Functions via graphs

Lesson #3

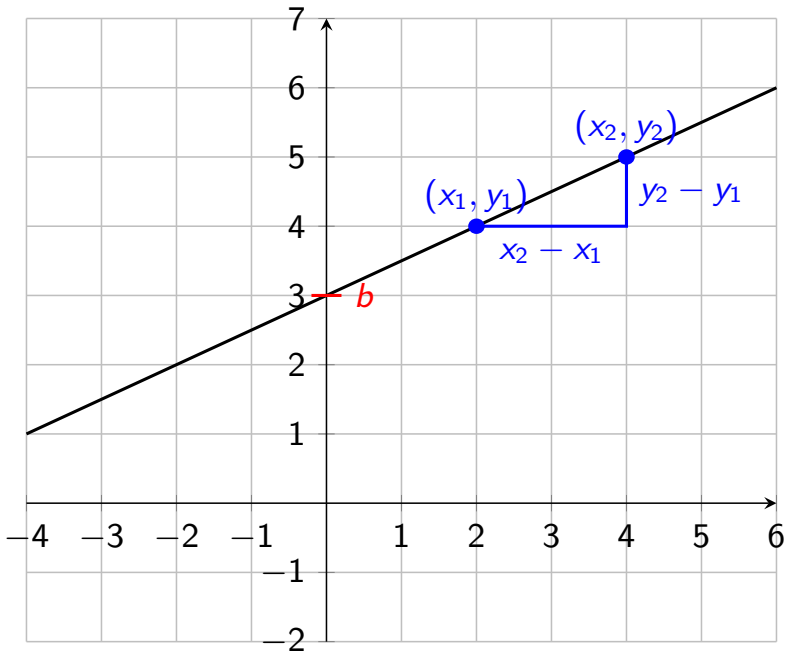
MAT 1375 Precalculus

New York City College of Technology CUNY

Lines - review

Recall one of the most fundamental type of functions: **linear functions**

$$\boxed{f(x) = m \cdot x + b}$$



Slope-intercept form of the line:

$$\boxed{y = m \cdot x + b}$$

y-intercept: b

slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$

Point-slope form of the line:

Fix (x_1, y_1). Then $(x_2 - x_1) \cdot m = (y_2 - y_1)$ must be true for all (x_2, y_2) on the line. Call it (x, y):

$$\boxed{y - y_1 = m \cdot (x - x_1)}$$

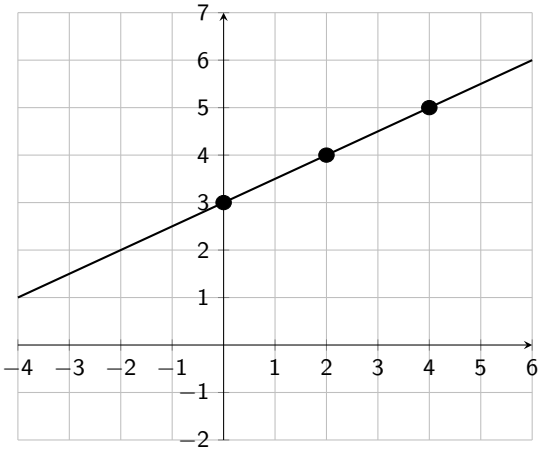
point: (x_1, y_1)

slope: m

Lines - exercises

Find the slope, and the equation of the line.

1



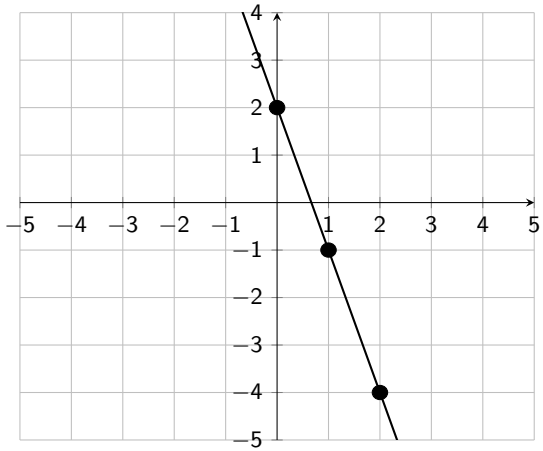
$(x_1, y_1) = (2, 4)$, $(x_2, y_2) = (4, 5)$

$m = \frac{5-4}{4-2} = \frac{1}{2}$

$b = 3$

$\Longrightarrow y = \frac{1}{2} \cdot x + 3$

2



$(x_1, y_1) = (1, -1)$, $(x_2, y_2) = (2, -4)$

$m = \frac{(-4)-(-1)}{2-1} = \frac{-3}{1} = -3$

$b = 2$

$\Longrightarrow y = -3 \cdot x + 2$

3

Find the equation of the line passing through the point $(3, 5)$ with slope $m = -\frac{2}{5}$.

