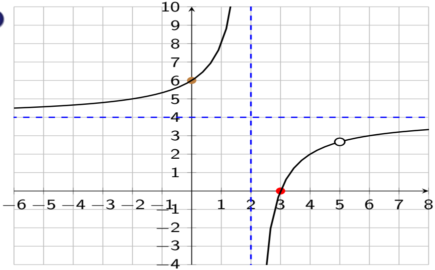
A: For $x \neq 5$: $\quad f(x) = \frac{4\cdot(x-3)}{(x-2)}$

So, as x approaches 5, $\quad x \to 5$

$y = f(x)$ approaches

$y \to$

Find the intercepts and the asymptotes of the rational function displayed below. What is its domain? What is its formula?

5.


x-int.:	$x = 3$	y-int.:	$y = 6$
VA:	$x = 2$	HA:	$y = 4$
Hole:	$x = 5$		

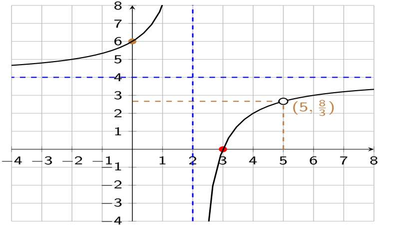
Domain: $D = (-\infty, 2) \cup (2, 5) \cup (5, \infty)$

$$f(x) = \frac{a\cdot(x-3)\cdot(x-5)}{(x-2)\cdot(x-5)}$$

$$f(0) = \frac{3a}{2} \overset{!}{=} 6 \implies a = \frac{2}{3}\cdot 6 = 4$$

$$\implies f(x) = \frac{4\cdot(x-3)\cdot(x-5)}{(x-2)\cdot(x-5)}$$

6. From previous exercise:



$f(x) = \frac{4\cdot(x-3)\cdot(x-5)}{(x-2)\cdot(x-5)}$ has hole at $x = 5$.

Q: What is the y-coordinate of the removable singularity?

A: For $x \neq 5$: $\quad f(x) = \frac{4\cdot(x-3)}{(x-2)}$

So, as x approaches 5, $\quad x \to 5$

$y = f(x)$ approaches

$$y \to \frac{4\cdot(5-3)}{(5-2)} = \frac{4\cdot 2}{3} = \frac{8}{3} \approx 2.667$$

$\implies$ The singularity is at $(x, y) = (5, \frac{8}{3})$.

Asymptotes of a rational function $f(x)$

Using a long division $\frac{f(x)}{q(x)}$, we can write $f(x) = g(x) + \frac{p(x)}{q(x)}$ for polynomials $g(x)$ and $p(x)$. When $\deg(p) < \deg(q)$, the rational function $\frac{p(x)}{q(x)}$ has a horizontal asymptote $y = 0$, and so:

$$\text{for } x \to \pm\infty: \quad f(x) \approx g(x)$$

Asymptotes of a rational function $f(x)$

Using a long division $\frac{f(x)}{q(x)}$, we can write $f(x) = g(x) + \frac{p(x)}{q(x)}$ for polynomials $g(x)$ and $p(x)$. When $\deg(p) < \deg(q)$, the rational function $\frac{p(x)}{q(x)}$ has a horizontal asymptote $y = 0$, and so:

$$\text{for } x \to \pm\infty: \quad f(x) \approx g(x)$$

1. Find the slant asymptote of

$$f(x) = \frac{3x^2 - 5x + 2}{x - 4}.$$

Asymptotes of a rational function $f(x)$

Using a long division $\frac{f(x)}{q(x)}$, we can write $f(x) = g(x) + \frac{p(x)}{q(x)}$ for polynomials $g(x)$ and $p(x)$. When $\deg(p) < \deg(q)$, the rational function $\frac{p(x)}{q(x)}$ has a horizontal asymptote $y = 0$, and so:

$$\text{for } x \to \pm\infty: \quad f(x) \approx g(x)$$

1. Find the slant asymptote of

$$f(x) = \frac{3x^2-5x+2}{x-4}.$$

Solution: Use long division:

$$\begin{array}{r|lll} & & 3x & +7 \\ \hline x-4 & 3x^2 & -5x & +2 \\ & -(3x^2 & -12x) & \\ \hline & & 7x & +2 \\ & & -(7x & -28) \\ \hline & & & 30 \end{array}$$

Thus: $f(x) = \frac{3x^2-5x+2}{x-4} = 3x + 7 + \frac{30}{x-4}$
has slant asymptote:

Asymptotes of a rational function $f(x)$

Using a long division $\frac{f(x)}{q(x)}$, we can write $f(x) = g(x) + \frac{p(x)}{q(x)}$ for polynomials $g(x)$ and $p(x)$. When $\deg(p) < \deg(q)$, the rational function $\frac{p(x)}{q(x)}$ has a horizontal asymptote $y = 0$, and so:

$$\text{for } x \to \pm\infty: \quad f(x) \approx g(x)$$

1. Find the slant asymptote of

$$f(x) = \frac{3x^2 - 5x + 2}{x - 4}.$$

Solution: Use long division:

$$\begin{array}{r|lll} & & 3x & +7 \\ \hline x-4 & 3x^2 & -5x & +2 \\ & -(3x^2 & -12x) & \\ \hline & & 7x & +2 \\ & & -(7x & -28) \\ \hline & & & 30 \end{array}$$

