

# Functions via graphs

## Lesson #3

### MAT 1375 Precalculus

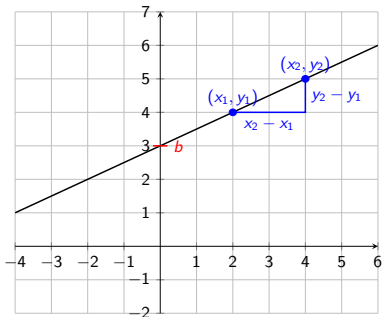
New York City College of Technology CUNY



# Lines - review

Recall one of the most fundamental type of functions: **linear functions**

$$f(x) = m \cdot x + b$$



**Slope-intercept form of the line:**

$$y = m \cdot x + b$$

y-intercept:

$b$

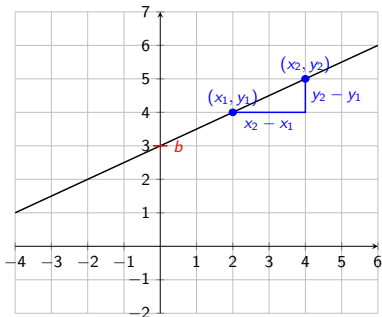
slope:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

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y-intercept:  $b$

slope:  $m = \frac{y_2 - y_1}{x_2 - x_1}$

**Point-slope form of the line:**

Fix  $(x_1, y_1)$ . Then  $(x_2 - x_1) \cdot m = (y_2 - y_1)$  must be true for all  $(x_2, y_2)$  on the line.

Call it  $(x, y)$ :

$$y - y_1 = m \cdot (x - x_1)$$

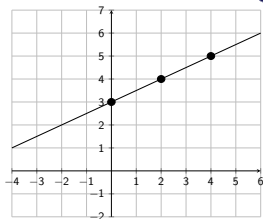
point:  $(x_1, y_1)$

slope:  $m$

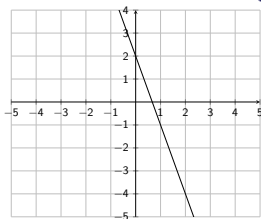
# Lines - exercises

Find the slope, and the equation of the line.

1



2



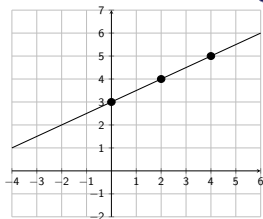
3

Find the equation of the line passing through the point  $(3, 5)$  with slope  $m = -\frac{2}{5}$ .

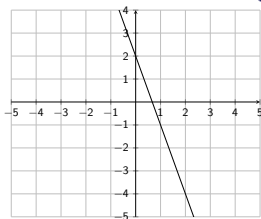
# Lines - exercises

Find the slope, and the equation of the line.

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Find the equation of the line passing through the point  $(3, 5)$  with slope  $m = -\frac{2}{5}$ .

$$(x_1, y_1) = (2, 4), (x_2, y_2) = (4, 5)$$

$$m = \frac{5-4}{4-2} = \frac{1}{2}$$

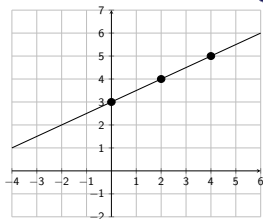
$$b = 3$$

$$\implies y = \frac{1}{2} \cdot x + 3$$

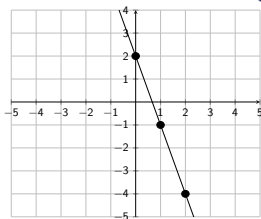
# Lines - exercises

Find the slope, and the equation of the line.

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Find the equation of the line passing through the point  $(3, 5)$  with slope  $m = -\frac{2}{5}$ .

$$(x_1, y_1) = (2, 4), (x_2, y_2) = (4, 5)$$

$$m = \frac{5-4}{4-2} = \frac{1}{2}$$

$$b = 3$$

$$\Rightarrow y = \frac{1}{2} \cdot x + 3$$

$$(x_1, y_1) = (1, -1), (x_2, y_2) = (2, -4)$$

$$m = \frac{-4-(-1)}{2-1} = \frac{-3}{1} = -3$$

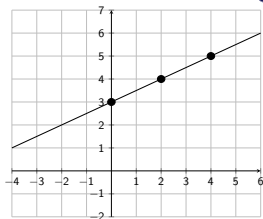
$$b = 2$$

$$\Rightarrow y = -3 \cdot x + 2$$

# Lines - exercises

Find the slope, and the equation of the line.

