Graphing polynomials

Lesson #8

MAT 1375 Precalculus

New York City College of Technology CUNY

Factor completely - exercise

1. Find all roots and find all factors of $f(x) = 3x^3 + 4x^2 - 5x - 2$

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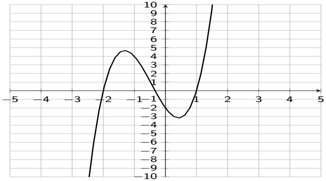

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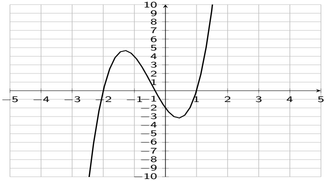

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Therefore: divide $f(x)$ by

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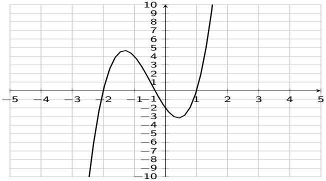

Observe: 1 and -2 are roots of $f(x)$.
Therefore: divide $f(x)$ by either $(x+2)$ or $(x-1)$ (choose one)

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$$x+2 \,\overline{\big)\; 3x^3 \quad +4x^2 \quad -5x \quad -2}$$

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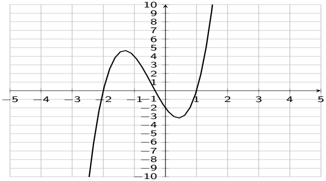

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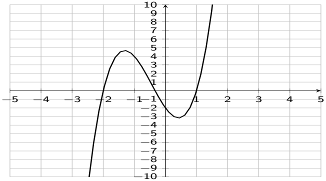

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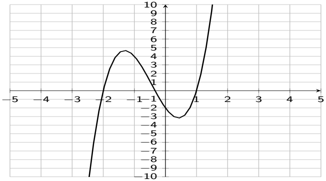

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     $3x^2 - 2x - 1 = (3x + 1)(x - 1)$

     $\Rightarrow x^3 + 5x^2 - x - 14 =$

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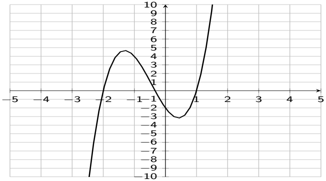

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$$\Rightarrow x^3+5x^2-x-14 = (3x^2-2x-1)(x+2) = (3x+1)(x-1)(x+2) = 3\left(x+\frac{1}{3}\right)(x-1)(x+2)$$

   - Solution: Factors: $(x+\frac{1}{3})$, $(x-1)$, $(x+2)$, Roots: $-\frac{1}{3}$, 1, -2

Graphs of polynomials

degree 1	degree 2	degree 3	degree 4	degree 5
$y = ax + b$	$y = ax^2 + bx + c$	$y = ax^3 + bx^2 + cx + d$	$y = ax^4 + bx^3 + cx^2$ $+dx + e$	$y = ax^5 + bx^4 + cx^3$ $+dx^2 + ex + f$

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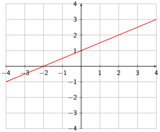 $a > 0$				
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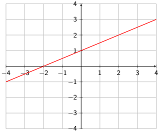
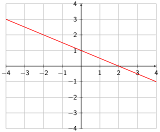
 $a > 0$				
 $a < 0$				

