Rational functions

Lesson #10

MAT 1375 Precalculus

New York City College of Technology CUNY

Rational function, domain, VA, hole

A **rational function** is a function $f(x) = \frac{p(x)}{q(x)}$ where $p(x)$ and $q(x)$ are polynomials.

Example

Graph $f(x) = \frac{x+1}{(x+2)(x-4)}$

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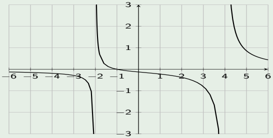

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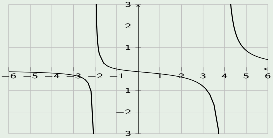

Domain: $D = (-\infty, -2) \cup (-2, 4) \cup (4, \infty)$

Domain, vertical asymptotes, holes

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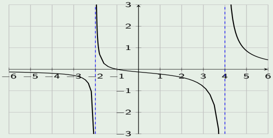

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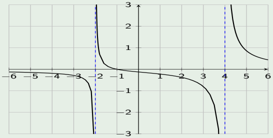

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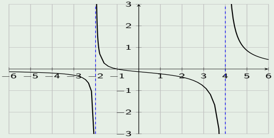

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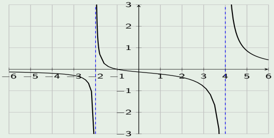

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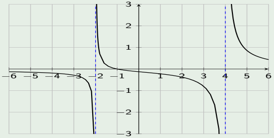

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Domain: $D = (-\infty, 3) \cup (3, 5) \cup (5, \infty)$
vertical asymptote:

Domain, vertical asymptotes, holes

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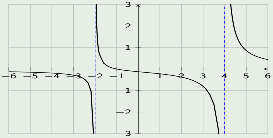

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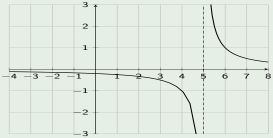

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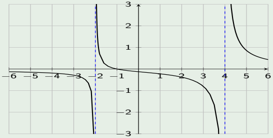

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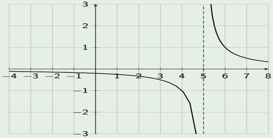

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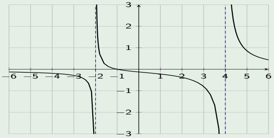

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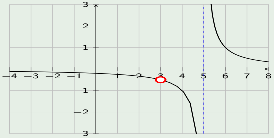

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hole: $x = 3$

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Domain: $D = \{x : q(x) \neq 0\}$
Vertical asymptotes (VA): $x = c$ for roots c of $q(x)$
Hole or removable singularity:
If the factors $(x - c)$ of $q(x)$ cancel with factors of $p(x)$, then $x = c$ is a hole.

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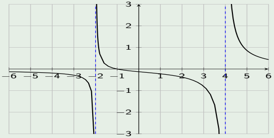

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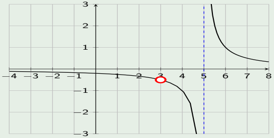

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Domain, vertical asymptotes, holes - exercises

Find the domain, vertical asymptotes, and holes of the rational function.

1. $f(x) = \frac{x+2}{(x-1)(x+4)}$
   Domain:
   VA:
   holes:

2. $f(x) = \frac{3}{x^2-4}$
   Domain:
   VA:
   holes:

3. $f(x) = \frac{2x^2+4x}{x^2+5x+6}$
   Domain:
   VA:
   holes:

4. $f(x) = \frac{x^2-7x+12}{x^2-x-12}$
   Domain:
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   holes:

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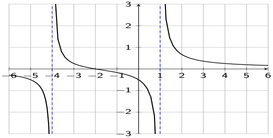
   holes: no holes



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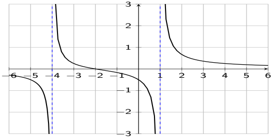

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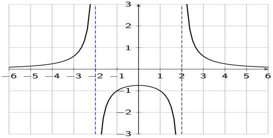