Solution:

$y - y_1 = m \cdot (x - x_1)$

$\Longrightarrow y - 5 = -\frac{2}{5} \cdot (x - 3)$

$\Longrightarrow y = -\frac{2}{5} \cdot x + \frac{6}{5} + 5$

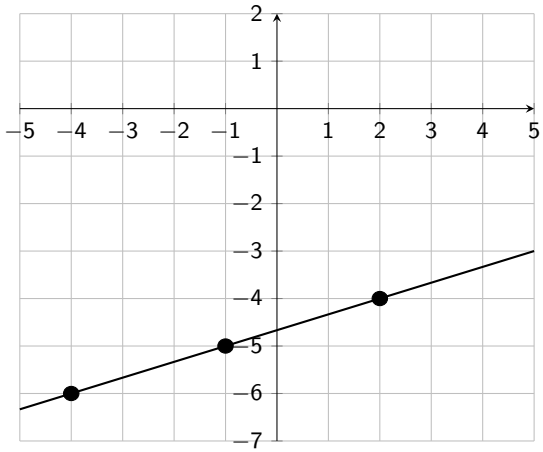
$\Longrightarrow y = -\frac{2}{5} \cdot x + \frac{6}{5} + \frac{25}{5}$

$\Longrightarrow y = -\frac{2}{5} \cdot x + \frac{31}{5}$

Lines - exercises

Find the equation of the line.

4



$(x_1, y_1) = (2, -4)$, $(x_2, y_2) = (-4, -6)$

$m = \frac{(-6)-(-4)}{(-4)-2} = \frac{-2}{-6} = \frac{1}{3}$

b is unknown

Use $y - y_1 = m(x - x_1)$

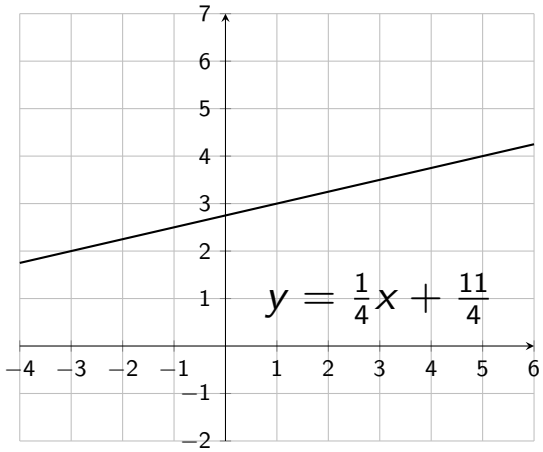
$\Longrightarrow y - (-4) = \frac{1}{3}(x - 2)$

$\Longrightarrow y + 4 = \frac{1}{3}x - \frac{2}{3}$

$\Longrightarrow y = \frac{1}{3}x - \frac{2}{3} - 4$

$\Longrightarrow y = \frac{1}{3}x - \frac{14}{3}$

5



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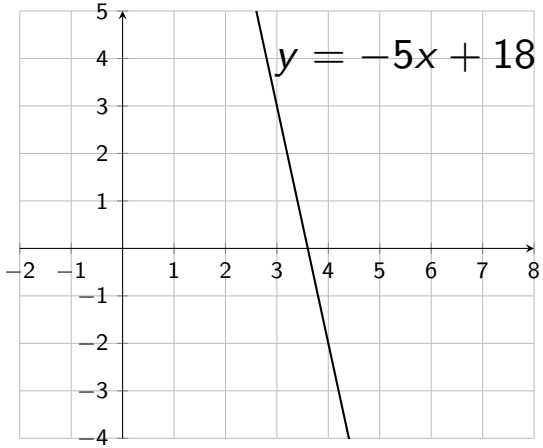

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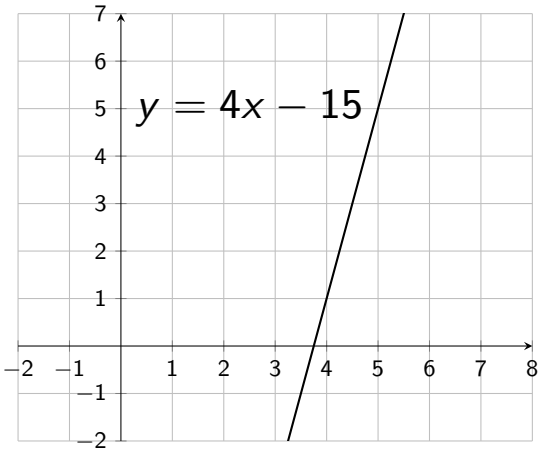

