Basic functions and transformations

Lesson #4

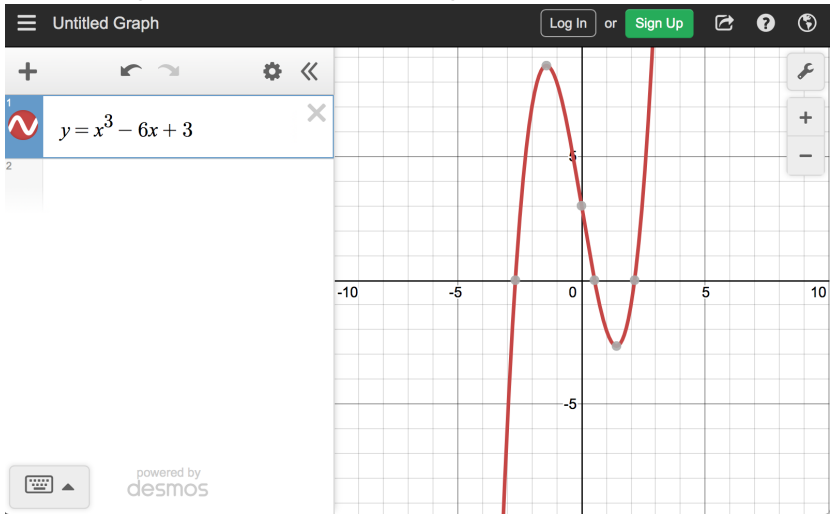
MAT 1375 Precalculus

New York City College of Technology CUNY

Introduction to desmos

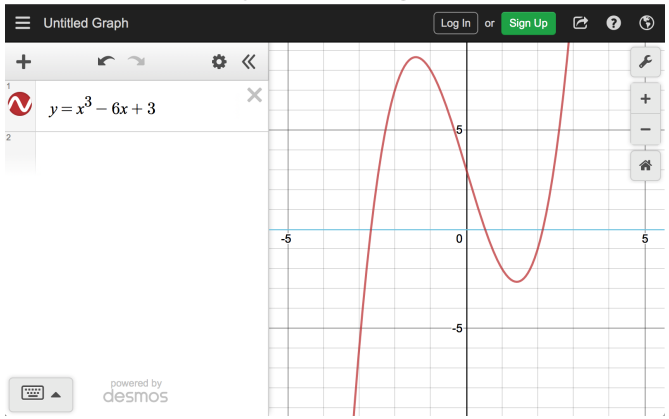
Download the desmos app or go to www.desmos.com (click on "Graphing Calculator")