1



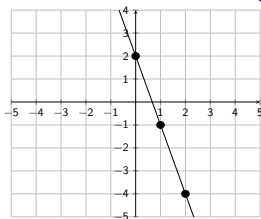
$$(x_1, y_1) = (2, 4), (x_2, y_2) = (4, 5)$$

$$m = \frac{5-4}{4-2} = \frac{1}{2}$$

$$b = 3$$

$$\Rightarrow y = \frac{1}{2} \cdot x + 3$$

2



$$(x_1, y_1) = (1, -1), (x_2, y_2) = (2, -4)$$

$$m = \frac{(-4)-(-1)}{2-1} = \frac{-3}{1} = -3$$

$$b = 2$$

$$\Rightarrow y = -3 \cdot x + 2$$

3

Find the equation of the line passing through the point  $(3, 5)$  with slope  $m = -\frac{2}{5}$ .

Solution:

$$y - y_1 = m \cdot (x - x_1)$$
$$\Rightarrow y - 5 = -\frac{2}{5} \cdot (x - 3)$$

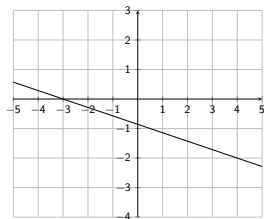
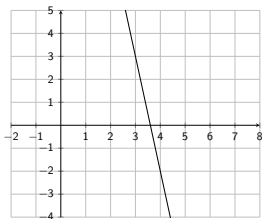
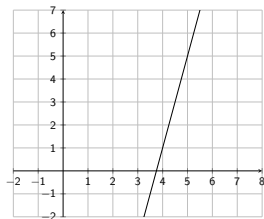
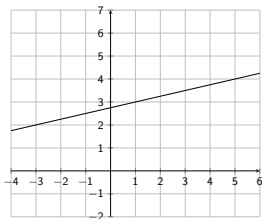
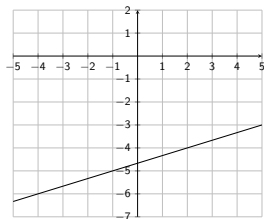
$$\Rightarrow y = -\frac{2}{5} \cdot x + \frac{6}{5} + 5$$

$$\Rightarrow y = -\frac{2}{5} \cdot x + \frac{6}{5} + \frac{25}{5}$$

$$\Rightarrow y = -\frac{2}{5} \cdot x + \frac{31}{5}$$

# Lines - exercises

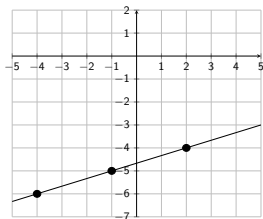
Find the equation of the line.





# Lines - exercises

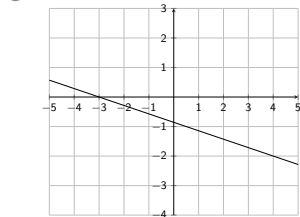
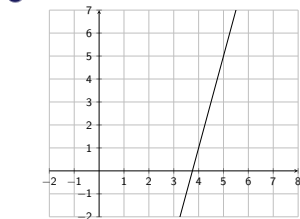
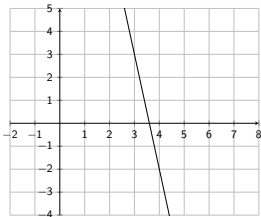
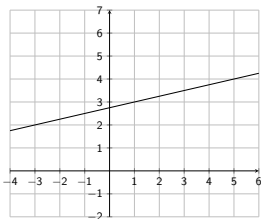
Find the equation of the line.



$$(x_1, y_1) = (2, -4), (x_2, y_2) = (-4, -6)$$

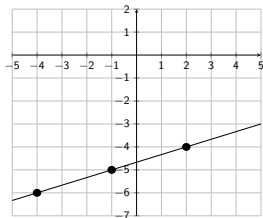
$$m = \frac{(-6) - (-4)}{(-4) - 2} = \frac{-2}{-6} = \frac{1}{3}$$

$b$  is unknown



# Lines - exercises

Find the equation of the line.