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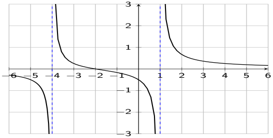

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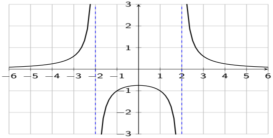

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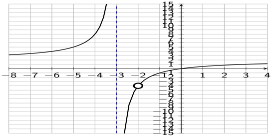

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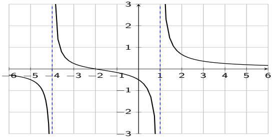

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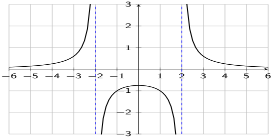

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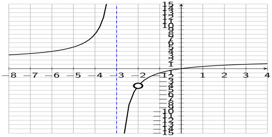

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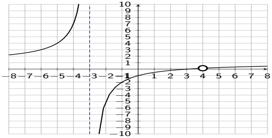

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$y = 0 \quad \Rightarrow f(x) = 0 \quad \Rightarrow \frac{p(x)}{q(x)} = 0 \quad \Rightarrow p(x) = 0 \qquad$ (but $q(x) \neq 0$)

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x-intercepts, y-intercept, horizontal asymptote

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Horizontal asymptote of $f(x) = \frac{p(x)}{q(x)}$

$f(x) = \frac{2x-3}{2x^2-4x-6}$	$f(x) = \frac{6x^2-7x+1}{2x^2-4x-6}$	$f(x) = \frac{2x^3-3x^2+4x-3}{2x^2-4x-6}$

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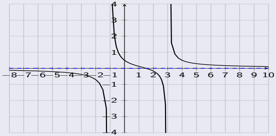		

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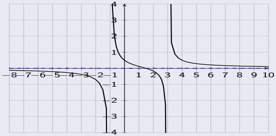	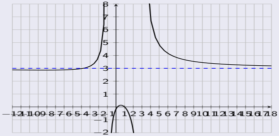	

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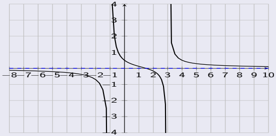
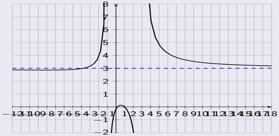
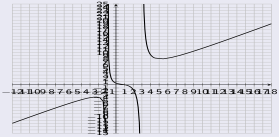
		

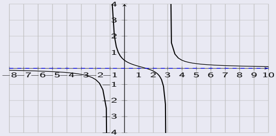
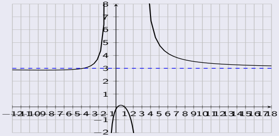
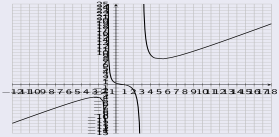
x-intercept(s)

$y = 0 \quad \Rightarrow f(x) = 0 \quad \Rightarrow \frac{p(x)}{q(x)} = 0 \quad \Rightarrow p(x) = 0 \qquad$ (but $q(x) \neq 0$)

y-intercept

$x = 0 \quad \Rightarrow y = f(0)$

Horizontal asymptote of $f(x) = \frac{p(x)}{q(x)}$

$f(x) = \frac{2x-3}{2x^2-4x-6}$	$f(x) = \frac{6x^2-7x+1}{2x^2-4x-6}$	$f(x) = \frac{2x^3-3x^2+4x-3}{2x^2-4x-6}$
		
q has higher degree HA: $y = 0$		

x-intercepts, y-intercept, horizontal asymptote

x-intercept(s)

$y = 0 \quad \Rightarrow f(x) = 0 \quad \Rightarrow \frac{p(x)}{q(x)} = 0 \quad \Rightarrow p(x) = 0 \qquad$ (but $q(x) \neq 0$)