- Display the function $y = x^3 - 6x + 3$

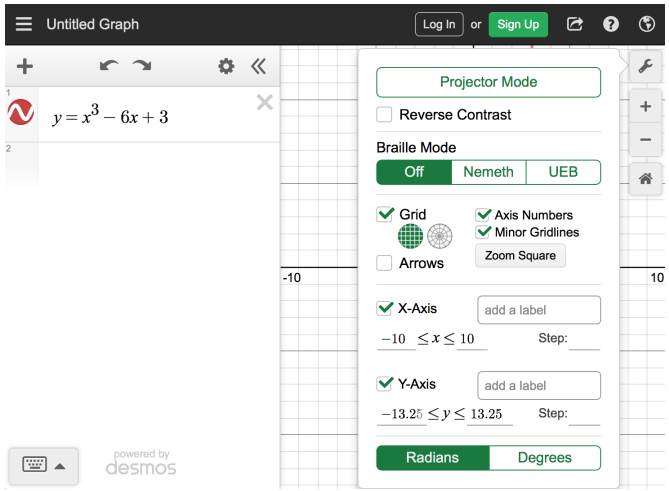


- Change scale by zooming in or out

Zoom axis by holding "shift" + zoom axis

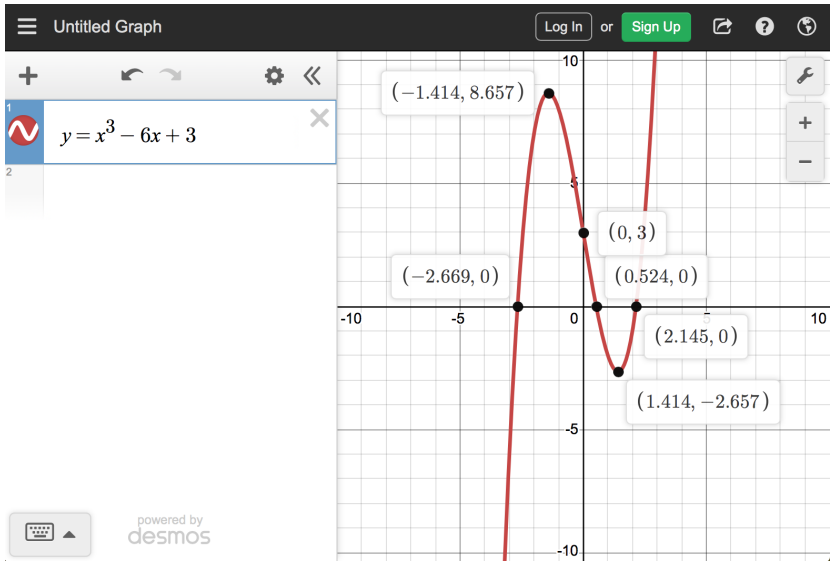


Or under settings: X-Axis and Y-Axis



Introduction to desmos

- Find points of interest



Approximate coordinates found by clicking

local maximum: $(x, y) \approx (-1.414, 8.657)$

local minimum: $(x, y) \approx (1.414, -2.657)$

x-intercepts or roots or zeros:

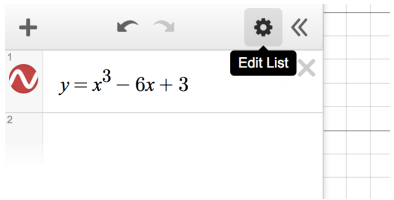
$(x, y) \approx (-2.669, 0)$

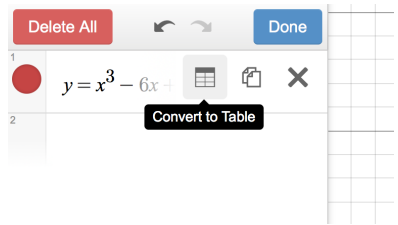
$(x, y) \approx (0.524, 0)$

$(x, y) \approx (2.145, 0)$

y-intercept: $(x, y) = (0, 3)$

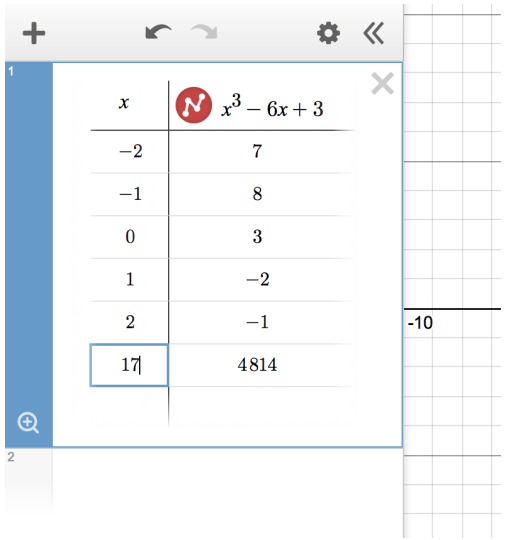
- Create a table





Find function values for specified inputs x:

