Dividing polynomials

Lesson #7

MAT 1375 Precalculus

New York City College of Technology CUNY

Long division

- Recall long division of integers.
  Divide: $\frac{3571}{11}$

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$$11 \overline{\big)\;3571}$$

Long division

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$$\begin{array}{r|l} & 3 \\ \hline 11 & 3571 \\ & \underline{-33} \end{array}$$

Long division

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  Divide: $\frac{3571}{11}$

$$\begin{array}{r|l} & 3 \\ \hline 11 & 3571 \\ & -33 \\ \cline{2-2} & 27 \end{array}$$

Long division

- Recall long division of integers.
  Divide: $\frac{3571}{11}$

$$
\begin{array}{r|l}
 & 32 \\
\hline
11 & 3571 \\
 & \underline{-33} \\
 & 27 \\
 & \underline{-22}
\end{array}
$$

- Recall long division of integers.
  Divide: $\frac{3571}{11}$

$$\begin{array}{r|l} & 32 \\ \hline 11 & 3571 \\ & \underline{-33} \\ & 27 \\ & \underline{-22} \\ & 51 \end{array}$$

- Recall long division of integers.
  Divide: $\frac{3571}{11}$

$$
\begin{array}{r|l}
 & 324 \\ \hline
11 & 3571 \\
 & \underline{-33} \\
 & 27 \\
 & \underline{-22} \\
 & 51 \\
 & \underline{-44}
\end{array}
$$

Long division

- Recall long division of integers.
  Divide: $\frac{3571}{11} = \frac{\text{dividend}}{\text{divisor}}$

$$
\begin{array}{r|l}
 & \quad 324 \quad = \text{quotient} \\
\hline
11 & \quad 3571 \\
 & -33 \\
\hline
 & \quad 27 \\
 & -22 \\
\hline
 & \quad\; 51 \\
 & \quad -44 \\
\hline
 & \quad\quad 7 \quad = \text{remainder}
\end{array}
$$

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  Divide: $\frac{3571}{11} \quad = \frac{\text{dividend}}{\text{divisor}}$

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324 & = \text{quotient} \\
11\,\overline{)\,3571} & \\
\underline{-33} & \\
27 & \\
\underline{-22} & \\
51 & \\
\underline{-44} & \\
7 & = \text{remainder}
\end{array}
$$

Answer:

$$\frac{3571}{11} = 324 + \frac{7}{11}$$

Or rewrite without fractions:

$$3571 = 324 \cdot 11 + 7$$

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$$x+3 \;\overline{\big)\; x^3 \quad +5x^2 \quad +4x \quad +2}$$

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$$
\begin{array}{r l}
& x^2 \\
x+3 & \overline{)\ x^3 \quad +5x^2 \quad +4x \quad +2} \\
& -(x^3 \quad +3x^2)
\end{array}
$$

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$$
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 & x^2 & +2x & & \\
\hline
x+3 & x^3 \;\; +5x^2 & +4x & +2 \\
 & -(x^3 \;\; +3x^2) & & \\
\cline{2-2}
 & 2x^2 & +4x & \\
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Answer:

$$\frac{x^3+5x^2+4x+2}{x+3} = x^2 + 2x - 2 + \frac{8}{x+3}$$

Or:

$$x^3 + 5x^2 + 4x + 2 = (x^2 + 2x - 2) \cdot (x+3) + 8$$

Long division - exercises

1. Divide: $\frac{x^3-3x^2-4x+9}{x+2}$
2. Divide: $\frac{x^3-5x^2-2x+16}{x-4}$

Long division - exercises

1. Divide: $\frac{x^3-3x^2-4x+9}{x+2}$

$$
\begin{array}{r|rrrr}
 & x^2 & -5x & +6 & \\
\hline
x+2 & x^3 & -3x^2 & -4x & +9 \\
 & -(x^3 & +2x^2) & & \\
\cline{2-3}
 & & -5x^2 & -4x & \\
 & & -(-5x^2 & -10x) & \\
\cline{3-4}
 & & & 6x & +9 \\
 & & & -(6x & +12) \\
\cline{4-5}
 & & & & -3
\end{array}
$$