Thus: $f(x) = \frac{3x^2-5x+2}{x-4} = 3x + 7 + \frac{30}{x-4}$
has slant asymptote:

$$y = 3x + 7$$

Asymptotic behavior at infinity

Asymptotes of a rational function $f(x)$

Using a long division $\frac{f(x)}{q(x)}$, we can write $f(x) = g(x) + \frac{p(x)}{q(x)}$ for polynomials $g(x)$ and $p(x)$. When $\deg(p) < \deg(q)$, the rational function $\frac{p(x)}{q(x)}$ has a horizontal asymptote $y = 0$, and so:

$$\text{for } x \to \pm\infty: \quad f(x) \approx g(x)$$

1. Find the slant asymptote of

$$f(x) = \frac{3x^2-5x+2}{x-4}.$$

Solution: Use long division:

$$\begin{array}{r|lll} & & 3x & +7 \\ \hline x-4 & 3x^2 & -5x & +2 \\ & -(3x^2 & -12x) & \\ & & 7x & +2 \\ & & -(7x & -28) \\ & & & 30 \end{array}$$

Thus: $f(x) = \frac{3x^2-5x+2}{x-4} = 3x + 7 + \frac{30}{x-4}$
has slant asymptote:

$$y=3x+7$$

2. Which polynomial does

$$f(x) = \frac{x^3+4x^2-5x+7}{x+2}$$

asymptotically approach?

Asymptotes of a rational function $f(x)$

Using a long division $\frac{f(x)}{q(x)}$, we can write $f(x) = g(x) + \frac{p(x)}{q(x)}$ for polynomials $g(x)$ and $p(x)$. When $\deg(p) < \deg(q)$, the rational function $\frac{p(x)}{q(x)}$ has a horizontal asymptote $y = 0$, and so:

$$\text{for } x \to \pm\infty: \quad f(x) \approx g(x)$$

1. Find the slant asymptote of

$$f(x) = \frac{3x^2-5x+2}{x-4}.$$

Solution: Use long division:

$$\begin{array}{r|lll} & & 3x & +7 \\ \hline x-4 & 3x^2 & -5x & +2 \\ & -(3x^2 & -12x) & \\ \hline & & 7x & +2 \\ & & -(7x & -28) \\ \hline & & & 30 \end{array}$$

Thus: $f(x) = \frac{3x^2-5x+2}{x-4} = 3x + 7 + \frac{30}{x-4}$ has slant asymptote:

$$y=3x+7$$

2. Which polynomial does

$$f(x) = \frac{x^3+4x^2-5x+7}{x+2}$$

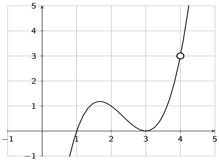
asymptotically approach?

Solution:

$$\begin{array}{r|llll} & & x^2 & +2x & -9 \\ \hline x+2 & x^3 & +4x^2 & -5x & +7 \\ & -(x^3 & +2x^2) & & \\ \hline & & 2x^2 & -5x & \\ & & -(2x^2 & +4x) & \\ \hline & & & -9x & +7 \\ & & & -(-9x & -18) \\ \hline & & & & 25 \end{array}$$

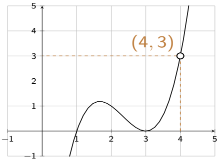
Therefore $f(x) = \frac{x^3+4x^2-5x+7}{x+2}$ approaches: $\quad y = x^2 + 2x - 9$

1. Assuming the following graph has a hole at (x, y) where x and y are integer values.



As $x \to 4$, what does y approach?

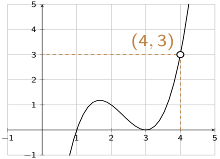
1. Assuming the following graph has a hole at (x, y) where x and y are integer values.



As $x \to 4$, what does y approach?

Function values approaching a discontinuity - exercises

1. Assuming the following graph has a hole at (x, y) where x and y are integer values.

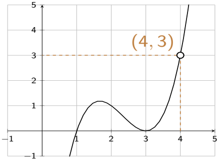


As $x \to 4$, what does y approach?

$y \to 3$

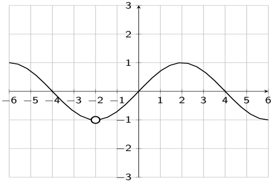
Function values approaching a discontinuity - exercises

1. Assuming the following graph has a hole at (x, y) where x and y are integer values.



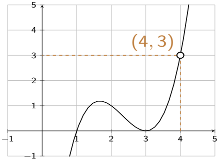
As $x \to 4$, what does y approach?

$y \to 3$

2. 

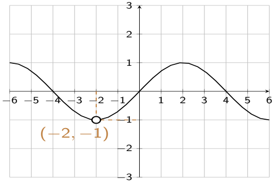
As $x \to -2$, what does y approach?

1. Assuming the following graph has a hole at (x, y) where x and y are integer values.



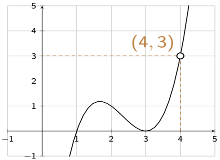
As $x \to 4$, what does y approach?

$y \to 3$

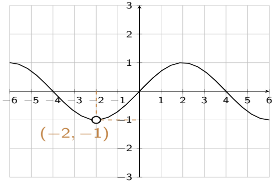
2. 

As $x \to -2$, what does y approach?

1. Assuming the following graph has a hole at (x, y) where x and y are integer values.

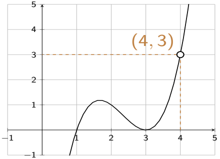


As $x \to 4$, what does y approach?
$y \to 3$

2. 