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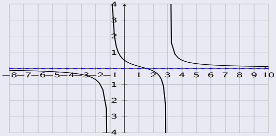	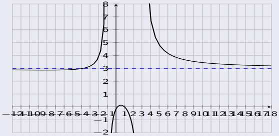	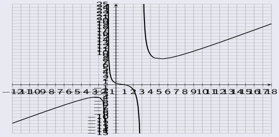
q has higher degree	p and q have same degree	
	$f(x) = \frac{ax^n + \ldots(\text{lower terms})\ldots}{bx^n + \ldots(\text{lower terms})\ldots}$	
HA: $\quad y = 0$	HA: $\quad y = \frac{a}{b}$	

x-intercept(s)

$y = 0 \quad \Rightarrow f(x) = 0 \quad \Rightarrow \frac{p(x)}{q(x)} = 0 \quad \Rightarrow p(x) = 0 \qquad$ (but $q(x) \neq 0$)

y-intercept

$x = 0 \quad \Rightarrow y = f(0)$

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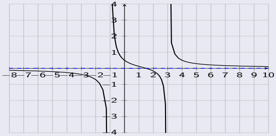
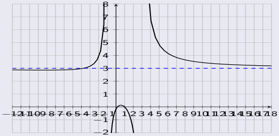
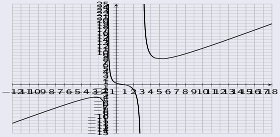
$f(x) = \frac{2x-3}{2x^2-4x-6}$	$f(x) = \frac{6x^2-7x+1}{2x^2-4x-6}$	$f(x) = \frac{2x^3-3x^2+4x-3}{2x^2-4x-6}$
		
q has higher degree	p and q have same degree $f(x) = \frac{ax^n + \ldots\text{(lower terms)}\ldots}{bx^n + \ldots\text{(lower terms)}\ldots}$	p has higher degree
HA: $y = 0$	HA: $y = \frac{a}{b}$	HA: there is no HA

Find the horizontal asymptotes, x-intercepts, and y-intercept of the rational function.

1. $f(x) = \frac{4x^2-4}{8x^2-12x+3}$
   x-int.:
   y-int.:
   HA:

2. $f(x) = \frac{x^2+5x-6}{18-4x^2}$
   x-int.:
   y-int.:
   HA:

3. $f(x) = \frac{3x+7}{x^2-2x-5}$
   x-int.:
   y-int.:
   HA:

4. $f(x) = \frac{x^4-16}{3x-4}$
   x-int.:
   y-int.:
   HA:

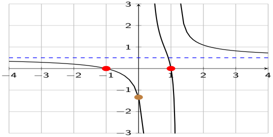
Find the horizontal asymptotes, x-intercepts, and y-intercept of the rational function.

1. $f(x) = \frac{4x^2-4}{8x^2-12x+3}$

   x-int.: $4x^2 - 4 = 0 \Rightarrow 4(x+1)(x-1) = 0 \Rightarrow x = \pm 1$

   y-int.: $y = f(0) = \frac{-4}{3}$

   HA: $y = \frac{4}{8} = \frac{1}{2}$

2. $f(x) = \frac{x^2+5x-6}{18-4x^2}$

   x-int.:

   y-int.:

   HA:

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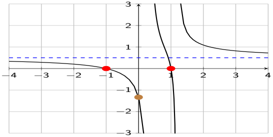


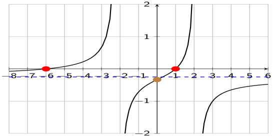
2. $f(x) = \frac{x^2+5x-6}{18-4x^2}$

   x-int.: $x^2 + 5x - 6 = 0 \Rightarrow (x-1)(x+6) = 0$

   $\Rightarrow x = 1, x = -6$

   y-int.: $y = f(0) = \frac{-6}{18} = -\frac{1}{3}$

   HA: $y = \frac{1}{-4}$



3. $f(x) = \frac{3x+7}{x^2-2x-5}$

   x-int.:

   y-int.:

   HA:

4. $f(x) = \frac{x^4-16}{3x-4}$

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Find the horizontal asymptotes, x-intercepts, and y-intercept of the rational function.