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$$m = \frac{(-6) - (-4)}{(-4) - 2} = \frac{-2}{-6} = \frac{1}{3}$$

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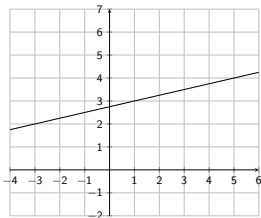
$$\text{Use } y - y_1 = m(x - x_1)$$

$$\implies y - (-4) = \frac{1}{3}(x - 2)$$

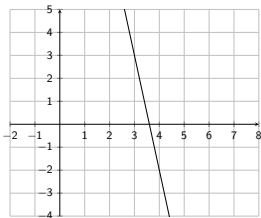
$$\implies y + 4 = \frac{1}{3}x - \frac{2}{3}$$

$$\implies y = \frac{1}{3}x - \frac{2}{3} - 4$$

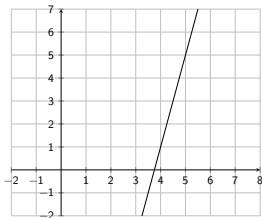
$$\implies y = \frac{1}{3}x - \frac{14}{3}$$



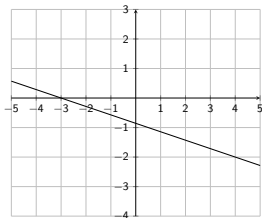
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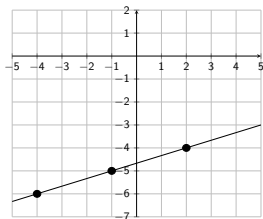


8



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Find the equation of the line.



$$(x_1, y_1) = (2, -4), (x_2, y_2) = (-6, -4)$$

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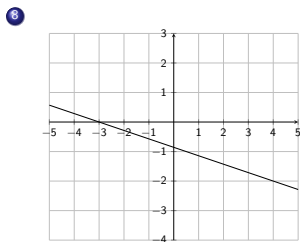
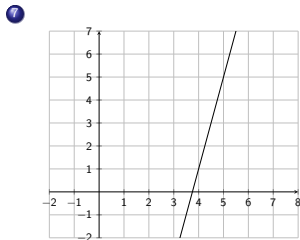
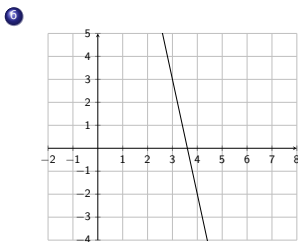
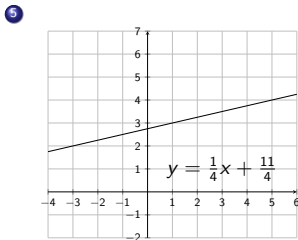
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$$\implies y - (-4) = \frac{1}{3}(x - 2)$$

$$\implies y + 4 = \frac{1}{3}x - \frac{2}{3}$$

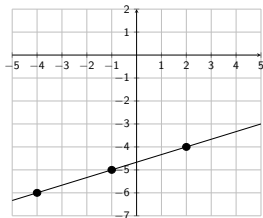
$$\implies y = \frac{1}{3}x - \frac{2}{3} - 4$$

$$\implies y = \frac{1}{3}x - \frac{14}{3}$$



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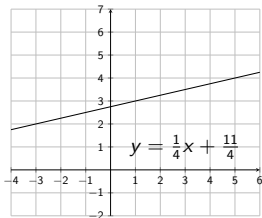
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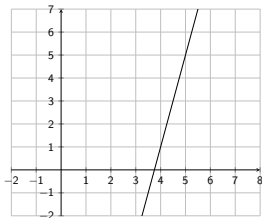
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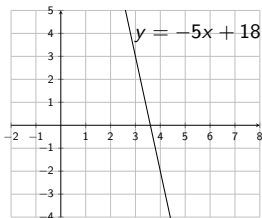
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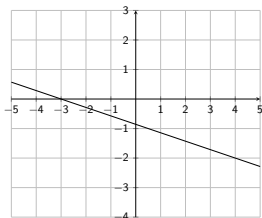
7



6

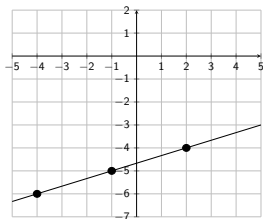


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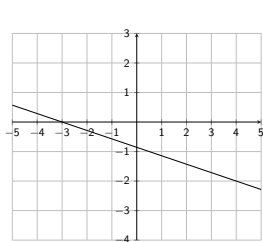
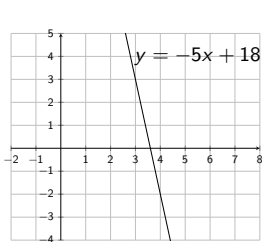
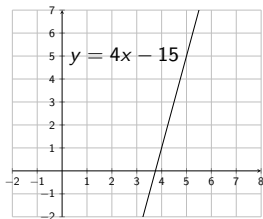
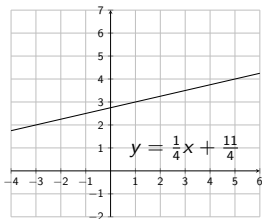
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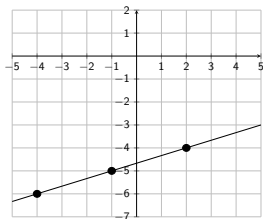
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$$\Rightarrow y = \frac{1}{3}x - \frac{14}{3}$$