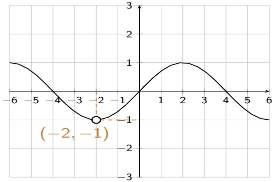
As $x \to -2$, what does y approach?
$y \to -1$

1. Assuming the following graph has a hole at (x, y) where x and y are integer values.



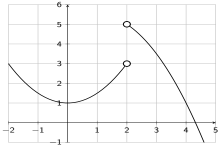
As $x \to 4$, what does y approach?

$y \to 3$

2. 

As $x \to -2$, what does y approach?

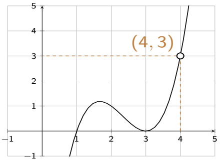
$y \to -1$

3. 

When $x \to 2^+ \implies y \to$

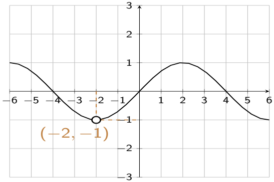
When $x \to 2^- \implies y \to$

1. Assuming the following graph has a hole at (x, y) where x and y are integer values.



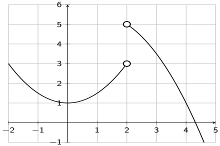
As $x \to 4$, what does y approach?

$y \to 3$

2. 

As $x \to -2$, what does y approach?

$y \to -1$

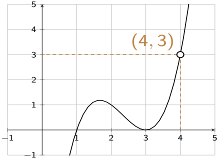
3. 

When $x \to 2^+ \implies y \to 5$

When $x \to 2^- \implies y \to 3$

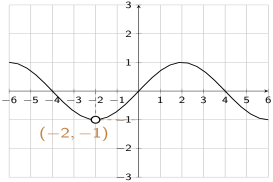
Function values approaching a discontinuity - exercises

1. Assuming the following graph has a hole at (x, y) where x and y are integer values.



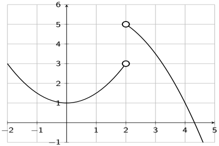
As $x \to 4$, what does y approach?

$y \to 3$

2. 

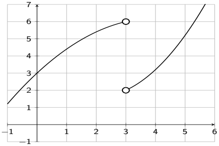
As $x \to -2$, what does y approach?

$y \to -1$

3. 

When $x \to 2^+ \implies y \to 5$

When $x \to 2^- \implies y \to 3$

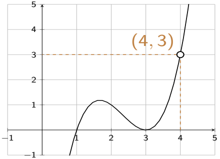
4. 

When $x \to 3^+ \implies y \to$

When $x \to 3^- \implies y \to$

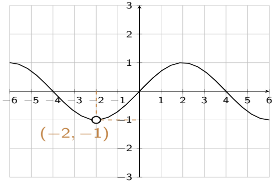
Function values approaching a discontinuity - exercises

1. Assuming the following graph has a hole at (x, y) where x and y are integer values.



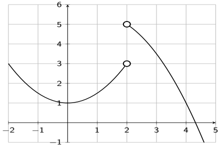
As $x \to 4$, what does y approach?

$y \to 3$

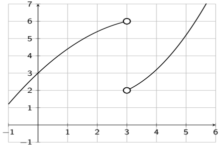
2. 

As $x \to -2$, what does y approach?

$y \to -1$

3. 

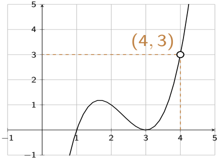
When $x \to 2^+ \implies y \to 5$
When $x \to 2^- \implies y \to 3$

4. 

When $x \to 3^+ \implies y \to 2$
When $x \to 3^- \implies y \to 6$

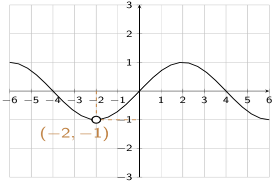
Function values approaching a discontinuity - exercises

1. Assuming the following graph has a hole at (x, y) where x and y are integer values.



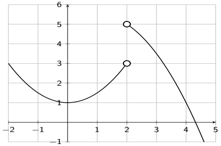
As $x \to 4$, what does y approach?

$y \to 3$

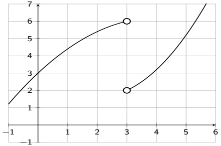
2. 

As $x \to -2$, what does y approach?

$y \to -1$

3. 

When $x \to 2^+ \implies y \to 5$
When $x \to 2^- \implies y \to 3$

4. 

When $x \to 3^+ \implies y \to 2$
When $x \to 3^- \implies y \to 6$

Limit

In calculus, this will be written as:

$$\lim_{x \to 3^+} f(x) = 2 \quad \text{and} \quad \lim_{x \to 3^-} f(x) = 6$$

5. Let $f(x) = \frac{(x-2)}{2x \cdot |x-2|}$
   When $x \to 2^{+} \implies y \to$
   When $x \to 2^{-} \implies y \to$

5. Let $f(x) = \frac{(x-2)}{2x \cdot |x-2|}$

   When $x \to 2^+ \implies y \to$

   When $x \to 2^- \implies y \to$

   Use calculator to find values close to 2:

x	y
2.1	0.2380...
2.01	0.2487...
2.001	0.2498...

x	y
1.9	-0.2631...
1.99	-0.2512...
1.999	-0.2501...

Function values approaching a discontinuity using a formula - exercises

5. Let $f(x) = \frac{(x-2)}{2x \cdot |x-2|}$

   When $x \to 2^+ \implies y \to 0.25$

   When $x \to 2^- \implies y \to -0.25$

   Use calculator to find values close to 2:

x	y
2.1	0.2380...
2.01	0.2487...
2.001	0.2498...

x	y
1.9	-0.2631...
1.99	-0.2512...
1.999	-0.2501...