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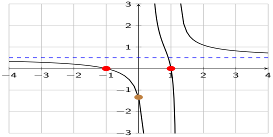

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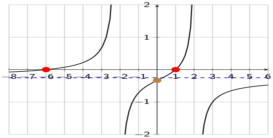
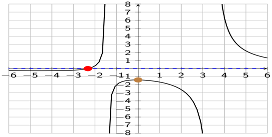


3. $f(x) = \frac{3x+7}{x^2-2x-5}$

   x-int.: $3x + 7 = 0 \quad \Rightarrow x = -\frac{7}{3}$

   y-int.: $y = f(0) = \frac{7}{-5}$

   HA: $y = 0$



4. $f(x) = \frac{x^4-16}{3x-4}$

   x-int.:

   y-int.:

   HA:

Find the horizontal asymptotes, x-intercepts, and y-intercept of the rational function.

1. $f(x) = \frac{4x^2-4}{8x^2-12x+3}$

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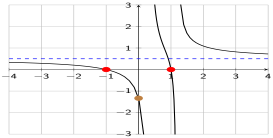

2. $f(x) = \frac{x^2+5x-6}{18-4x^2}$

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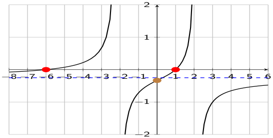
   y-int.: $y = f(0) = \frac{-6}{18} = -\frac{1}{3}$

   HA: $y = \frac{1}{-4}$



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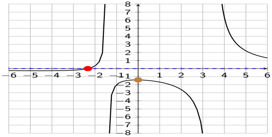
   HA: $y = 0$

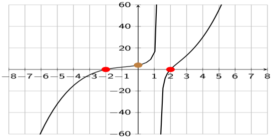


4. $f(x) = \frac{x^4-16}{3x-4}$

   x-int.: $x^4 - 16 = 0 \quad \Rightarrow x^4 = 16 \quad \Rightarrow x = \pm\sqrt[4]{16} = \pm 2$

   y-int.: $y = f(0) = \frac{-16}{-4} = 4$

   HA: no HA



Rational function - exercises

Find the domain, VA, holes, x-intercepts, y-intercept, and HA of the rational function.

1. $f(x) = \frac{2x-6}{x^2-3x-10}$

   Domain:
   VA:
   holes:
   x-int.:
   y-int.:
   HA:

2. $f(x) = \frac{2x^2+2x-4}{x^2+x-12}$

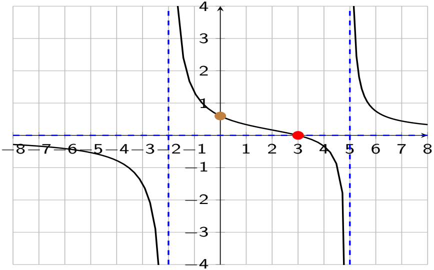
   Domain:
   VA:
   holes:
   x-int.:
   y-int.:
   HA:

Rational function - exercises

Find the domain, VA, holes, x-intercepts, y-intercept, and HA of the rational function.

1. $f(x) = \frac{2x-6}{x^2-3x-10} = \frac{2(x-3)}{(x-5)(x+2)}$

   Domain: $D = (-\infty, -2) \cup (-2, 5) \cup (5, \infty)$
   VA: $x = -2, x = 5$
   holes: no holes
   x-int.: $2x - 6 = 0 \Rightarrow 2(x-3) = 0 \Rightarrow x = 3$
   y-int.: $y = f(0) = \frac{-6}{-10} = \frac{3}{5}$
   HA: $y = 0$



2. $f(x) = \frac{2x^2+2x-4}{x^2+x-12}$

   Domain:
   VA:
   holes:
   x-int.:
   y-int.:
   HA:

Find the domain, VA, holes, x-intercepts, y-intercept, and HA of the rational function.