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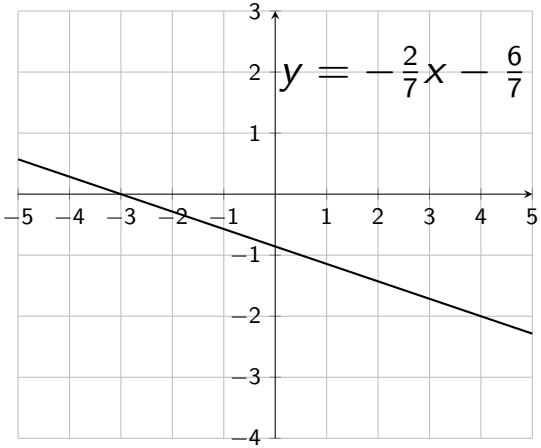
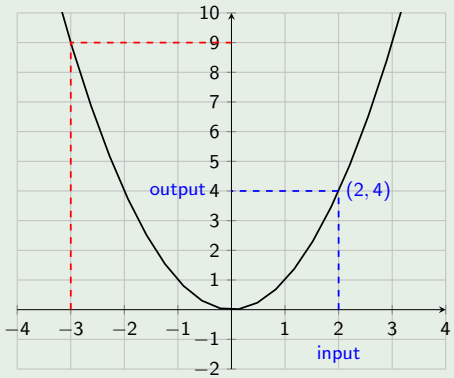


Functions by graphs

Example

Graph $f(x) = x^2$

x	-3	-2	-1	0	1	2	3
y	9	4	1	0	1	4	9

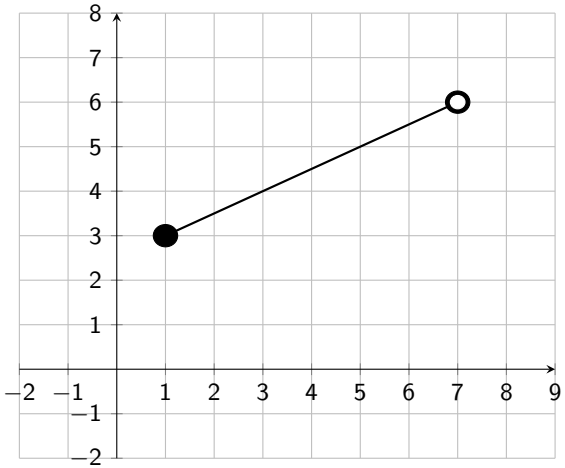


$$f(2) = 4$$
$$f(-3) = 9$$

Domain: $D = \mathbb{R}$

Range: $R = [0, \infty)$

1. State domain and range and function values:

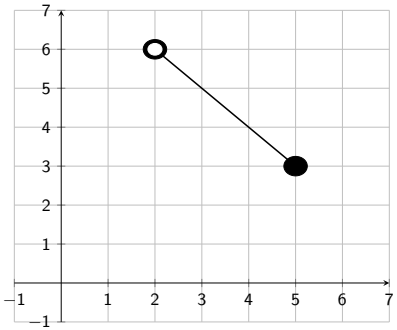


$f(3)$ =4
$f(5)$ =5
$f(1)$ =3
$f(8)$ =undefined
$f(7)$ =undefined
$f(6)$ =5.5
Domain D =$[1, 7)$
Range R =$[3, 6)$

Functions by graphs - exercises

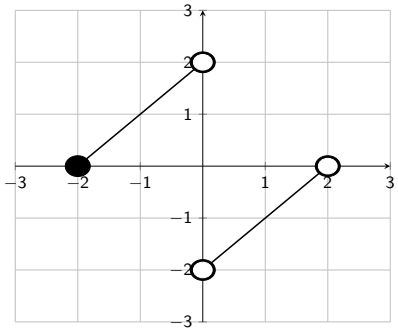
Find the function values, and the domain and range.

2



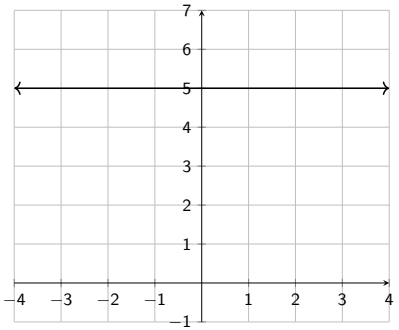
$f(3) = 5 \quad f(2) = \text{undef.}$
$D = (2, 5]$
$R = [3, 6)$