Function values approaching a discontinuity using a formula - exercises

5. Let $f(x) = \frac{(x-2)}{2x\cdot|x-2|}$
   When $x \to 2^+ \implies y \to 0.25$
   When $x \to 2^- \implies y \to -0.25$
   Use calculator to find values close to 2:

x	y
2.1	0.2380...
2.01	0.2487...
2.001	0.2498...

x	y
1.9	-0.2631...
1.99	-0.2512...
1.999	-0.2501...

$\lim_{x\to 2^+} f(x) = 0.25 \quad \lim_{x\to 2^-} f(x) = -0.25$

Function values approaching a discontinuity using a formula - exercises

5. Let $f(x) = \frac{(x-2)}{2x \cdot |x-2|}$
   When $x \to 2^+ \implies y \to 0.25$
   When $x \to 2^- \implies y \to -0.25$
   Use calculator to find values close to 2:

x	y
2.1	0.2380...
2.01	0.2487...
2.001	0.2498...

x	y
1.9	-0.2631...
1.99	-0.2512...
1.999	-0.2501...

$\lim_{x \to 2^+} f(x) = 0.25 \quad \lim_{x \to 2^-} f(x) = -0.25$

6. Let $f(x) = \frac{\sqrt{x}-1}{x-1}$
   When $x \to 1^+ \implies y \to$
   When $x \to 1^- \implies y \to$

Function values approaching a discontinuity using a formula - exercises

5. Let $f(x) = \frac{(x-2)}{2x \cdot |x-2|}$

   When $x \to 2^+ \implies y \to 0.25$

   When $x \to 2^- \implies y \to -0.25$

   Use calculator to find values close to 2:

x	y
2.1	0.2380...
2.01	0.2487...
2.001	0.2498...

x	y
1.9	-0.2631...
1.99	-0.2512...
1.999	-0.2501...

$\lim_{x \to 2^+} f(x) = 0.25 \quad \lim_{x \to 2^-} f(x) = -0.25$

6. Let $f(x) = \frac{\sqrt{x}-1}{x-1}$

   When $x \to 1^+ \implies y \to$

   When $x \to 1^- \implies y \to$

   Use calculator to find values close to 1:

x	y
1.1	0.4880...
1.01	0.4987...
1.001	0.4998...

x	y
0.9	0.5131...
0.99	0.5012...
0.999	0.5001...

5. Let $f(x) = \frac{(x-2)}{2x \cdot |x-2|}$
   When $x \to 2^+ \implies y \to 0.25$
   When $x \to 2^- \implies y \to -0.25$
   Use calculator to find values close to 2:

x	y
2.1	0.2380...
2.01	0.2487...
2.001	0.2498...

x	y
1.9	-0.2631...
1.99	-0.2512...
1.999	-0.2501...

$\lim_{x \to 2^+} f(x) = 0.25 \quad \lim_{x \to 2^-} f(x) = -0.25$

6. Let $f(x) = \frac{\sqrt{x}-1}{x-1}$
   When $x \to 1^+ \implies y \to 0.5$
   When $x \to 1^- \implies y \to 0.5$
   Use calculator to find values close to 1:

x	y
1.1	0.4880...
1.01	0.4987...
1.001	0.4998...

x	y
0.9	0.5131...
0.99	0.5012...
0.999	0.5001...

Function values approaching a discontinuity using a formula - exercises

5. Let $f(x) = \frac{(x-2)}{2x\cdot|x-2|}$

   When $x \to 2^+ \implies y \to 0.25$

   When $x \to 2^- \implies y \to -0.25$

   Use calculator to find values close to 2:

x	y
2.1	0.2380...
2.01	0.2487...
2.001	0.2498...

x	y
1.9	-0.2631...
1.99	-0.2512...
1.999	-0.2501...

$\lim_{x\to 2^+} f(x) = 0.25 \quad \lim_{x\to 2^-} f(x) = -0.25$

6. Let $f(x) = \frac{\sqrt{x}-1}{x-1}$

   When $x \to 1^+ \implies y \to 0.5$

   When $x \to 1^- \implies y \to 0.5$

   Use calculator to find values close to 1:

x	y
1.1	0.4880...
1.01	0.4987...
1.001	0.4998...

x	y
0.9	0.5131...
0.99	0.5012...
0.999	0.5001...

We write: $\lim_{x\to 1^+} f(x) = 0.5$

$\lim_{x\to 1^-} f(x) = 0.5$

Thus: $\lim_{x\to 1} f(x) = 0.5$

Function values approaching a discontinuity using a formula - exercises

5. Let $f(x) = \frac{(x-2)}{2x \cdot |x-2|}$

   When $x \to 2^+ \implies y \to 0.25$

   When $x \to 2^- \implies y \to -0.25$

   Use calculator to find values close to 2:

x	y
2.1	0.2380...
2.01	0.2487...
2.001	0.2498...

x	y
1.9	-0.2631...
1.99	-0.2512...
1.999	-0.2501...

   $\lim_{x \to 2^+} f(x) = 0.25 \quad \lim_{x \to 2^-} f(x) = -0.25$

6. Let $f(x) = \frac{\sqrt{x}-1}{x-1}$

   When $x \to 1^+ \implies y \to 0.5$

   When $x \to 1^- \implies y \to 0.5$

   Use calculator to find values close to 1:

x	y
1.1	0.4880...
1.01	0.4987...
1.001	0.4998...

x	y
0.9	0.5131...
0.99	0.5012...
0.999	0.5001...