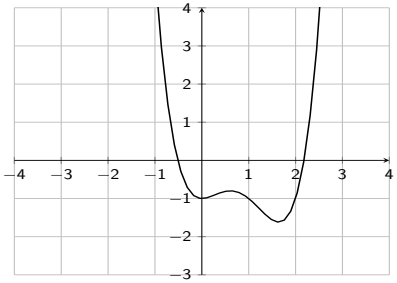
$$f(17) = 4814$$



Extrema, roots, and y-intercept with desmos - exercises

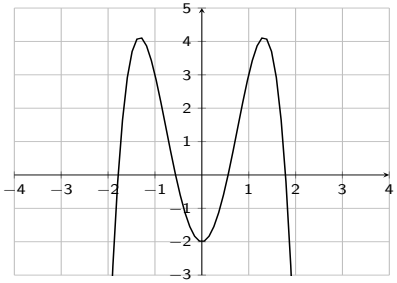
Find all maxima, minima and specify if they are local of global. Find the roots and the y-intercept.

1. $f(x) = x^4 - 3x^3 + 2x^2 - 1$



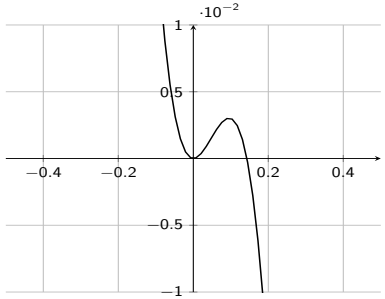
local max: $(x, y) \approx (0.61, -0.798)$
local min: $(x, y) = (0, -1)$
global min: $(x, y) \approx (1.64, -1.62)$
roots: $(x, y) \approx (-0.513, 0)$
$(x, y) \approx (2.179, 0)$
y-intercept: $(x, y) = (0, -1)$

2. $f(x) = -2x^4 + 7x^2 - 2$



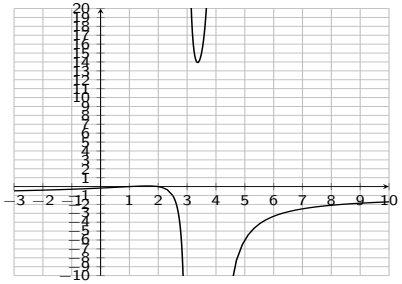
global max: $(x, y) \approx (-1.323, 4.125)$
global max: $(x, y) \approx (1.323, 4.125)$
local min: $(x, y) = (0, -2)$
roots: $(x, y) \approx (\pm 1.785, 0)$
$(x, y) \approx (\pm 0.56, 0)$
y-intercept: $(x, y) = (0, -2)$

3. $f(x) = x^2 - 7x^3$



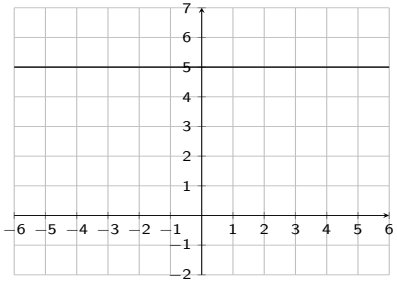
local max: $(x, y) \approx (0.0952, 0.003)$
local min: $(x, y) = (0, 0)$
roots: $(x, y) = (0, 0)$
$(x, y) \approx (0.1429, 0)$
y-intercept: $(x, y) = (0, 0)$

4. $f(x) = \frac{2}{x-3} - \frac{x+2}{x-4}$



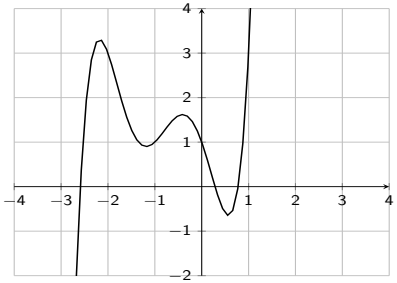
local max: $(x, y) \approx (1.634, 0.072)$
local min: $(x, y) \approx (3.366, 13.928)$
roots: $(x, y) = (1, 0)$
$(x, y) = (2, 0)$
y-intercept: $(x, y) \approx (0, -0.167)$

5. $f(x) = 5$



global maxima: all points $(x, 5)$
global minima: all points $(x, 5)$
roots: no roots
y-intercept: $(x, y) = (0, 5)$