Answer:

$$\frac{x^3-3x^2-4x+9}{x+2} = x^2-5x+6+\frac{-3}{x+2}$$

Or:

$$x^3-3x^2-4x+9 = (x^2-5x+6)\cdot(x+2)-3$$

2. Divide: $\frac{x^3-5x^2-2x+16}{x-4}$

Long division - exercises

1. Divide: $\frac{x^3-3x^2-4x+9}{x+2}$

$$\begin{array}{r|rrrr} & x^2 & -5x & +6 & \\ \hline x+2 & x^3 & -3x^2 & -4x & +9 \\ & -(x^3 & +2x^2) & & \\ \hline & & -5x^2 & -4x & \\ & & -(-5x^2 & -10x) & \\ \hline & & & 6x & +9 \\ & & & -(6x & +12) \\ \hline & & & & -3 \end{array}$$

Answer:

$$\frac{x^3-3x^2-4x+9}{x+2} = x^2-5x+6+\frac{-3}{x+2}$$

Or:

$$x^3-3x^2-4x+9 = (x^2-5x+6)\cdot(x+2)-3$$

2. Divide: $\frac{x^3-5x^2-2x+16}{x-4}$

$$\begin{array}{r|rrrr} & x^2 & -x & -6 & \\ \hline x-4 & x^3 & -5x^2 & -2x & +16 \\ & -(x^3 & -4x^2) & & \\ \hline & & -x^2 & -2x & \\ & & -(-x^2 & +4x) & \\ \hline & & & -6x & +16 \\ & & & -(-6x & +24) \\ \hline & & & & -8 \end{array}$$

Answer:

$$\frac{x^3-5x^2-2x+16}{x-4} = x^2-x-6-\frac{8}{x-4}$$

Or:

$$x^3-5x^2-2x+16 = (x^2-x-6)\cdot(x-4)-8$$

Long division - exercises

3. Divide: $\frac{6x^3+16x^2-7x+4}{3x+2}$

4. Divide: $\frac{x^3+2x^2-5x+3}{x-2}$

Long division - exercises

3. Divide: $\frac{6x^3+16x^2-7x+4}{3x+2}$

$$
\begin{array}{r|rrrr}
 & 2x^2 & +4x & -5 & \\
\hline
3x+2 & 6x^3 & +16x^2 & -7x & +4 \\
 & -(6x^3 & +4x^2) & & \\
\cline{2-3}
 & & 12x^2 & -7x & \\
 & & -(12x^2 & +8x) & \\
\cline{3-4}
 & & & -15x & +4 \\
 & & & -(-15x & -10) \\
\cline{4-5}
 & & & & 14
\end{array}
$$

Answer:

$$\frac{6x^3+16x^2-7x+4}{3x+2} = 2x^2 + 4x - 5 + \frac{14}{3x+2}$$

Or:

$$6x^3 + 16x^2 - 7x + 4 = (2x^2 + 4x - 5) \cdot (3x + 2) + 14$$

4. Divide: $\frac{x^3+2x^2-5x+3}{x-2}$

Long division - exercises

3. Divide: $\frac{6x^3+16x^2-7x+4}{3x+2}$

$$
\begin{array}{r|rrrr}
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\hline
3x+2 & 6x^3 & +16x^2 & -7x & +4 \\
 & -(6x^3 & +4x^2) & & \\
\hline
 & & 12x^2 & -7x & \\
 & & -(12x^2 & +8x) & \\
\hline
 & & & -15x & +4 \\
 & & & -(-15x & -10) \\
\hline
 & & & & 14
\end{array}
$$

Answer:

$$\frac{6x^3+16x^2-7x+4}{3x+2} = 2x^2+4x-5+\frac{14}{3x+2}$$

Or:

$$6x^3+16x^2-7x+4 = (2x^2+4x-5)\cdot(3x+2)+14$$

4. Divide: $\frac{x^3+2x^2-5x+3}{x-2}$

$$
\begin{array}{r|rrrr}
 & x^2 & +4x & +3 & \\
\hline
x-2 & x^3 & +2x^2 & -5x & +3 \\
 & -(x^3 & -2x^2) & & \\
\hline
 & & 4x^2 & -5x & \\
 & & -(4x^2 & -8x) & \\
\hline
 & & & 3x & +3 \\
 & & & -(3x & -6) \\
\hline
 & & & & 9
\end{array}
$$

Answer:

$$\frac{x^3+2x^2-5x+3}{x-2} = x^2+4x+3+\frac{9}{x-2}$$

Or:

$$x^3+2x^2-5x+3 = (x^2+4x+3)\cdot(x-2)+9$$

From the last exercise:

$$\underbrace{x^3 + 2x^2 - 5x + 3}_{=f(x)} = (\underbrace{x^2 + 4x + 3}_{=q(x)}) \cdot (x - 2) + 9$$

$$\Rightarrow f(x) = q(x) \cdot (x - 2) + 9$$

$$\Rightarrow f(2) = q(2) \cdot (\underbrace{2 - 2}_{=0}) + 9 = 9$$

Remainder theorem and factor theorem

From the last exercise:

$$\underbrace{x^3 + 2x^2 - 5x + 3}_{=f(x)} = (\underbrace{x^2 + 4x + 3}_{=q(x)}) \cdot (x - 2) + 9$$