   We write: $\lim_{x \to 1^+} f(x) = 0.5$

   $\lim_{x \to 1^-} f(x) = 0.5$

   Thus: $\lim_{x \to 1} f(x) = 0.5$

7. Let $f(x) = \frac{x-5}{x-4}$

   When $x \to 4^+ \implies y \to$

   When $x \to 4^- \implies y \to$

Function values approaching a discontinuity using a formula - exercises

5. Let $f(x) = \frac{(x-2)}{2x \cdot |x-2|}$

   When $x \to 2^+ \implies y \to 0.25$

   When $x \to 2^- \implies y \to -0.25$

   Use calculator to find values close to 2:

x	y
2.1	0.2380...
2.01	0.2487...
2.001	0.2498...

x	y
1.9	-0.2631...
1.99	-0.2512...
1.999	-0.2501...

   $\lim_{x\to 2^+} f(x) = 0.25 \quad \lim_{x\to 2^-} f(x) = -0.25$

6. Let $f(x) = \frac{\sqrt{x}-1}{x-1}$

   When $x \to 1^+ \implies y \to 0.5$

   When $x \to 1^- \implies y \to 0.5$

   Use calculator to find values close to 1:

x	y
1.1	0.4880...
1.01	0.4987...
1.001	0.4998...

x	y
0.9	0.5131...
0.99	0.5012...
0.999	0.5001...

   We write: $\lim_{x\to 1^+} f(x) = 0.5$

   $\lim_{x\to 1^-} f(x) = 0.5$

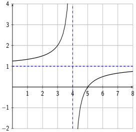
   Thus: $\lim_{x\to 1} f(x) = 0.5$

7. Let $f(x) = \frac{x-5}{x-4}$

   When $x \to 4^+ \implies y \to$

   When $x \to 4^- \implies y \to$

   Use calculator to find values close to 4:



x	y
4.1	-9
4.01	-99
4.001	-999

x	y
3.9	11
3.99	101
3.999	1001

Function values approaching a discontinuity using a formula - exercises

5. Let $f(x) = \frac{(x-2)}{2x \cdot |x-2|}$

When $x \to 2^+ \implies y \to 0.25$

When $x \to 2^- \implies y \to -0.25$

Use calculator to find values close to 2:

x	y
2.1	0.2380...
2.01	0.2487...
2.001	0.2498...

x	y
1.9	-0.2631...
1.99	-0.2512...
1.999	-0.2501...

$\lim_{x\to 2^+} f(x) = 0.25 \quad \lim_{x\to 2^-} f(x) = -0.25$

6. Let $f(x) = \frac{\sqrt{x}-1}{x-1}$

When $x \to 1^+ \implies y \to 0.5$

When $x \to 1^- \implies y \to 0.5$

Use calculator to find values close to 1:

x	y
1.1	0.4880...
1.01	0.4987...
1.001	0.4998...

x	y
0.9	0.5131...
0.99	0.5012...
0.999	0.5001...

We write: $\lim_{x\to 1^+} f(x) = 0.5$

$\lim_{x\to 1^-} f(x) = 0.5$

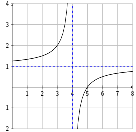
Thus: $\lim_{x\to 1} f(x) = 0.5$

7. Let $f(x) = \frac{x-5}{x-4}$

When $x \to 4^+ \implies y \to -\infty$

When $x \to 4^- \implies y \to +\infty$

Use calculator to find values close to 4:



x	y
4.1	-9
4.01	-99
4.001	-999

x	y
3.9	11
3.99	101
3.999	1001

Function values approaching a discontinuity using a formula - exercises

5. Let $f(x) = \frac{(x-2)}{2x \cdot |x-2|}$

   When $x \to 2^+ \implies y \to 0.25$

   When $x \to 2^- \implies y \to -0.25$

   Use calculator to find values close to 2:

x	y
2.1	0.2380...
2.01	0.2487...
2.001	0.2498...

x	y
1.9	-0.2631...
1.99	-0.2512...
1.999	-0.2501...

$\lim_{x\to 2^+} f(x) = 0.25 \quad \lim_{x\to 2^-} f(x) = -0.25$

6. Let $f(x) = \frac{\sqrt{x}-1}{x-1}$

   When $x \to 1^+ \implies y \to 0.5$

   When $x \to 1^- \implies y \to 0.5$

   Use calculator to find values close to 1:

x	y
1.1	0.4880...
1.01	0.4987...
1.001	0.4998...

x	y
0.9	0.5131...
0.99	0.5012...
0.999	0.5001...

We write: $\lim_{x\to 1^+} f(x) = 0.5$

$\lim_{x\to 1^-} f(x) = 0.5$

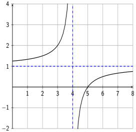
Thus: $\lim_{x\to 1} f(x) = 0.5$

7. Let $f(x) = \frac{x-5}{x-4}$

   When $x \to 4^+ \implies y \to -\infty$

   When $x \to 4^- \implies y \to +\infty$

   Use calculator to find values close to 4:



x	y
4.1	-9
4.01	-99
4.001	-999

x	y
3.9	11
3.99	101
3.999	1001

$\lim_{x\to 4^+} f(x) = -\infty \qquad \lim_{x\to 4^-} f(x) = +\infty$

Function values approaching a discontinuity using a formula - exercises

5. Let $f(x) = \frac{(x-2)}{2x\cdot|x-2|}$

   When $x \to 2^+ \implies y \to 0.25$

   When $x \to 2^- \implies y \to -0.25$

   Use calculator to find values close to 2:

x	y
2.1	0.2380...
2.01	0.2487...
2.001	0.2498...

x	y
1.9	-0.2631...
1.99	-0.2512...
1.999	-0.2501...