6. $f(x) = x^5 + 4x^4 + 3x^3 - 3x^2 - 3x + 1$



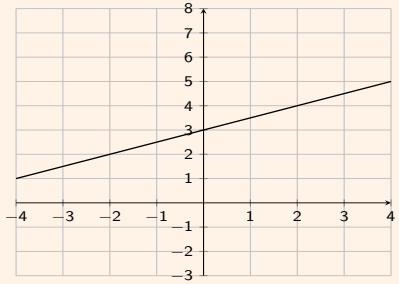
local max: $(x, y) \approx (-2.179, 3.307)$
local max: $(x, y) \approx (-0.408, 1.62)$
local min: $(x, y) \approx (-1.183, 0.901)$
local min: $(x, y) \approx (0.57, -0.647)$
roots: $(x, y) \approx (-2.589, 0)$
$(x, y) \approx (0.285, 0)$
$(x, y) \approx (0.776, 0)$
y-intercept: $(x, y) = (0, 1)$

Basic Functions

Use desmos to explore the graphs of the basic functions below. Memorize their graphs!

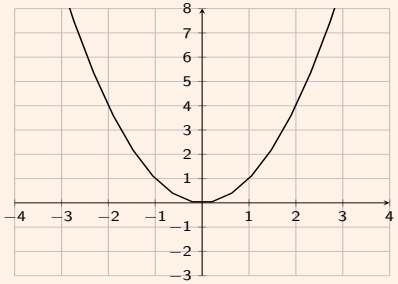
Graphs of basic functions

$y = mx + b$



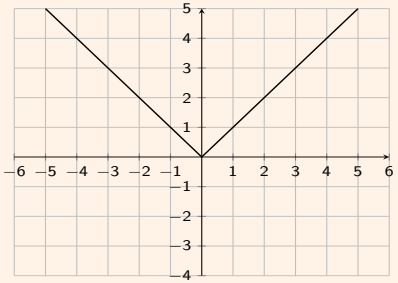
$D = \mathbb{R}$

$y = x^2$



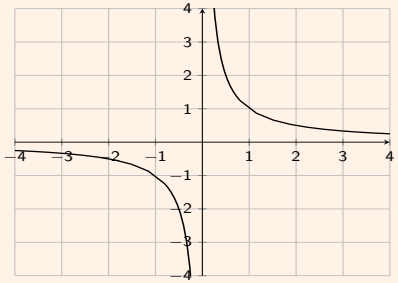
$D = \mathbb{R}$

$y = |x|$



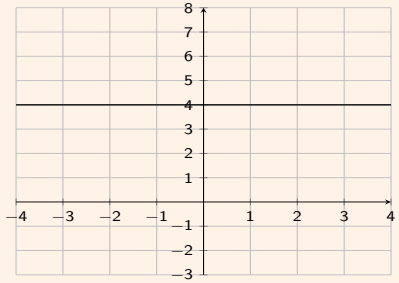
$D = \mathbb{R}$

$y = \frac{1}{x}$



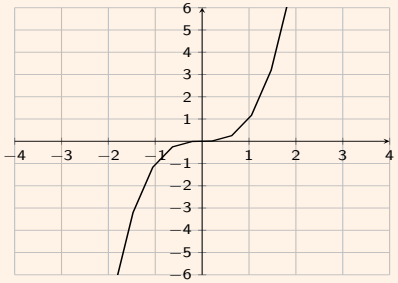
$D = \mathbb{R} - \{0\}$

$y = c$



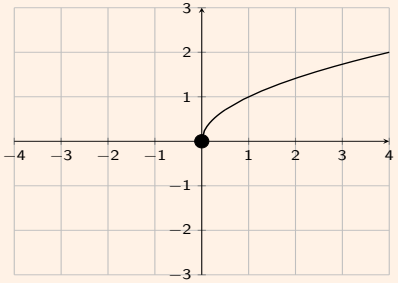
$D = \mathbb{R}$

$y = x^3$



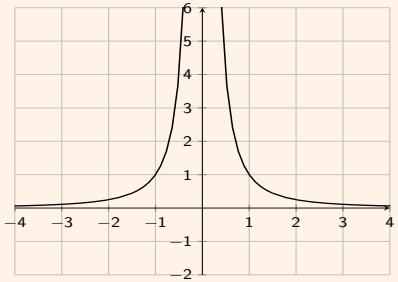
$D = \mathbb{R}$

$y = \sqrt{x}$



$D = [0, \infty)$

