Roots of polynomials

Lesson #9

MAT 1375 Precalculus

New York City College of Technology CUNY

The fundamental theorem of algebra

Theorem (The fundamental theorem of algebra)

Every polynomial, which is not constant, has a root.

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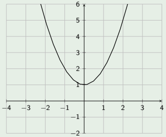

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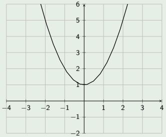

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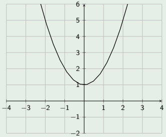

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$$= \frac{-0 \pm \sqrt{0^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1} = \frac{\pm\sqrt{-4}}{2}$$
$$= \frac{\pm\sqrt{4}\sqrt{-1}}{2} = \frac{\pm 2i}{2} = \pm i$$

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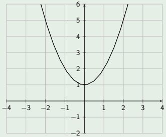

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f has roots i and $-i$
f has factors
$(x - i)$ and $(x + i)$
Check:
$(x + i) \cdot (x - i) =$

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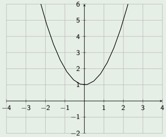

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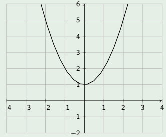

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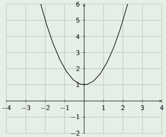

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Fact: If the polynomial f with real coefficients has the complex root $c = a + bi$, then the complex conjugate $\bar{c} = a - bi$ is also a root!

Factoring completely

Factor polynomials completely

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Example:
$f(x) = (x-2)^3 \cdot (x+4)^2$
root 2 has multiplicity 3
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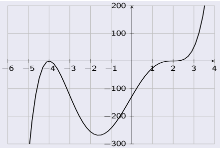

Find a polynomial that fits the given data.

1. degree 3
   roots $2, -3, 1$

2. degree 3
   roots $-1, +1, -4$

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Polynomials theory - exercises

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   Therefore, $-6a = 24 \quad \Rightarrow a = -4$
   $$\begin{aligned} f(x) &= (-4)\cdot x(x+1)\cdot(x-2)(x+2) \\ &= (-4x^2-4x)\cdot(x^2-4) \\ &= -4x^4-4x^3+16x^2+16x \end{aligned}$$

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5. degree 2
   roots $2 + 3i$, $2 - 3i$

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Find a polynomial that fits the given data.

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Factor completely - exercises

1. Find all roots and find all factors of $f(x) = x^3 - 5x^2 - 7x + 51$

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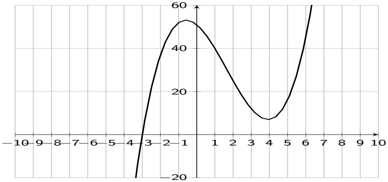

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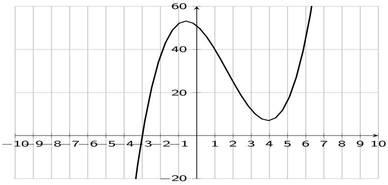

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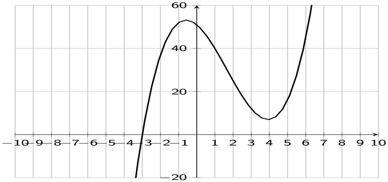

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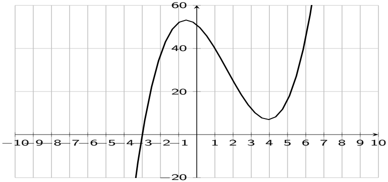

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$x^3 - 5x^2 - 7x + 51 = (x^2 - 8x + 17) \cdot (x + 3)$

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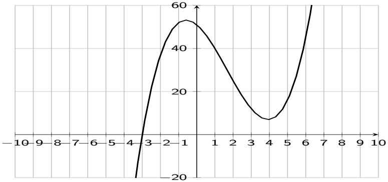

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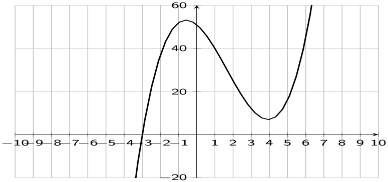

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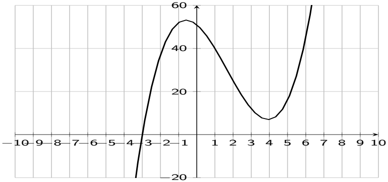

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Factor completely - exercises

2. a) Find a real number C so that the polynomial

$$f(x) = 4x^3 - 12x^2 + 5x + C$$

has a root at $x = 2$.

b) For this C, find all remaining roots of the polynomial algebraically and factor the polynomial completely.

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