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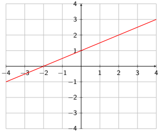
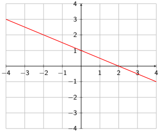
 $a > 0$				
 $a < 0$				
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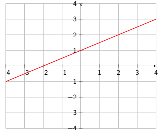
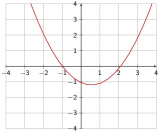
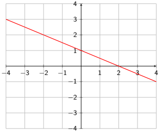
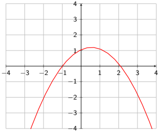
 $a > 0$	 $a > 0$			
 $a < 0$	 $a < 0$			
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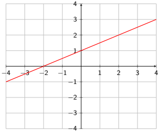
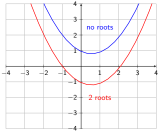
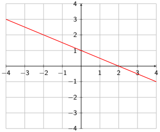
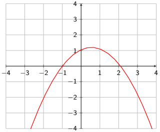
 $a > 0$	 $a > 0$			
 $a < 0$	 $a < 0$			
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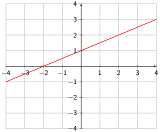
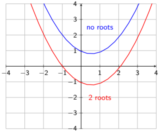
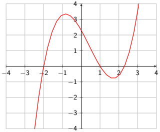
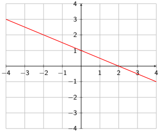
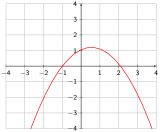
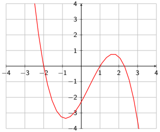
 $a > 0$	 $a > 0$	 $a > 0$		
 $a < 0$	 $a < 0$	 $a < 0$		
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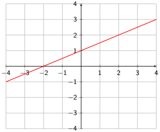
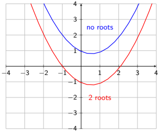
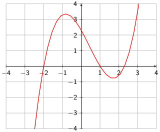
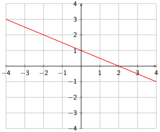
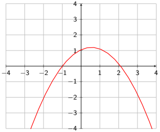
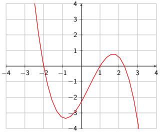
 $a > 0$	 $a > 0$	 $a > 0$		
 $a < 0$	 $a < 0$	 $a < 0$		
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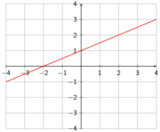
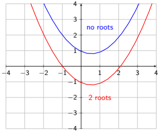
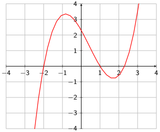
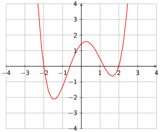
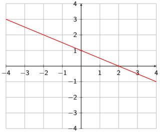
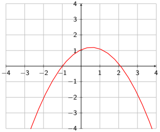
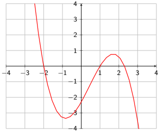
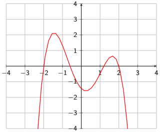
 $a > 0$	 $a > 0$	 $a > 0$	 $a > 0$	
 $a < 0$	 $a < 0$	 $a < 0$	 $a < 0$	
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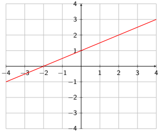
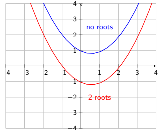
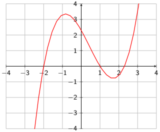
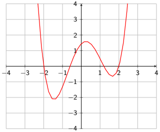
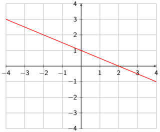
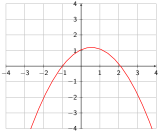
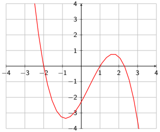
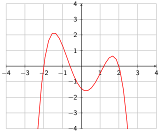
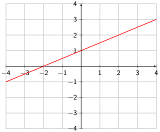
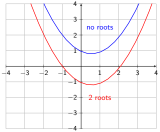
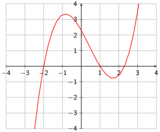
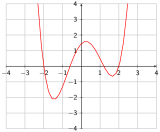
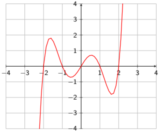
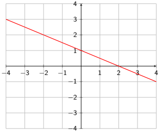
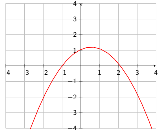
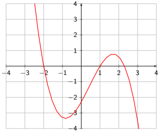
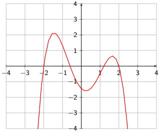
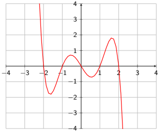
 $a > 0$	 $a > 0$	 $a > 0$	 $a > 0$	
 $a < 0$	 $a < 0$	 $a < 0$	 $a < 0$	
1 real root	at most 2 real root	at most 3 real root	at most 4 real root	
no max. or min.	1 max. or min.	at most 2 max. or min.	at most 3 max. or min.	