$y = \frac{1}{x^2}$



$D = \mathbb{R} - \{0\}$

Transformations: shift left/right/up/down, reflection, stretch/compress

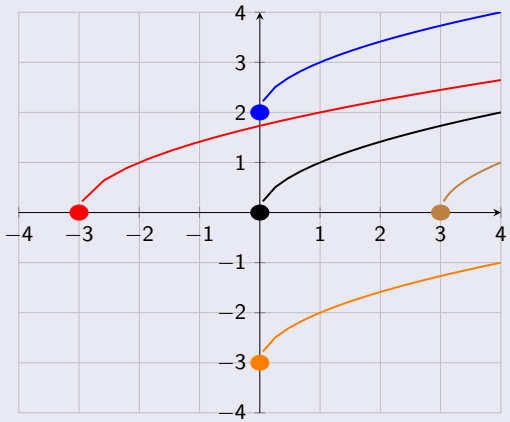
Explore with desmos:

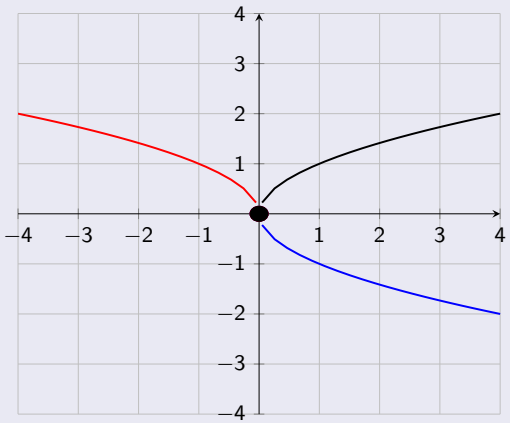
- $y = \sqrt{x} + a$ (shift the slider a up and down)
- $y = \sqrt{x + a}$
- $y = -\sqrt{x}$ and $y = \sqrt{-x}$
- $y = x^3 + 2$ and $y = a \cdot (x^3 + 2)$ and $y = (a \cdot x)^3 + 2$

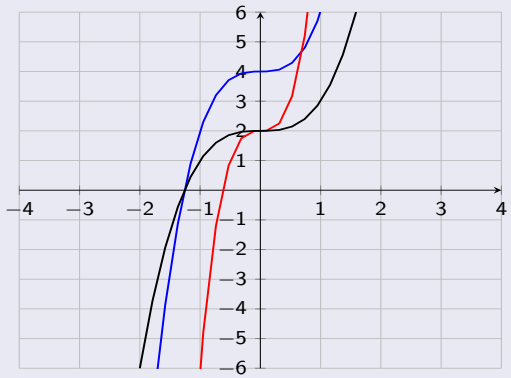
Summary of graph transformations

For $a > 0$, the graph of $y = f(x)$ is

- shifted up/down by $y = f(x) \pm a$
- shifted left/right by $y = f(x \pm a)$
- reflected about the x-axis by $y = -f(x)$
- reflected about the y-axis by $y = f(-x)$
- stretched away ($a > 1$) or compressed towards ($0 < a < 1$) the x-axis by $y = a \cdot f(x)$
- compressed towards ($a > 1$) or stretched away ($0 < a < 1$) the y-axis by $y = f(a \cdot x)$





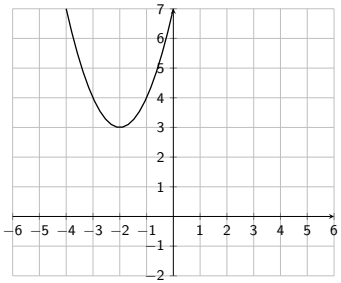


Transformations - exercises

Use sliders in desmos to explore how the graph differs from a basic graph.