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Theorem (Remainder theorem)

Divide $f(x)$ by $(x - c)$ gives $f(x) = q(x) \cdot (x - c) + R$, where the remainder R is given by

$$f(c) = R$$

From the last exercise:

$$\underbrace{x^3 + 2x^2 - 5x + 3}_{=f(x)} = (\underbrace{x^2 + 4x + 3}_{=q(x)}) \cdot (x - 2) + 9$$

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Theorem (Remainder theorem)

Divide $f(x)$ by $(x - c)$ gives $f(x) = q(x) \cdot (x - c) + R$, where the remainder R is given by

$$f(c) = R$$

Find the remainder using the remainder theorem.

1. $\frac{x^3 - 4x^2 + 2x + 8}{x - 3}$

2. $\frac{x^3 + 6x^2 + 11x + 6}{x + 2}$

From the last exercise:

$$\underbrace{x^3+2x^2-5x+3}_{=f(x)} = (\underbrace{x^2+4x+3}_{=q(x)})\cdot(x-2)+9$$

$$\Rightarrow f(x) = q(x)\cdot(x-2)+9$$

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Find the remainder using the remainder theorem.

1. $\frac{x^3-4x^2+2x+8}{x-3}$ $\quad c=3$
2. $\frac{x^3+6x^2+11x+6}{x+2}$

From the last exercise:

$$\underbrace{x^3+2x^2-5x+3}_{=f(x)} = (\underbrace{x^2+4x+3}_{=q(x)})\cdot(x-2)+9$$

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Find the remainder using the remainder theorem.

1. $\frac{x^3-4x^2+2x+8}{x-3}$ $\quad c=3$

$$\begin{aligned} R = f(3) &= 3^3-4\cdot 3^2+2\cdot 3+8 \\ &= 27-4\cdot 9+6+8 \\ &= 27-36+6+8 \\ &= 5 \end{aligned}$$

2. $\frac{x^3+6x^2+11x+6}{x+2}$

From the last exercise:

$$\underbrace{x^3 + 2x^2 - 5x + 3}_{=f(x)} = (\underbrace{x^2 + 4x + 3}_{=q(x)}) \cdot (x-2) + 9$$

$$\Rightarrow f(x) = q(x) \cdot (x-2) + 9$$

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Find the remainder using the remainder theorem.

1. $\frac{x^3-4x^2+2x+8}{x-3}$ $\quad c = 3$

$$\begin{aligned} R = f(3) &= 3^3 - 4\cdot 3^2 + 2\cdot 3 + 8 \\ &= 27 - 4 \cdot 9 + 6 + 8 \\ &= 27 - 36 + 6 + 8 \\ &= 5 \end{aligned}$$

2. $\frac{x^3+6x^2+11x+6}{x+2}$ $\quad c = -2$

From the last exercise:

$$\underbrace{x^3+2x^2-5x+3}_{=f(x)} = (\underbrace{x^2+4x+3}_{=q(x)})\cdot(x-2)+9$$

$$\Rightarrow f(x) = q(x)\cdot(x-2)+9$$

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Theorem (Remainder theorem)

Divide $f(x)$ by $(x-c)$ gives $f(x) = q(x)\cdot(x-c)+R$, where the remainder R is given by

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Find the remainder using the remainder theorem.

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$$\begin{aligned} R = f(3) &= 3^3-4\cdot 3^2+2\cdot 3+8 \\ &= 27-4\cdot 9+6+8 \\ &= 27-36+6+8 \\ &= 5 \end{aligned}$$

2. $\frac{x^3+6x^2+11x+6}{x+2}$ $\quad c = -2$

$$\begin{aligned} R &= f(-2) \\ &= (-2)^3+6\cdot(-2)^2+11\cdot(-2)+6 \\ &= -8+6\cdot 4-22+6 \\ &= -8+24-22+6 \\ &= 0 \end{aligned}$$

Remainder theorem and factor theorem

From the last exercise:

$$\underbrace{x^3+2x^2-5x+3}_{=f(x)} = (\underbrace{x^2+4x+3}_{=q(x)})\cdot(x-2)+9$$

$$\Rightarrow f(x) = q(x)\cdot(x-2)+9$$
$$\Rightarrow f(2) = q(2)\cdot(\underbrace{2-2}_{=0})+9 = 9$$

Theorem (Remainder theorem)

Divide $f(x)$ by $(x-c)$ gives $f(x) = q(x)\cdot(x-c)+R$, where the remainder R is given by

$$f(c) = R$$

Find the remainder using the remainder theorem.