Graphs of polynomials

degree 1	degree 2	degree 3	degree 4	degree 5
$y = ax + b$	$y = ax^2 + bx + c$	$y = ax^3 + bx^2 + cx + d$	$y = ax^4 + bx^3 + cx^2$ $+dx + e$	$y = ax^5 + bx^4 + cx^3$ $+dx^2 + ex + f$
				
$a > 0$	$a > 0$	$a > 0$	$a > 0$	$a > 0$
				
$a < 0$	$a < 0$	$a < 0$	$a < 0$	$a < 0$
1 real root	at most 2 real root	at most 3 real root	at most 4 real root	
no max. or min.	1 max. or min.	at most 2 max. or min.	at most 3 max. or min.	

Graphs of polynomials

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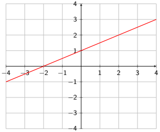
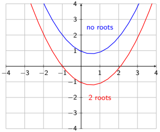
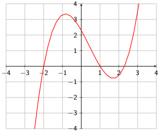
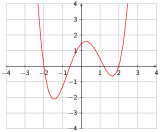
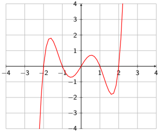
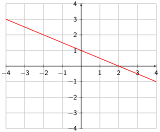
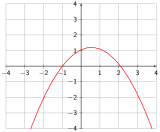
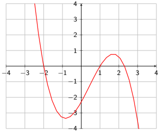
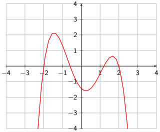
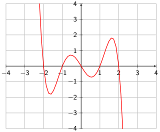
 $a > 0$	 $a > 0$	 $a > 0$	 $a > 0$	 $a > 0$
 $a < 0$	 $a < 0$	 $a < 0$	 $a < 0$	 $a < 0$
1 real root	at most 2 real root	at most 3 real root	at most 4 real root	at most 5 real root
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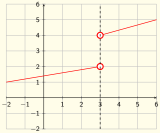
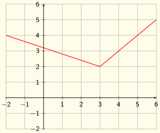
Polynomials do ***NOT*** have the following:

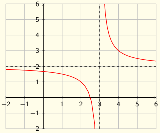
discontinuity:



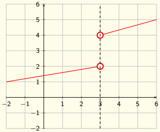
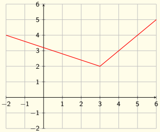
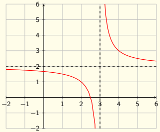
corner:



asymptotes:

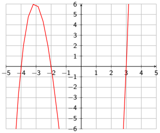
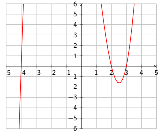
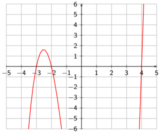


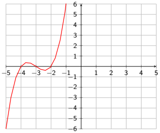
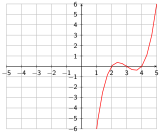
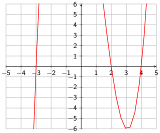
Polynomials do ***NOT*** have the following:

discontinuity:  corner:  asymptotes: 

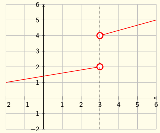
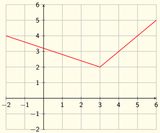
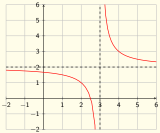
1. Match the functions with their graphs:

$$f(x) = (x-2)(x-3)(x+4) \qquad h(x) = (x-2)(x-3)(x-4)$$
$$g(x) = (x+2)(x+3)(x+4) \qquad k(x) = (x+2)(x-3)(x+4)$$

(A)  (C)  (E) 

(B)  (D)  (F) 

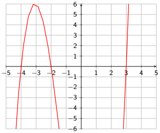
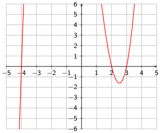
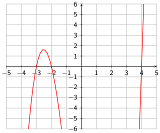
Polynomials do ***NOT*** have the following:

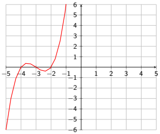
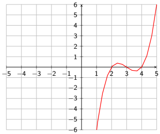
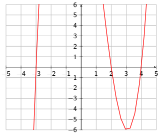
discontinuity:  corner:  asymptotes: 

1. Match the functions with their graphs:

$(C)\quad f(x) = (x-2)(x-3)(x+4)$ $\qquad (D)\quad h(x) = (x-2)(x-3)(x-4)$

$(B)\quad g(x) = (x+2)(x+3)(x+4)$ $\qquad (A)\quad k(x) = (x+2)(x-3)(x+4)$

(A)  (C)  (E) 