   $\lim_{x\to 2^+} f(x) = 0.25 \quad \lim_{x\to 2^-} f(x) = -0.25$

6. Let $f(x) = \frac{\sqrt{x}-1}{x-1}$

   When $x \to 1^+ \implies y \to 0.5$

   When $x \to 1^- \implies y \to 0.5$

   Use calculator to find values close to 1:

x	y
1.1	0.4880...
1.01	0.4987...
1.001	0.4998...

x	y
0.9	0.5131...
0.99	0.5012...
0.999	0.5001...

   We write: $\lim_{x\to 1^+} f(x) = 0.5$

   $\lim_{x\to 1^-} f(x) = 0.5$

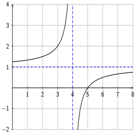
   Thus: $\lim_{x\to 1} f(x) = 0.5$

7. Let $f(x) = \frac{x-5}{x-4}$

   When $x \to 4^+ \implies y \to -\infty$

   When $x \to 4^- \implies y \to +\infty$

   Use calculator to find values close to 4:



x	y
4.1	-9
4.01	-99
4.001	-999

x	y
3.9	11
3.99	101
3.999	1001

   $\lim_{x\to 4^+} f(x) = -\infty \quad \lim_{x\to 4^-} f(x) = +\infty$

8. Let $f(x) = \frac{x+3}{x+2}$

   When $x \to -2^+ \implies y \to$

   When $x \to -2^- \implies y \to$

Function values approaching a discontinuity using a formula - exercises

5. Let $f(x) = \frac{(x-2)}{2x \cdot |x-2|}$

   When $x \to 2^+ \implies y \to 0.25$

   When $x \to 2^- \implies y \to -0.25$

   Use calculator to find values close to 2:

x	y
2.1	0.2380...
2.01	0.2487...
2.001	0.2498...

x	y
1.9	-0.2631...
1.99	-0.2512...
1.999	-0.2501...

$\lim_{x\to 2^+} f(x) = 0.25 \quad \lim_{x\to 2^-} f(x) = -0.25$

6. Let $f(x) = \frac{\sqrt{x}-1}{x-1}$

   When $x \to 1^+ \implies y \to 0.5$

   When $x \to 1^- \implies y \to 0.5$

   Use calculator to find values close to 1:

x	y
1.1	0.4880...
1.01	0.4987...
1.001	0.4998...

x	y
0.9	0.5131...
0.99	0.5012...
0.999	0.5001...

We write: $\lim_{x\to 1^+} f(x) = 0.5$

$\lim_{x\to 1^-} f(x) = 0.5$

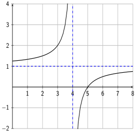
Thus: $\lim_{x\to 1} f(x) = 0.5$

7. Let $f(x) = \frac{x-5}{x-4}$

   When $x \to 4^+ \implies y \to -\infty$

   When $x \to 4^- \implies y \to +\infty$

   Use calculator to find values close to 4:



x	y
4.1	-9
4.01	-99
4.001	-999

x	y
3.9	11
3.99	101
3.999	1001

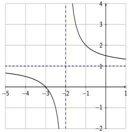
$\lim_{x\to 4^+} f(x) = -\infty \quad \lim_{x\to 4^-} f(x) = +\infty$

8. Let $f(x) = \frac{x+3}{x+2}$

   When $x \to -2^+ \implies y \to$

   When $x \to -2^- \implies y \to$

   Use calculator to find values close to -2:



x	y
-1.9	11
-1.99	101
-1.999	1001

x	y
-2.1	-9
-2.01	-99
-2.01	-999

Function values approaching a discontinuity using a formula - exercises

5. Let $f(x) = \frac{(x-2)}{2x \cdot |x-2|}$

When $x \to 2^+ \implies y \to 0.25$

When $x \to 2^- \implies y \to -0.25$

Use calculator to find values close to 2:

x	y
2.1	0.2380...
2.01	0.2487...
2.001	0.2498...

x	y
1.9	-0.2631...
1.99	-0.2512...
1.999	-0.2501...

$\lim_{x\to 2^+} f(x) = 0.25 \quad \lim_{x\to 2^-} f(x) = -0.25$

6. Let $f(x) = \frac{\sqrt{x}-1}{x-1}$

When $x \to 1^+ \implies y \to 0.5$

When $x \to 1^- \implies y \to 0.5$

Use calculator to find values close to 1:

x	y
1.1	0.4880...
1.01	0.4987...
1.001	0.4998...

x	y
0.9	0.5131...
0.99	0.5012...
0.999	0.5001...

We write: $\lim_{x\to 1^+} f(x) = 0.5$

$\lim_{x\to 1^-} f(x) = 0.5$

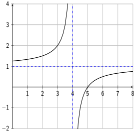
Thus: $\lim_{x\to 1} f(x) = 0.5$

7. Let $f(x) = \frac{x-5}{x-4}$

When $x \to 4^+ \implies y \to -\infty$

When $x \to 4^- \implies y \to +\infty$

Use calculator to find values close to 4:



x	y
4.1	-9
4.01	-99
4.001	-999

x	y
3.9	11
3.99	101
3.999	1001

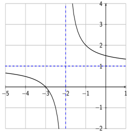
$\lim_{x\to 4^+} f(x) = -\infty \quad \lim_{x\to 4^-} f(x) = +\infty$