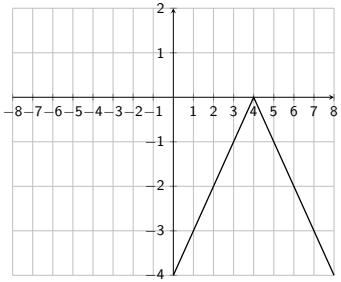
1. $f(x) = (x+2)^2 + 3$

   Use $y = (x+a)^2 + b$ with sliders a and b.

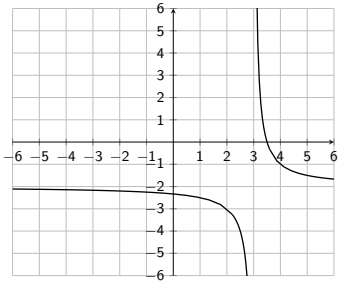
   shift up by 3, left by 2

2. $f(x) = -|x-4|$

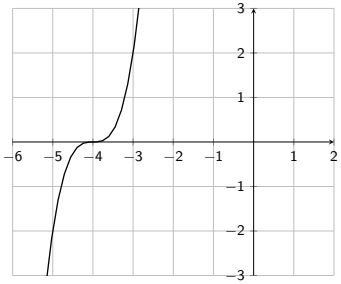
   shift right by 4, reflect down

3. $f(x) = \frac{1}{x-3} - 2$

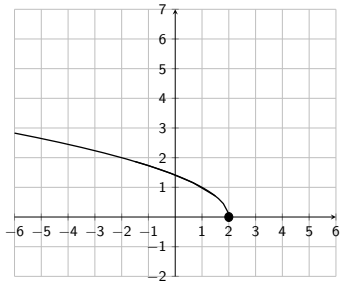
   shift right by 3, down by 2

4. $f(x) = 2 \cdot (x+4)^3$

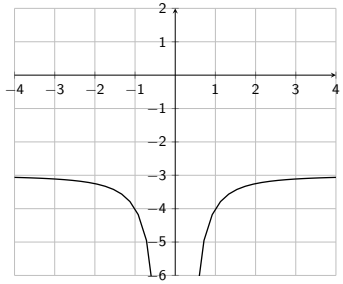
   shift left by 4,
   stretch away from x-axis

5. $f(x) = \sqrt{-x+2}$

   reflect to the left,
   shift right by 2

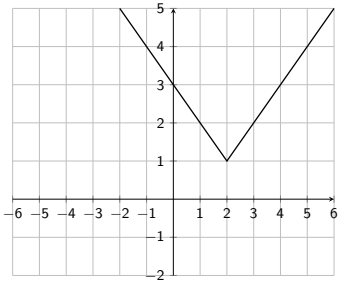
6. $f(x) = -(\frac{1}{x^2} + 3)$

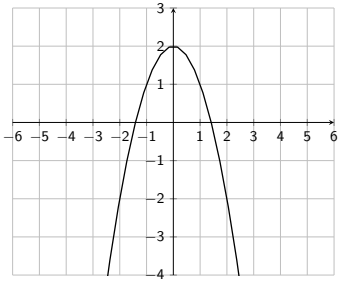
   reflect down,
   shift down by 3

Transformations - exercises

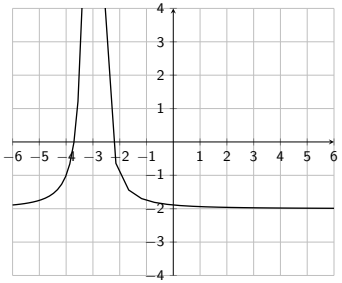
Find a possible formula for the graph displayed below.

1. 