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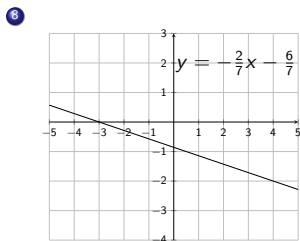
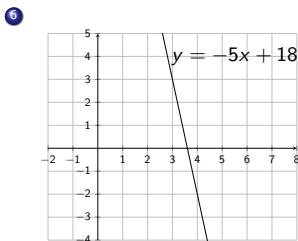
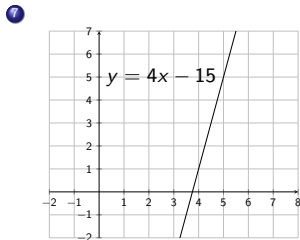
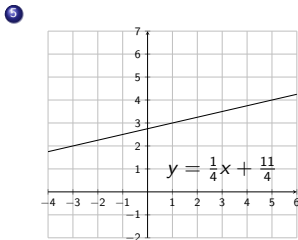
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## Example

Graph  $f(x) = x^2$

x	-3	-2	-1	0	1	2	3
y							

## Example

Graph  $f(x) = x^2$

x	-3	-2	-1	0	1	2	3
y	9	4	1	0	1	4	9

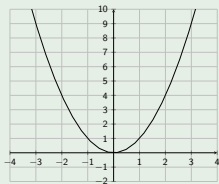


# Functions by graphs

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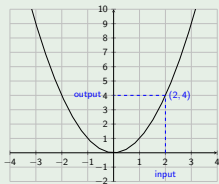


$$f(2) =$$

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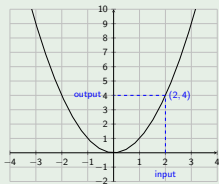


$$f(2) = 4$$

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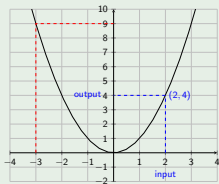
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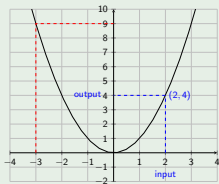
Domain:

Range:

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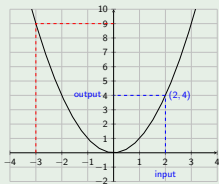
Range:  $R = [0, \infty)$

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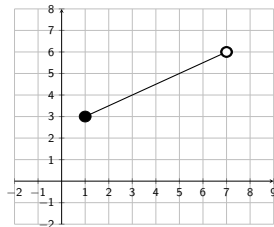
$$f(2) = 4$$

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Domain:  $D = \mathbb{R}$

Range:  $R = [0, \infty)$

- 1 State domain and range and function values:



$$f(3) =$$

$$f(5) =$$

$$f(1) =$$

$$f(8) =$$

$$f(7) =$$

$$f(6) =$$

Domain  $D =$

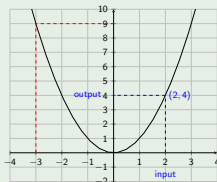
Range  $R =$

# Functions by graphs

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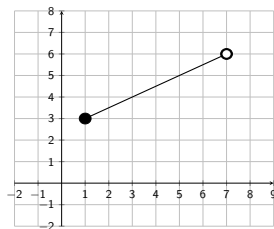
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Domain:  $D = \mathbb{R}$

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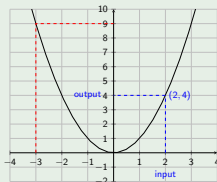
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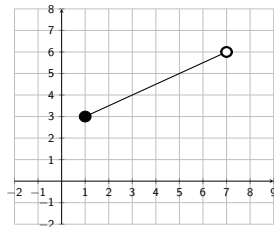
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Domain:  $D = \mathbb{R}$