4



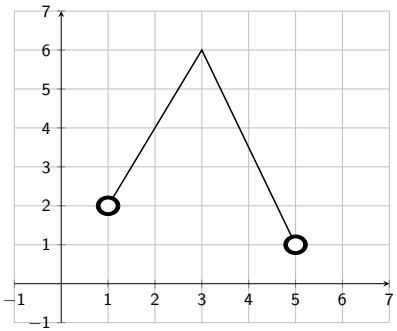
$f(0) = \text{undef.} \quad f(-2) = 0$
$D = [-2, 0) \cup (0, 2)$
$R = (-2, 2)$

6



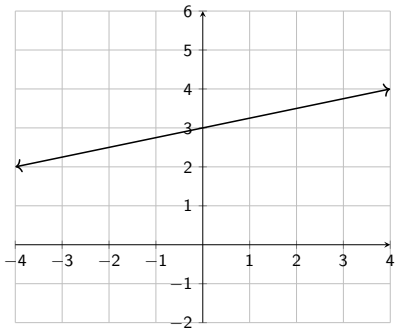
$y = 5$
$f(-3) = 5 \quad f(\sqrt{17}) = 5$
$D = \mathbb{R} \quad R = \{5\}$

3



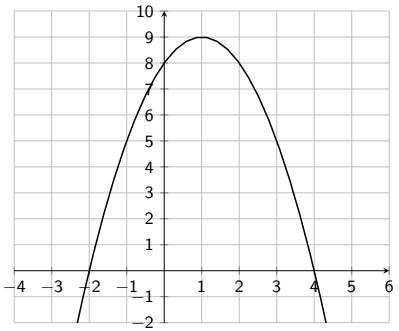
$f(3) = 6 \quad f(2) = 4$
$D = (1, 5)$
$R = (1, 6]$

5



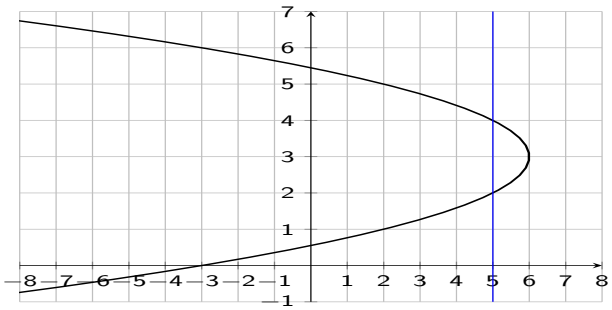
$y = \frac{1}{4}x + 3$
$f(3) = 3.75 \quad f(24) = 9$
$D = \mathbb{R}$
$R = \mathbb{R}$

7



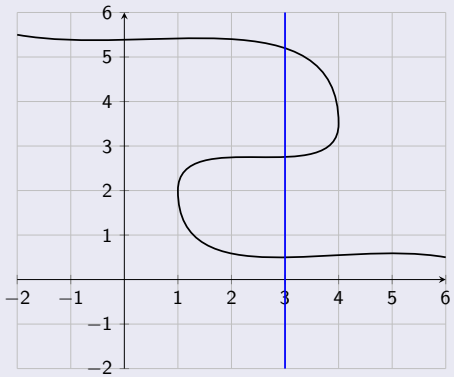
$y = -x^2 + 2x + 8$
$f(1) = 9 \quad f(5) = -7$
$D = \mathbb{R}$
$R = (-\infty, 9]$

Vertical line test



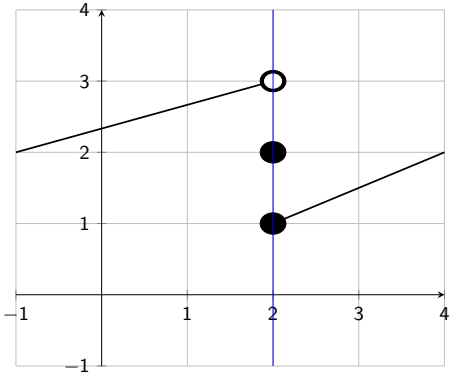
Find $f(5) =$??? 2 or 4 ???
The input 5 has multiple outputs!
$\Rightarrow$ f is not a function! ✗ (f is a relation.)

Vertical line test

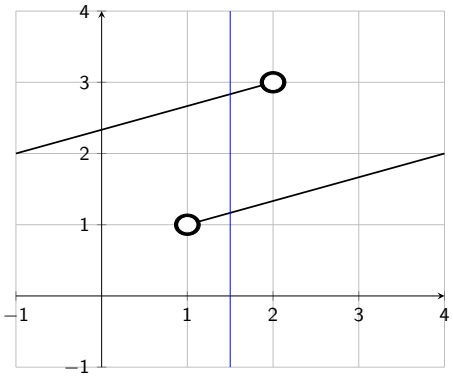


If there is a vertical line, which intersects the graph in more than one point, then the graph is not the graph of a function.

Are these graphs of a function?

1. 

No!

2. 

No!