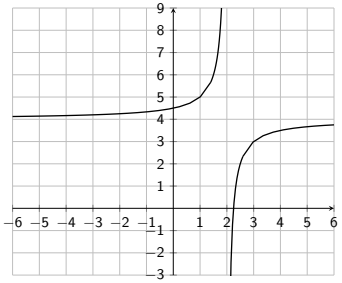
$f(x) = |x-2|+1$

2. 

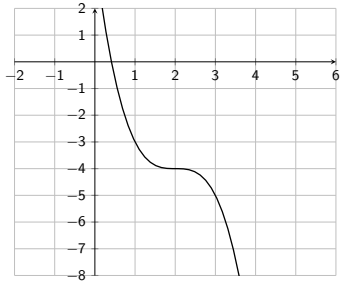
$f(x) = -x^2+2$

3. 

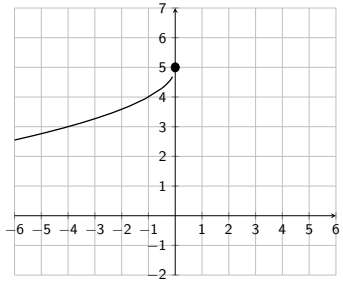
$f(x) = \frac{1}{(x+3)^2} - 2$

4. 

$f(x) = -\frac{1}{x-2} + 4$

5. 

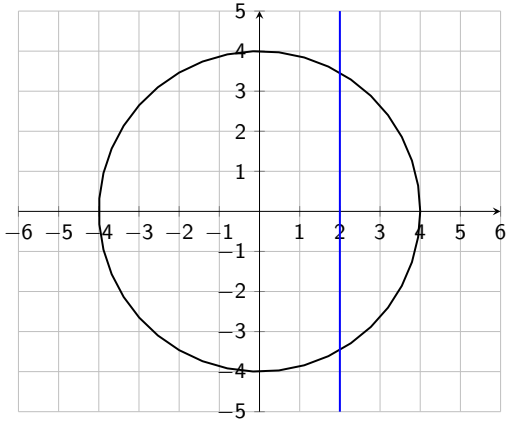
$f(x) = -(x-2)^3 - 4$

6. 

$f(x) = -\sqrt{-x} + 5$

Function vs. relation with desmos - exercises

Graph $x^2 + y^2 = 16$

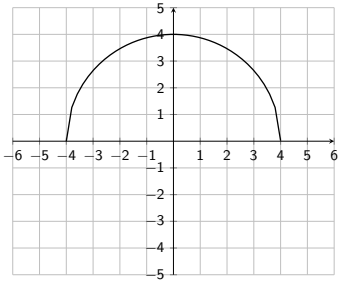


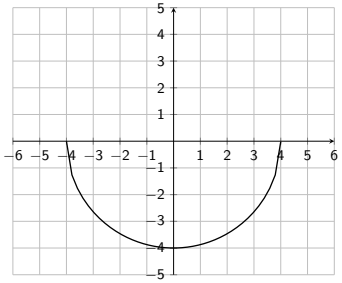
Circle with center $(0, 0)$ and radius 4
Is this the graph of a function?
No! ✗ Use Vertical Line Test!
Solve for y: $\Rightarrow y^2 = 16 - x^2$

$$\Rightarrow y = \pm\sqrt{16 - x^2}$$

The "$\pm$" gives two separate *functions*:

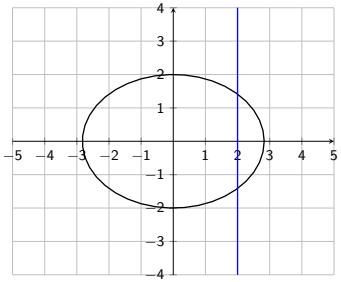
$$y = \sqrt{16 - x^2} \qquad y = -\sqrt{16 - x^2}$$





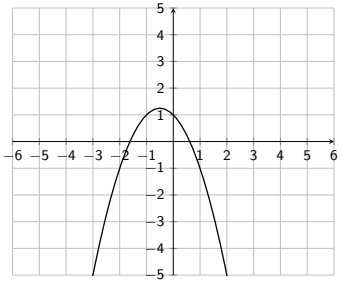
Use the vertical line test to determine if the given equation is a function or a relation.