1. $\frac{x^3-4x^2+2x+8}{x-3}$ $\quad c = 3$

$$\begin{aligned} R = f(3) &= 3^3-4\cdot 3^2+2\cdot 3+8 \\ &= 27-4\cdot 9+6+8 \\ &= 27-36+6+8 \\ &= 5 \end{aligned}$$

2. $\frac{x^3+6x^2+11x+6}{x+2}$ $\quad c = -2$

$$\begin{aligned} R &= f(-2) \\ &= (-2)^3+6\cdot(-2)^2+11\cdot(-2)+6 \\ &= -8+6\cdot 4-22+6 \\ &= -8+24-22+6 \\ &= 0 \end{aligned}$$

Therefore: $(x+2)$ is a factor of $x^3+6x^2+11x+6$:

$$x^3+6x^2+11x+6 = q(x)\cdot(x+2)$$

Find $q(x)$ via long division.

Remainder theorem and factor theorem

From the last exercise:

$$\underbrace{x^3 + 2x^2 - 5x + 3}_{=f(x)} = (\underbrace{x^2 + 4x + 3}_{=q(x)}) \cdot (x-2) + 9$$

$$\Rightarrow f(x) = q(x) \cdot (x-2) + 9$$
$$\Rightarrow f(2) = q(2) \cdot (\underbrace{2-2}_{=0}) + 9 = 9$$

Theorem (Remainder theorem)

Divide $f(x)$ by $(x-c)$ gives $f(x) = q(x) \cdot (x-c) + R$, where the remainder R is given by

$$f(c) = R$$

Note that $R = 0$, that is $f(c) = 0$, exactly when $f(x) = q(x) \cdot (x-c)$, that is we succeeded in factoring $f(x)$ with $(x-c)$ being one factor.

Theorem (Factor theorem)

$(x-c)$ is a factor of $f(x)$ exactly when $f(c) = 0$, that is exactly when c is a root of f.

Find the remainder using the remainder theorem.

1. $\frac{x^3 - 4x^2 + 2x + 8}{x-3}$ $\quad c = 3$

$$\begin{aligned} R = f(3) &= 3^3 - 4 \cdot 3^2 + 2 \cdot 3 + 8 \\ &= 27 - 4 \cdot 9 + 6 + 8 \\ &= 27 - 36 + 6 + 8 \\ &= 5 \end{aligned}$$

2. $\frac{x^3 + 6x^2 + 11x + 6}{x+2}$ $\quad c = -2$

$$\begin{aligned} R &= f(-2) \\ &= (-2)^3 + 6 \cdot (-2)^2 + 11 \cdot (-2) + 6 \\ &= -8 + 6 \cdot 4 - 22 + 6 \\ &= -8 + 24 - 22 + 6 \\ &= 0 \end{aligned}$$

Therefore: $(x+2)$ is a factor of $x^3 + 6x^2 + 11x + 6$:

$$x^3 + 6x^2 + 11x + 6 = q(x) \cdot (x+2)$$

Find $q(x)$ via long division.

Factoring completely - exercises

1. Factor completely: $f(x) = x^3 + 2x^2 - 17x + 6$

Factoring completely - exercises

1. Factor completely: $f(x) = x^3 + 2x^2 - 17x + 6$
   - Step 1: Find a root c of $f(x)$ with a graphing calculator

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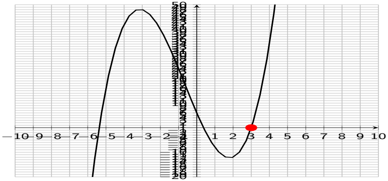

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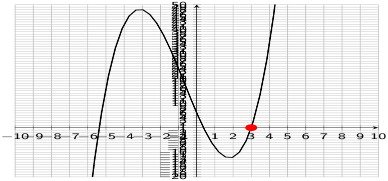

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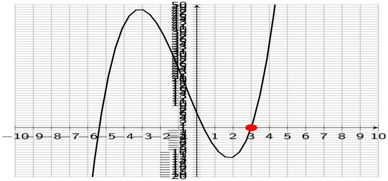