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$$f(5) = 5$$

$$f(1) = 3$$

$$f(8) =$$

$$f(7) =$$

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Domain  $D =$

Range  $R =$

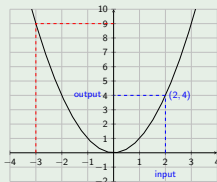


# Functions by graphs

## Example

Graph  $f(x) = x^2$

x	-3	-2	-1	0	1	2	3
y	9	4	1	0	1	4	9



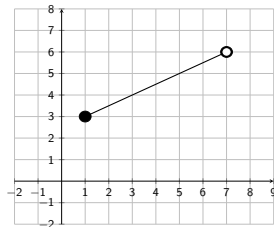
$$f(2) = 4$$

$$f(-3) = 9$$

Domain:  $D = \mathbb{R}$

Range:  $R = [0, \infty)$

- 1 State domain and range and function values:



$$f(3) = 4$$

$$f(5) = 5$$

$$f(1) = 3$$

$$f(8) = \text{undefined}$$

$$f(7) =$$

$$f(6) =$$

Domain  $D =$

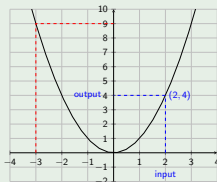
Range  $R =$

# Functions by graphs

## Example

Graph  $f(x) = x^2$

x	-3	-2	-1	0	1	2	3
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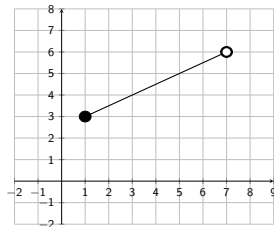
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$$f(3) = 4$$

$$f(5) = 5$$

$$f(1) = 3$$

$$f(8) = \text{undefined}$$

$$f(7) = \text{undefined}$$

$$f(6) =$$

Domain  $D =$

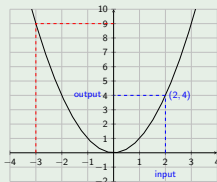
Range  $R =$

# Functions by graphs

## Example

Graph  $f(x) = x^2$

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y	9	4	1	0	1	4	9



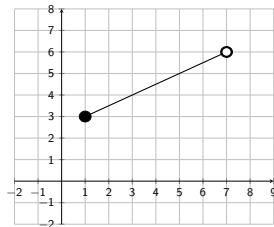
$$f(2) = 4$$

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Domain:  $D = \mathbb{R}$

Range:  $R = [0, \infty)$

- 1 State domain and range and function values:



$$f(3) = 4$$

$$f(5) = 5$$

$$f(1) = 3$$

$$f(8) = \text{undefined}$$

$$f(7) = \text{undefined}$$

$$f(6) = 5.5$$

Domain  $D =$

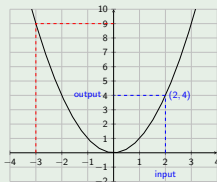
Range  $R =$

# Functions by graphs

## Example

Graph  $f(x) = x^2$

x	-3	-2	-1	0	1	2	3
y	9	4	1	0	1	4	9



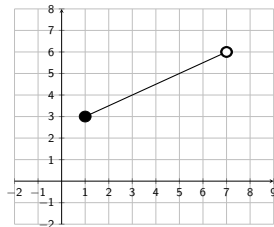
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Range:  $R = [0, \infty)$

- 1 State domain and range and function values:



$$f(3) = 4$$

$$f(5) = 5$$

$$f(1) = 3$$

$$f(8) = \text{undefined}$$

$$f(7) = \text{undefined}$$

$$f(6) = 5.5$$

Domain  $D = [1, 7)$

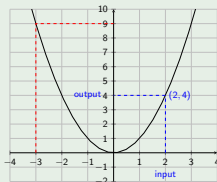
Range  $R =$

# Functions by graphs

## Example

Graph  $f(x) = x^2$

x	-3	-2	-1	0	1	2	3
y	9	4	1	0	1	4	9



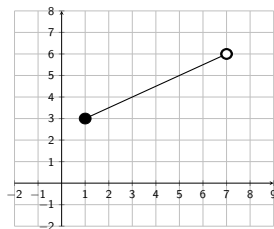
$$f(2) = 4$$

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Domain:  $D = \mathbb{R}$

Range:  $R = [0, \infty)$

- 1 State domain and range and function values:



$$f(3) = 4$$

$$f(5) = 5$$

$$f(1) = 3$$

$$f(8) = \text{undefined}$$

$$f(7) = \text{undefined}$$

$$f(6) = 5.5$$

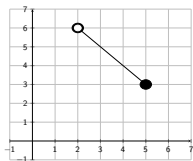
Domain  $D = [1, 7)$

Range  $R = [3, 6)$

# Functions by graphs - exercises

Find the function values, and the domain and range.