8. Let $f(x) = \frac{x+3}{x+2}$

When $x \to -2^+ \implies y \to +\infty$

When $x \to -2^- \implies y \to -\infty$

Use calculator to find values close to -2:



x	y
-1.9	11
-1.99	101
-1.999	1001

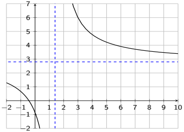
x	y
-2.1	-9
-2.01	-99
-2.01	-999

$\lim_{x\to -2^+} f(x) = +\infty \quad \lim_{x\to -2^-} f(x) = -\infty$

Approaching infinities - exercises

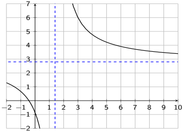
1. Let $f(x) = \frac{14x+6}{5x-7}$
   When $x \to +\infty \implies y \to$

1. Let $f(x) = \frac{14x+6}{5x-7}$
   When $x \to +\infty \implies y \to$
   Use calculator to find larger and larger function values:



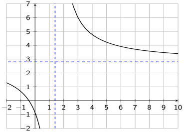
x	y
100	2.851...
1000	2.805...
10000	2.800...
100000	2.800...

1. Let $f(x) = \frac{14x+6}{5x-7}$
   When $x \to +\infty \implies y \to 2.8$
   Use calculator to find larger and larger function values:



x	y
100	2.851...
1000	2.805...
10000	2.800...
100000	2.800...

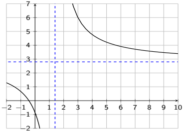
1. Let $f(x) = \frac{14x+6}{5x-7}$
   When $x \to +\infty \implies y \to 2.8$
   Use calculator to find larger and larger function values:



x	y
100	2.851...
1000	2.805...
10000	2.800...
100000	2.800...

$$\lim_{x \to +\infty} f(x) = 2.8 = \frac{14}{5}$$

1. Let $f(x) = \frac{14x+6}{5x-7}$
   When $x \to +\infty \implies y \to 2.8$
   Use calculator to find larger and larger function values:

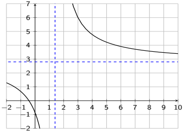


x	y
100	2.851...
1000	2.805...
10000	2.800...
100000	2.800...

$$\lim_{x \to +\infty} f(x) = 2.8 = \frac{14}{5}$$

2. Let $f(x) = \frac{3x^2-17x+4}{2x^2-6x-8}$
   When $x \to -\infty \implies y \to$

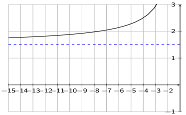
1. Let $f(x) = \frac{14x+6}{5x-7}$
   When $x \to +\infty \implies y \to 2.8$
   Use calculator to find larger and larger function values:



x	y
100	2.851...
1000	2.805...
10000	2.800...
100000	2.800...

$$\lim_{x\to+\infty} f(x) = 2.8 = \frac{14}{5}$$

2. Let $f(x) = \frac{3x^2-17x+4}{2x^2-6x-8}$
   When $x \to -\infty \implies y \to$
   Use calculator to find smaller and smaller function values:

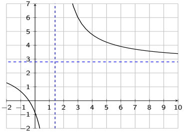


x	y
−100	1.539...
−1000	1.503...
−10000	1.500...
−100000	1.500...

1. Let $f(x) = \frac{14x+6}{5x-7}$

   When $x \to +\infty \implies y \to 2.8$

   Use calculator to find larger and larger function values:



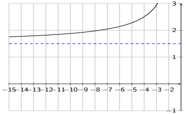
x	y
100	2.851...
1000	2.805...
10000	2.800...
100000	2.800...

$$\lim_{x \to +\infty} f(x) = 2.8 = \frac{14}{5}$$

2. Let $f(x) = \frac{3x^2-17x+4}{2x^2-6x-8}$

   When $x \to -\infty \implies y \to 1.5$

   Use calculator to find smaller and smaller function values:



x	y
−100	1.539...
−1000	1.503...
−10000	1.500...
−100000	1.500...

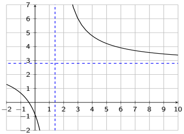
$$\lim_{x \to -\infty} f(x) = 1.5 = \frac{3}{2}$$

Approaching infinities - exercises

1. Let $f(x) = \frac{14x+6}{5x-7}$

   When $x \to +\infty \implies y \to 2.8$

   Use calculator to find larger and larger function values:



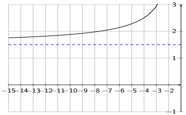
x	y
100	2.851...
1000	2.805...
10000	2.800...
100000	2.800...

$$\lim_{x \to +\infty} f(x) = 2.8 = \frac{14}{5}$$

2. Let $f(x) = \frac{3x^2-17x+4}{2x^2-6x-8}$

   When $x \to -\infty \implies y \to 1.5$

   Use calculator to find smaller and smaller function values:



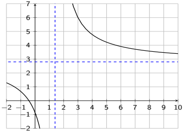
x	y
−100	1.539...
−1000	1.503...
−10000	1.500...
−100000	1.500...

$$\lim_{x \to -\infty} f(x) = 1.5 = \frac{3}{2}$$

3. Let $f(x) = \frac{-6x^2+7x-8}{3x+4}$

   When $x \to +\infty \implies y \to$

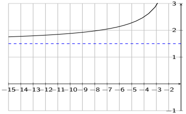
1. Let $f(x) = \frac{14x+6}{5x-7}$
When $x \to +\infty \implies y \to 2.8$
Use calculator to find larger and larger function values:



x	y
100	2.851...
1000	2.805...
10000	2.800...
100000	2.800...