1. $x^2 + 2y^2 = 8$



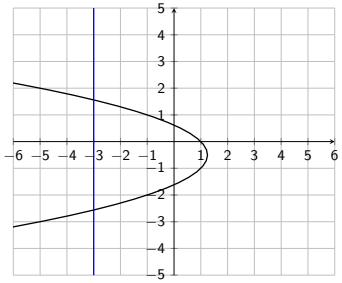
Not a function, but a relation

2. $x + y + x^2 = 1$



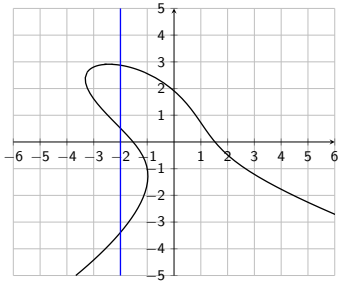
Function

3. $y + x + y^2 = 1$



Not a function, but a relation

4. $3x^2 + y^3 + 5xy = 7$

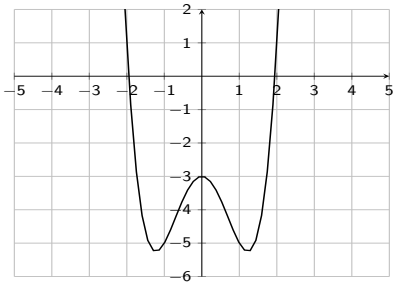


Not a function, but a relation

Domain and range with desmos - exercises

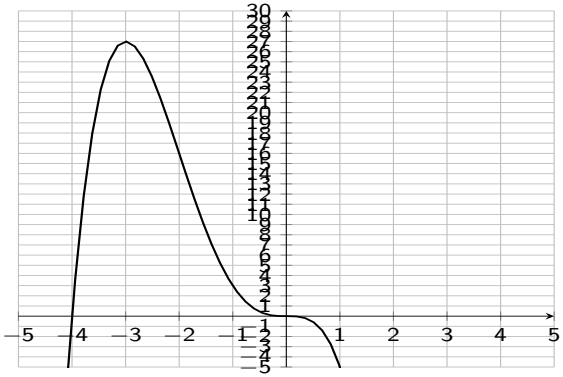
Find the domain and range of the function.

1. $f(x) = x^4 - 3x^2 - 3$



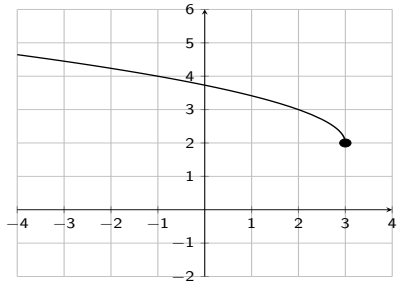
Domain: $D = \mathbb{R}$
Range: $R = [-5.25, \infty)$

2. $f(x) = -x^4 - 4x^3$



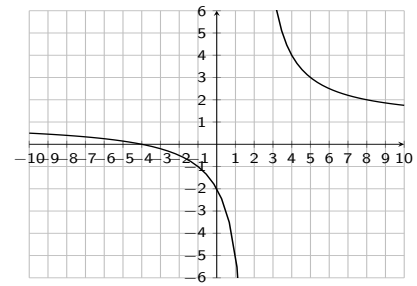
Domain: $D = \mathbb{R}$
Range: $R = (-\infty, 27]$

3. $f(x) = 2 + \sqrt{3 - x}$



Domain: $D = (-\infty, 3]$
Range: $R = [2, \infty)$
Note: $f(3) = 2$ so 3 is in the domain and 2 is in the range

4. $f(x) = \frac{x+4}{x-2}$



Domain: $D = \mathbb{R} - \{2\}$
Range: $R = \mathbb{R} - \{1\}$
Note: $f(2)$ is undefined