2

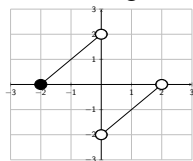


$$f(3) = \quad f(2) =$$

$$D =$$

$$R =$$

4

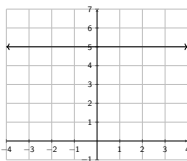


$$f(0) = \quad f(-2) =$$

$$D =$$

$$R =$$

6

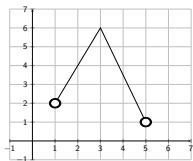


$$y = 5$$

$$f(-3) = \quad f(\sqrt{17}) =$$

$$D = \quad R =$$

3

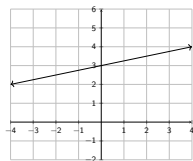


$$f(3) = \quad f(2) =$$

$$D =$$

$$R =$$

5



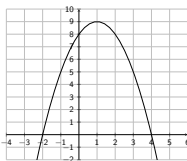
$$y = \frac{1}{4}x + 3$$

$$f(3) = \quad f(24) =$$

$$D =$$

$$R =$$

7



$$y = -x^2 + 2x + 8$$

$$f(1) = \quad f(5) =$$

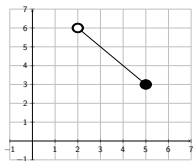
$$D =$$

$$R =$$

# Functions by graphs - exercises

Find the function values, and the domain and range.

2

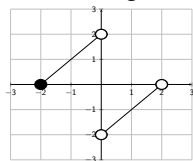


$$f(3) = 5 \quad f(2) = \text{undef.}$$

$$D = (2, 5]$$

$$R = [3, 6]$$

4

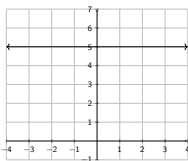


$$f(0) = \quad f(-2) =$$

$$D =$$

$$R =$$

6

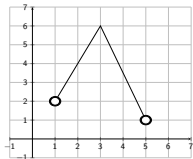


$$y = 5$$

$$f(-3) = \quad f(\sqrt{17}) =$$

$$D = \quad R =$$

3

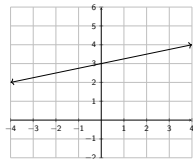


$$f(3) = \quad f(2) =$$

$$D =$$

$$R =$$

5



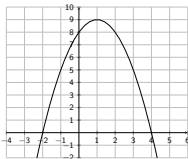
$$y = \frac{1}{4}x + 3$$

$$f(3) = \quad f(24) =$$

$$D =$$

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7



$$y = -x^2 + 2x + 8$$

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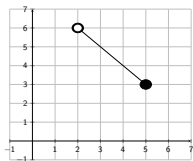
$$D =$$

$$R =$$

# Functions by graphs - exercises

Find the function values, and the domain and range.

2

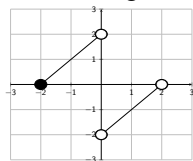


$$f(3) = 5 \quad f(2) = \text{undef.}$$

$$D = (2, 5]$$

$$R = [3, 6]$$

4

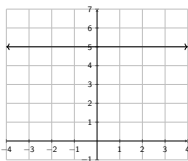


$$f(0) = \quad f(-2) =$$

$$D =$$

$$R =$$

6

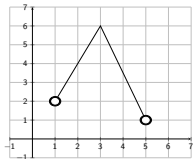


$$y = 5$$

$$f(-3) = \quad f(\sqrt{17}) =$$

$$D = \quad R =$$

3

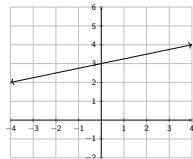


$$f(3) = 6 \quad f(2) = 4$$

$$D = (1, 5)$$

$$R = (1, 6]$$

5



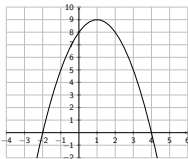
$$y = \frac{1}{4}x + 3$$

$$f(3) = \quad f(24) =$$

$$D =$$

$$R =$$

7



$$y = -x^2 + 2x + 8$$

$$f(1) = \quad f(5) =$$

$$D =$$

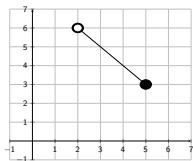
$$R =$$



# Functions by graphs - exercises

Find the function values, and the domain and range.