$$\lim_{x\to+\infty} f(x) = 2.8 = \frac{14}{5}$$

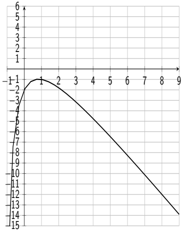
2. Let $f(x) = \frac{3x^2-17x+4}{2x^2-6x-8}$
When $x \to -\infty \implies y \to 1.5$
Use calculator to find smaller and smaller function values:



x	y
-100	1.539...
-1000	1.503...
-10000	1.500...
-100000	1.500...

$$\lim_{x\to-\infty} f(x) = 1.5 = \frac{3}{2}$$

3. Let $f(x) = \frac{-6x^2+7x-8}{3x+4}$
When $x \to +\infty \implies y \to$
Use calculator to find larger and larger function values:

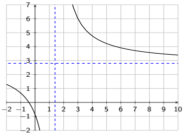


x	y
100	-195.1...
1000	-1995.0...
10000	-19995.0...
100000	-199995.0...

1. Let $f(x) = \frac{14x+6}{5x-7}$

   When $x \to +\infty \implies y \to 2.8$

   Use calculator to find larger and larger function values:



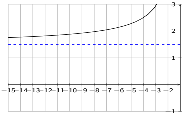
x	y
100	2.851...
1000	2.805...
10000	2.800...
100000	2.800...

$$\lim_{x\to+\infty} f(x) = 2.8 = \frac{14}{5}$$

2. Let $f(x) = \frac{3x^2-17x+4}{2x^2-6x-8}$

   When $x \to -\infty \implies y \to 1.5$

   Use calculator to find smaller and smaller function values:



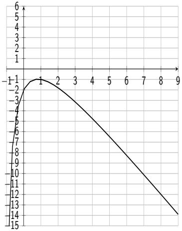
x	y
−100	1.539...
−1000	1.503...
−10000	1.500...
−100000	1.500...

$$\lim_{x\to-\infty} f(x) = 1.5 = \frac{3}{2}$$

3. Let $f(x) = \frac{-6x^2+7x-8}{3x+4}$

   When $x \to +\infty \implies y \to -\infty$

   Use calculator to find larger and larger function values:



x	y
100	-195.1...
1000	-1995.0...
10000	-19995.0...
100000	-199995.0...

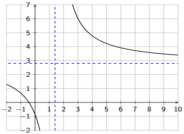
$$\lim_{x\to+\infty} f(x) = -\infty$$

Approaching infinities - exercises

1. Let $f(x) = \frac{14x+6}{5x-7}$

   When $x \to +\infty \implies y \to 2.8$

   Use calculator to find larger and larger function values:



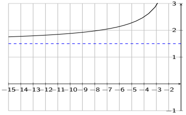
x	y
100	2.851...
1000	2.805...
10000	2.800...
100000	2.800...

$$\lim_{x\to+\infty} f(x) = 2.8 = \frac{14}{5}$$

2. Let $f(x) = \frac{3x^2-17x+4}{2x^2-6x-8}$

   When $x \to -\infty \implies y \to 1.5$

   Use calculator to find smaller and smaller function values:



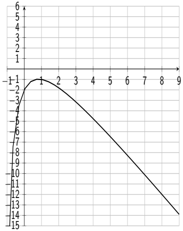
x	y
−100	1.539...
−1000	1.503...
−10000	1.500...
−100000	1.500...

$$\lim_{x\to-\infty} f(x) = 1.5 = \frac{3}{2}$$

3. Let $f(x) = \frac{-6x^2+7x-8}{3x+4}$

   When $x \to +\infty \implies y \to -\infty$

   Use calculator to find larger and larger function values:



x	y
100	-195.1...
1000	-1995.0...
10000	-19995.0...
100000	-199995.0...

$$\lim_{x\to+\infty} f(x) = -\infty$$

Note that we can compute the slant asymptote via a long division:

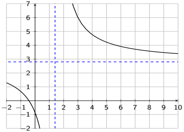
$$f(x) = -2x + 5 + \frac{-28}{3x+4}$$

Approaching infinities - exercises

1. Let $f(x) = \frac{14x+6}{5x-7}$

   When $x \to +\infty \implies y \to 2.8$

   Use calculator to find larger and larger function values:



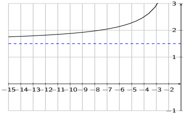
x	y
100	2.851...
1000	2.805...
10000	2.800...
100000	2.800...

$$\lim_{x\to+\infty} f(x) = 2.8 = \frac{14}{5}$$

2. Let $f(x) = \frac{3x^2-17x+4}{2x^2-6x-8}$

   When $x \to -\infty \implies y \to 1.5$

   Use calculator to find smaller and smaller function values:



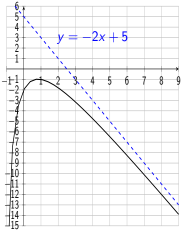
x	y
−100	1.539...
−1000	1.503...
−10000	1.500...
−100000	1.500...

$$\lim_{x\to-\infty} f(x) = 1.5 = \frac{3}{2}$$

3. Let $f(x) = \frac{-6x^2+7x-8}{3x+4}$

   When $x \to +\infty \implies y \to -\infty$

   Use calculator to find larger and larger function values:



x	y
100	-195.1...
1000	-1995.0...
10000	-19995.0...
100000	-199995.0...

$$\lim_{x\to+\infty} f(x) = -\infty$$

Note that we can compute the slant asymptote via a long division:

$$f(x) = -2x + 5 + \frac{-28}{3x+4}$$

$\implies$ slant asymptote is $y = -2x + 5$