(B)  (D)  (F) 

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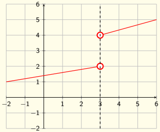
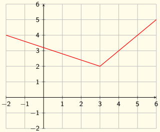
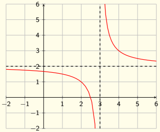
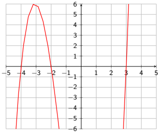
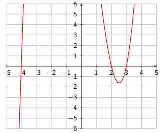
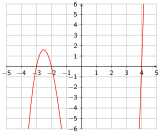
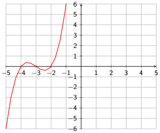
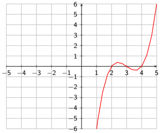
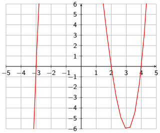
discontinuity:  corner:  asymptotes: 

1. Match the functions with their graphs:

   (C) $f(x) = (x-2)(x-3)(x+4)$ (D) $h(x) = (x-2)(x-3)(x-4)$

   (B) $g(x) = (x+2)(x+3)(x+4)$ (A) $k(x) = (x+2)(x-3)(x+4)$

(A)  (C)  (E) 

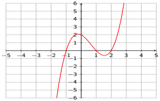
(B)  (D)  (F) 

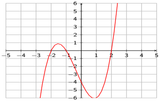
Notation: $f(x) = (x-2)(x-3)(x+4)$ has **factors**: $(x-2)$ and $(x-3)$ and $(x+4)$
and has **roots**: 2 and 3 and -4.

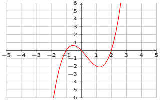
Match the functions with their graphs:

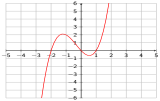
2. $f(x) = (x+1)(x-2)x$ $\quad$ $h(x) = (x-1)(x+1)x$

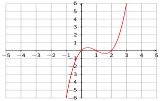
   $g(x) = (x-2)(x-1)x$ $\quad$ $k(x) = (x+1)(x-2)(x-1)$

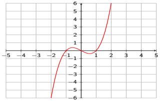
(A) 

(C) 

(E) 

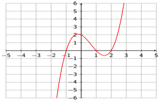
(B) 

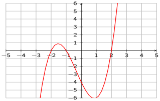
(D) 

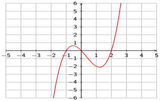
(F) 

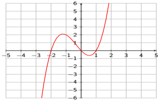
Match the functions with their graphs:

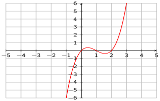
2. (E) $f(x) = (x+1)(x-2)x$ (F) $h(x) = (x-1)(x+1)x$
   (D) $g(x) = (x-2)(x-1)x$ (A) $k(x) = (x+1)(x-2)(x-1)$

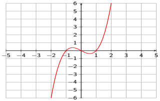
(A) 

(C) 

(E) 

(B) 

(D) 

(F) 

Graphs of polynomials - exercises

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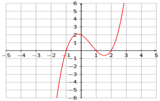
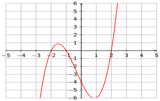
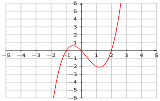
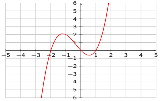
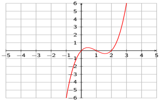
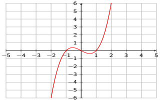
(A)  (C)  (E) 

(B)  (D)  (F) 

3. $f(x) = (x-1)^2(x+2)$ $h(x) = (x-2)^2(x+1)$
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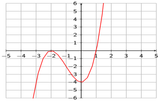
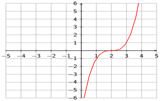
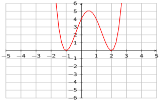
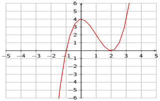
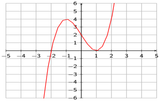
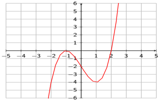
(A)  (C)  (E) 

(B)  (D)  (F) 