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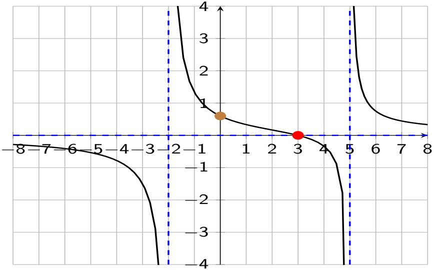
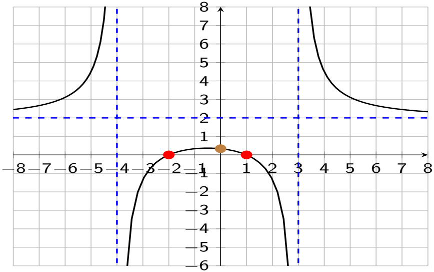
   VA: $x = -2, x = 5$
   holes: no holes
   x-int.: $2x - 6 = 0 \Rightarrow 2(x-3) = 0 \Rightarrow x = 3$
   y-int.: $y = f(0) = \frac{-6}{-10} = \frac{3}{5}$
   HA: $y = 0$



2. $f(x) = \frac{2x^2+2x-4}{x^2+x-12} = \frac{2(x-1)(x+2)}{(x-3)(x+4)}$

   Domain: $D = (-\infty, -4) \cup (-4, 3) \cup (3, \infty)$
   VA: $x = -4, x = 3$
   holes: no holes
   x-int.: $2(x-1)(x+2) = 0 \Rightarrow x = -2, x = 1$
   y-int.: $y = f(0) = \frac{-4}{-12} = \frac{1}{3}$
   HA: $y = \frac{2}{1} = 2$



Rational function - exercises

Find the domain, VA, holes, x-intercepts, y-intercept, and HA of the rational function.

3. $f(x) = \frac{x^2-4x-5}{x^2+2x-8}$

   Domain:
   VA:
   holes:
   x-int.:
   y-int.:
   HA:

4. $f(x) = \frac{6x^2-24x+18}{2x^2-4x-6}$

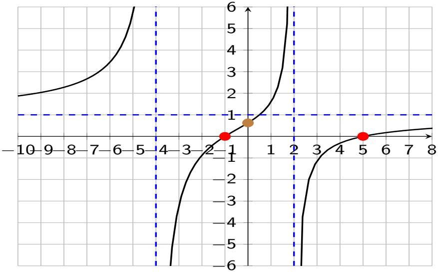
   Domain:
   VA:
   holes:
   x-int.:

   y-int.:
   HA:

Find the domain, VA, holes, x-intercepts, y-intercept, and HA of the rational function.

3. $f(x) = \frac{x^2-4x-5}{x^2+2x-8} = \frac{(x-5)(x+1)}{(x-2)(x+4)}$

Domain: $D = (-\infty, -4) \cup (-4, 2) \cup (2, \infty)$
VA: $x = -4, x = 2$
holes: no holes
x-int.: $(x-5)(x+1) = 0 \quad \Rightarrow x = 5, x = -1$
y-int.: $y = f(0) = \frac{-5}{-8} = \frac{5}{8}$
HA: $y = \frac{1}{1} = 1$



4. $f(x) = \frac{6x^2-24x+18}{2x^2-4x-6}$

Domain:
VA:
holes:
x-int.:

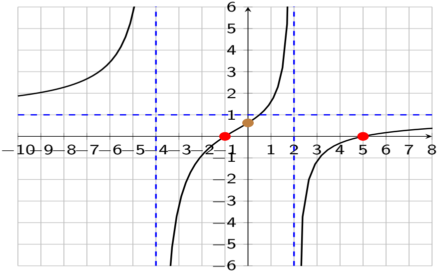
y-int.:
HA:

Rational function - exercises

Find the domain, VA, holes, x-intercepts, y-intercept, and HA of the rational function.