Observe: 3 is a root of $f(x)$.
Therefore: divide $f(x)$ by $(x - 3)$

   - Step 2: Divide $f(x)$ by $(x - c)$

$$x - 3 \;\overline{\big)\; x^3 \quad +2x^2 \quad -17x \quad +6}$$

Factoring completely - exercises

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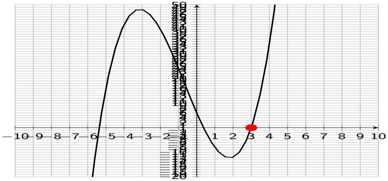

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		x^2	$+5x$	-2
$x-3$	x^3	$+2x^2$	$-17x$	$+6$
	$-(x^3$	$-3x^2)$		
		$5x^2$	$-17x$	
		$-(5x^2$	$-15x)$	
			$-2x$	$+6$
			$-(-2x$	$+6)$
				0 ✓

Thus:
$x^3 + 2x^2 - 17x + 6 = (x^2 + 5x - 2) \cdot (x - 3)$

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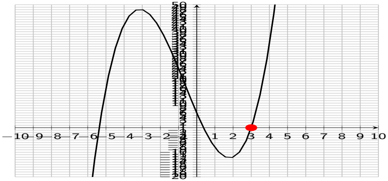

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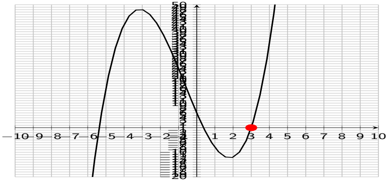

Observe: 3 is a root of $f(x)$.
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$$
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x-3 & x^3 & +2x^2 & -17x & +6 \\
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 & & 5x^2 & -17x & \\
 & & -(5x^2 & -15x) & \\
 & & & -2x & +6 \\
 & & & -(-2x & +6) \\
 & & & & 0 \checkmark
\end{array}
$$

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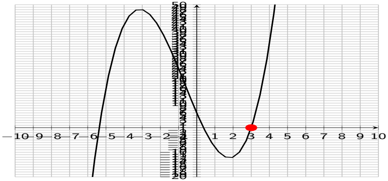

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$\Rightarrow x^3 + 2x^2 - 17x + 6 = (x-3)(x^2+5x-2) = (x-3)\left(x - \frac{-5+\sqrt{33}}{2}\right)\left(x - \frac{-5-\sqrt{33}}{2}\right)$

Factoring completely - exercises

2. Factor completely: $f(x) = x^3 + 5x^2 - x - 14$

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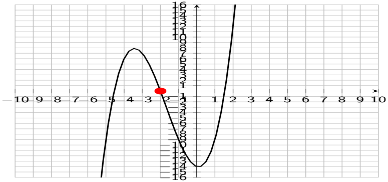

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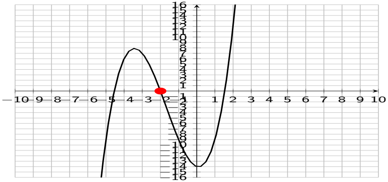

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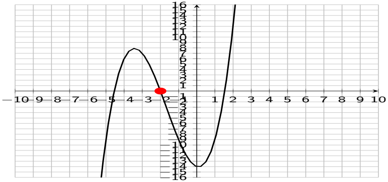

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Factoring completely - exercises

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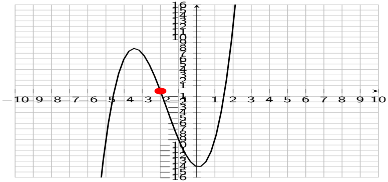

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\hline
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\hline
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Thus:
$x^3 + 5x^2 - x - 14 = (x^2 + 3x - 7) \cdot (x + 2)$

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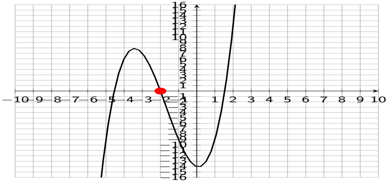

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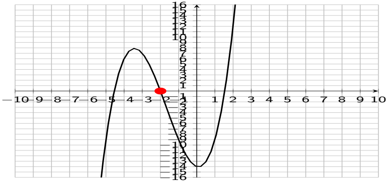

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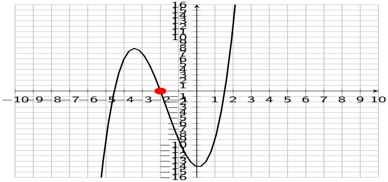

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