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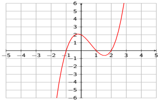
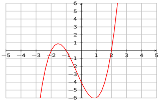
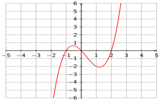
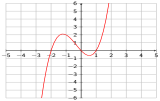
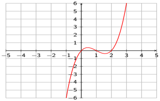
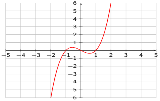
(A) 

(C) 

(E) 

(B) 

(D) 

(F) 

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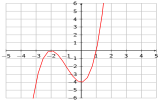
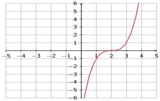
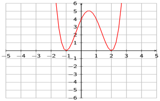
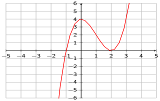
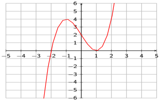
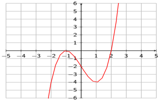
(A) 

(C) 

(E) 

(B) 

(D) 

(F) 

Graphs of polynomials - exercises

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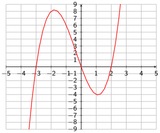
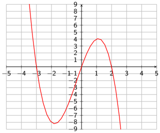
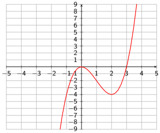
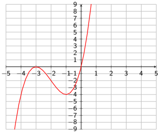
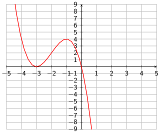
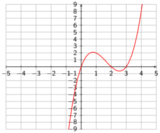
(A)  (C)  (E) 

(B)  (D)  (F) 

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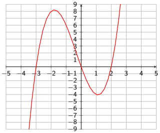
(A)


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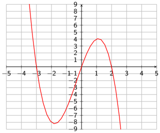

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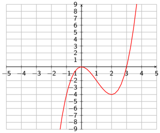

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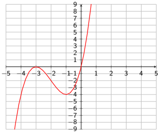

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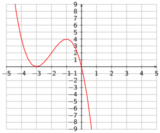

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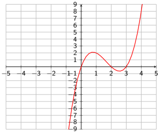

Factor the functions f, g, h, k:

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Graphs of polynomials - exercises

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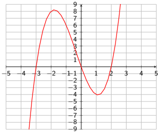
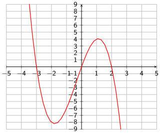
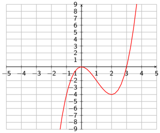
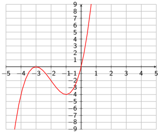
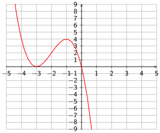
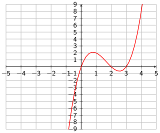
(A) 

(C) 

(E) 

(B) 

(D) 

(F) 

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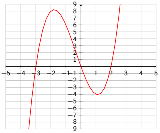

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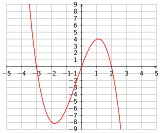

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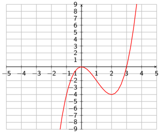

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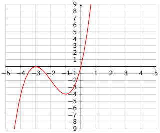
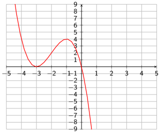


(D)


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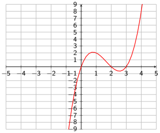

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Integer roots of polynomials

Example

- The polynomial $f(x) = (x + 3) \cdot (2x^2 + 4x + 5)$ has the *integer* root -3.
- Multiply this: $f(x) = 2x^3 + 6x^2 + 4x^2 + 12x + 5x + 15 = 2x^3 + 10x^2 + 17x + 15$.

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List all possible integer roots of the polynomial according to the above theorem.

1. $f(x) = x^3 + 2x^2 - 4x + 7$
2. $f(x) = x^3 - x^2 + 12$
3. $f(x) = 3x^5 + 23x^4 - 58x - 8$

Integer roots of polynomials

Example

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1. $f(x) = x^3+2x^2-4x+7$
   Answer:
   $+1, -1, +7, -7$
2. $f(x) = x^3-x^2+12$
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Integer roots of polynomials

Example

- The polynomial $f(x) = (x+3)\cdot(2x^2+4x+5)$ has the *integer* root -3.
- Multiply this: $f(x) = 2x^3+6x^2+4x^2+12x+5x+15 = 2x^3+10x^2+17x+15$.
- Indeed all integer roots of $f(x)$ have to be factors of the last coefficient 15.
- The possibilities are:

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