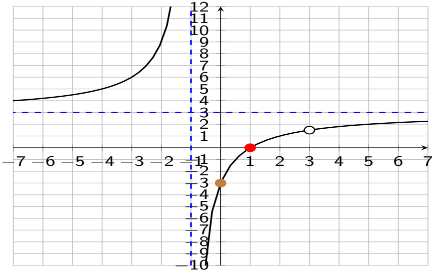
3. $f(x) = \frac{x^2-4x-5}{x^2+2x-8} = \frac{(x-5)(x+1)}{(x-2)(x+4)}$

Domain:	$D = (-\infty, -4) \cup (-4, 2) \cup (2, \infty)$
VA:	$x = -4, x = 2$
holes:	no holes
x-int.:	$(x-5)(x+1) = 0 \quad \Rightarrow x = 5, x = -1$
y-int.:	$y = f(0) = \frac{-5}{-8} = \frac{5}{8}$
HA:	$y = \frac{1}{1} = 1$



4. $f(x) = \frac{6x^2-24x+18}{2x^2-4x-6} = \frac{6(x-3)(x-1)}{2(x-3)(x+1)}$

Domain:	$D = (-\infty, -1) \cup (-1, 3) \cup (3, \infty)$
VA:	$x = -1$
holes:	$x = 3$
x-int.:	$6x^2 - 24x + 18 = 0 \Rightarrow 6(x-3)(x-1) = 0$
	$\Rightarrow x = 3$ ✗ $\qquad x = 1$ ✓
y-int.:	$y = f(0) = \frac{18}{-6} = -3$
HA:	$y = \frac{6}{2} = 3$



Find the domain, VA, holes, x-intercepts, y-intercept, and HA of the rational function.

5. $f(x) = \frac{x^2-3x-4}{x-1}$

   Domain:
   VA:
   holes:
   x-int.:
   y-int.:
   HA:

6. $f(x) = \frac{x+2}{x^3-4x^2}$

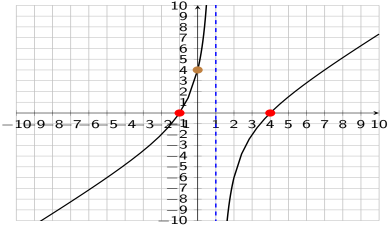
   Domain:
   VA:
   holes:
   x-int.:
   y-int.:

   HA:

Find the domain, VA, holes, x-intercepts, y-intercept, and HA of the rational function.

5. $f(x) = \frac{x^2-3x-4}{x-1} = \frac{(x-4)(x+1)}{x-1}$

Domain: $D = (-\infty, 1) \cup (1, \infty)$
VA: $x = 1$
holes: no holes
x-int.: $(x-4)(x+1) = 0 \Rightarrow x = 4, x = -1$
y-int.: $y = f(0) = \frac{-4}{-1} = 4$
HA: no HA



6. $f(x) = \frac{x+2}{x^3-4x^2}$

Domain:
VA:
holes:
x-int.:
y-int.:

HA:

Find the domain, VA, holes, x-intercepts, y-intercept, and HA of the rational function.

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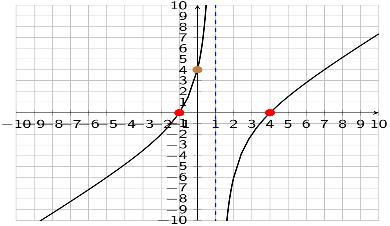
HA: no HA



Long division: $\frac{x^2-3x-4}{x-1} = x - 2 + \frac{-6}{x-1}$

6. $f(x) = \frac{x+2}{x^3-4x^2}$

Domain:
VA:
holes:
x-int.:
y-int.:

HA:

Find the domain, VA, holes, x-intercepts, y-intercept, and HA of the rational function.