2

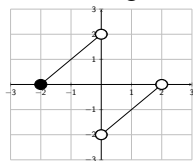


$$f(3) = 5 \quad f(2) = \text{undef.}$$

$$D = (2, 5]$$

$$R = [3, 6]$$

4

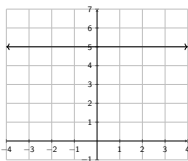


$$f(0) = \text{undef.} \quad f(-2) = 0$$

$$D = [-2, 0) \cup (0, 2)$$

$$R = (-2, 2)$$

6

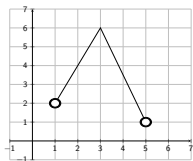


$$y = 5$$

$$f(-3) = \quad f(\sqrt{17}) =$$

$$D = \quad R =$$

3

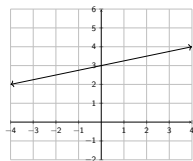


$$f(3) = 6 \quad f(2) = 4$$

$$D = (1, 5)$$

$$R = (1, 6]$$

5



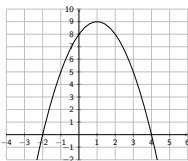
$$y = \frac{1}{4}x + 3$$

$$f(3) = \quad f(24) =$$

$$D =$$

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7



$$y = -x^2 + 2x + 8$$

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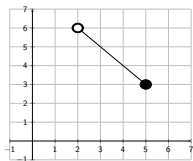
$$D =$$

$$R =$$

# Functions by graphs - exercises

Find the function values, and the domain and range.

2

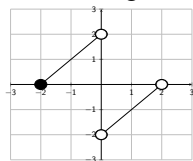


$$f(3) = 5 \quad f(2) = \text{undef.}$$

$$D = (2, 5]$$

$$R = [3, 6]$$

4

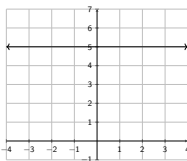


$$f(0) = \text{undef.} \quad f(-2) = 0$$

$$D = [-2, 0) \cup (0, 2)$$

$$R = (-2, 2)$$

6

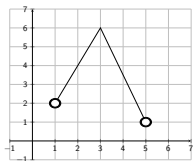


$$y = 5$$

$$f(-3) = \quad f(\sqrt{17}) =$$

$$D = \quad R =$$

3

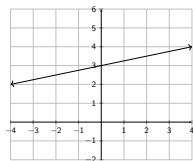


$$f(3) = 6 \quad f(2) = 4$$

$$D = (1, 5)$$

$$R = (1, 6]$$

5



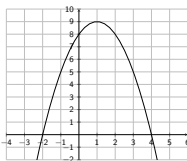
$$y = \frac{1}{4}x + 3$$

$$f(3) = 3.75 \quad f(24) = 9$$

$$D = \mathbb{R}$$

$$R = \mathbb{R}$$

7



$$y = -x^2 + 2x + 8$$

$$f(1) = \quad f(5) =$$

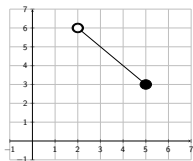
$$D =$$

$$R =$$

# Functions by graphs - exercises

Find the function values, and the domain and range.

2

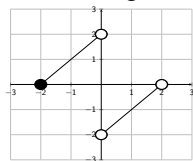


$$f(3) = 5 \quad f(2) = \text{undef.}$$

$$D = (2, 5]$$

$$R = [3, 6]$$

4

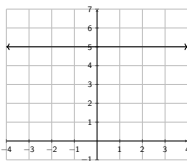


$$f(0) = \text{undef.} \quad f(-2) = 0$$

$$D = [-2, 0) \cup (0, 2)$$

$$R = (-2, 2)$$

6

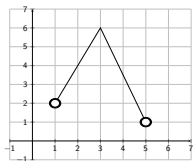


$$y = 5$$

$$f(-3) = 5 \quad f(\sqrt{17}) = 5$$

$$D = \mathbb{R} \quad R = \{5\}$$

3

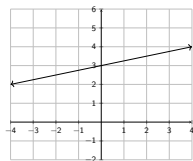


$$f(3) = 6 \quad f(2) = 4$$

$$D = (1, 5)$$

$$R = (1, 6]$$

5



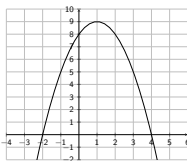
$$y = \frac{1}{4}x + 3$$

$$f(3) = 3.75 \quad f(24) = 9$$

$$D = \mathbb{R}$$

$$R = \mathbb{R}$$

7



$$y = -x^2 + 2x + 8$$

$$f(1) = \quad f(5) =$$

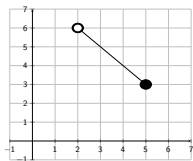
$$D =$$

$$R =$$

# Functions by graphs - exercises

Find the function values, and the domain and range.