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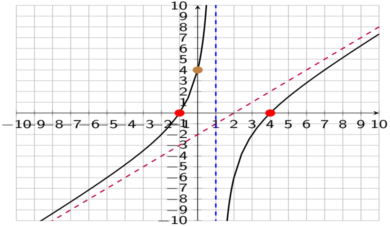
Domain: $D = (-\infty, 1) \cup (1, \infty)$
VA: $x = 1$
holes: no holes
x-int.: $(x-4)(x+1) = 0 \Rightarrow x = 4, x = -1$
y-int.: $y = f(0) = \frac{-4}{-1} = 4$
HA: no HA



Long division: $\frac{x^2-3x-4}{x-1} = x - 2 + \frac{-6}{x-1}$

$\Rightarrow$ slant asymptote: $y = x - 2$

6. $f(x) = \frac{x+2}{x^3-4x^2}$

Domain:
VA:
holes:
x-int.:
y-int.:

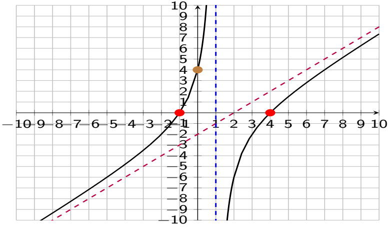
HA:

Rational function - exercises

Find the domain, VA, holes, x-intercepts, y-intercept, and HA of the rational function.

5. $f(x) = \frac{x^2-3x-4}{x-1} = \frac{(x-4)(x+1)}{x-1}$

Domain: $D = (-\infty, 1) \cup (1, \infty)$
VA: $x = 1$
holes: no holes
x-int.: $(x-4)(x+1) = 0 \Rightarrow x = 4, x = -1$
y-int.: $y = f(0) = \frac{-4}{-1} = 4$
HA: no HA

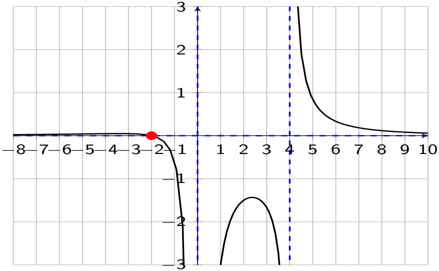


Long division: $\frac{x^2-3x-4}{x-1} = x - 2 + \frac{-6}{x-1}$

$\Rightarrow$ slant asymptote: $y = x - 2$

6. $f(x) = \frac{x+2}{x^3-4x^2} = \frac{x+2}{x^2(x-4)}$

Domain: $D = (-\infty, 0) \cup (0, 4) \cup (4, \infty)$
VA: $x = 0, x = 4$
holes: no holes
x-int.: $x + 2 = 0 \quad \Rightarrow x = -2$
y-int.: $y = f(0) = \frac{2}{0}$ is undefined ✗
$\Rightarrow$ no y-intercept
HA: $y = 0$



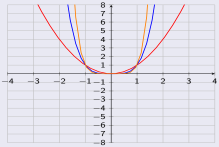
Graphs of $f(x) = x^n$ and $f(x) = \frac{1}{x^n}$

Explore the graphs $y = x^n$ and $y = \frac{1}{x^n}$ with desmos with n even or odd natural numbers.

Graphs of $f(x) = x^n$ and $f(x) = \frac{1}{x^n}$

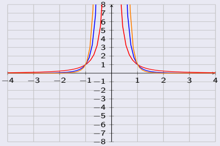
Explore the graphs $y = x^n$ and $y = \frac{1}{x^n}$ with desmos with n even or odd natural numbers.

$y = x^n$ with n even



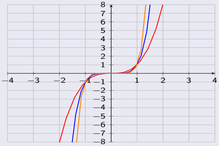
$y = x^2$
$y = x^4$
$y = x^6$

$y = \frac{1}{x^n}$ with n even



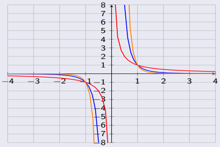
$y = \frac{1}{x^2}$
$y = \frac{1}{x^4}$
$y = \frac{1}{x^6}$

$y = x^n$ with n odd



$y = x^3$
$y = x^5$
$y = x^7$

$y = \frac{1}{x^n}$ with n odd



$y = \frac{1}{x}$
$y = \frac{1}{x^3}$
$y = \frac{1}{x^5}$