2

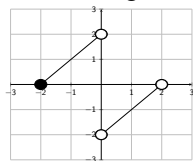


$$f(3) = 5 \quad f(2) = \text{undef.}$$

$$D = (2, 5]$$

$$R = [3, 6]$$

4

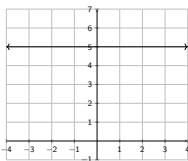


$$f(0) = \text{undef.} \quad f(-2) = 0$$

$$D = [-2, 0) \cup (0, 2)$$

$$R = (-2, 2)$$

6

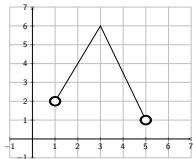


$$y = 5$$

$$f(-3) = 5 \quad f(\sqrt{17}) = 5$$

$$D = \mathbb{R} \quad R = \{5\}$$

3

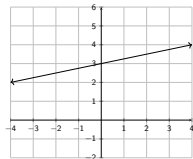


$$f(3) = 6 \quad f(2) = 4$$

$$D = (1, 5)$$

$$R = (1, 6]$$

5



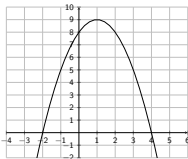
$$y = \frac{1}{4}x + 3$$

$$f(3) = 3.75 \quad f(24) = 9$$

$$D = \mathbb{R}$$

$$R = \mathbb{R}$$

7



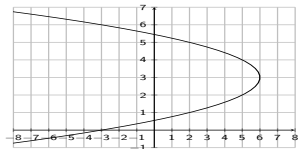
$$y = -x^2 + 2x + 8$$

$$f(1) = 9 \quad f(5) = -7$$

$$D = \mathbb{R}$$

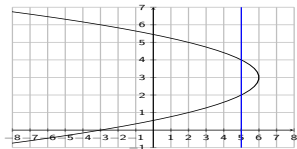
$$R = (-\infty, 9]$$

# Vertical line test



Find  $f(5) =$

# Vertical line test

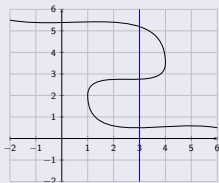


Find  $f(5) = ???$  2 or 4 ???

The input 5 has multiple outputs!

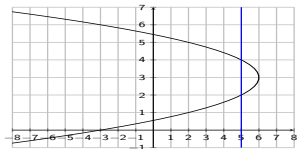
$\Rightarrow f$  is not a function! **X** ( $f$  is a relation.)

## Vertical line test



If there is a vertical line, which intersects the graph in more than one point, then the graph is not the graph of a function.

# Vertical line test

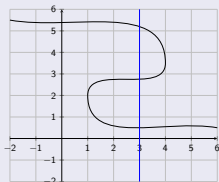


Find  $f(5) = ???$  2 or 4 ???

The input 5 has multiple outputs!

$\Rightarrow f$  is not a function! **X** ( $f$  is a relation.)

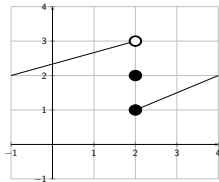
## Vertical line test



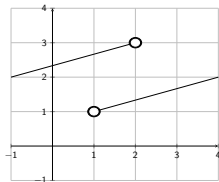
If there is a vertical line, which intersects the graph in more than one point, then the graph is not the graph of a function.

Are these graphs of a function?

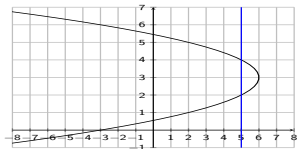
1



2



# Vertical line test



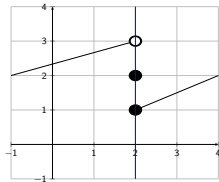
Find  $f(5) = ???$  2 or 4 ???

The input 5 has multiple outputs!

$\Rightarrow f$  is not a function! **X** ( $f$  is a relation.)

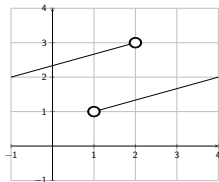
Are these graphs of a function?

1

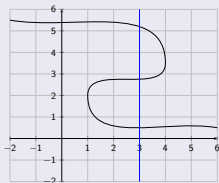


No!

2



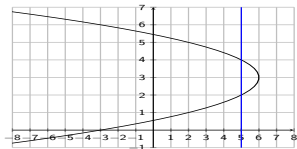
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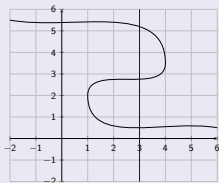


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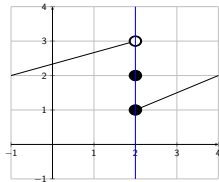
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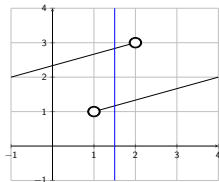
Are these graphs of a function?

1



No!

2



No!

