Functions via formulas

Lesson #2

MAT 1375 Precalculus

New York City College of Technology CUNY

Functions by formulas

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   f is a *piecewise defined* function!
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   $f(a+5) = (a+5)^2 + 3(a+5)$
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   Find $f(2) = 3 \cdot 2 + 2 = 6 + 2 = 8$
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   Find $f(4) =$ undefined
   Find $f(5) = 5^2 - 5 = 25 - 5 = 20$
   Find $f(7) = 7^2 - 7 = 49 - 7 = 42$
   Find $f(1) =$ undefined
7. $f(x) = \begin{cases} |x| & \text{for } -6 \leq x < 2 \\ \sqrt{x-1} & \text{for } 2 < x \leq 7 \\ x+2 & \text{for } 7 < x < 8 \end{cases}$
   Find $f(3) = \sqrt{3-1} = \sqrt{2}$
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8. Let $f(x) = x^2 + 3x$. Find $f(a+5)$
   $f(a+5) = (a+5)^2 + 3(a+5)$
   $= a^2 + 2 \cdot a \cdot 5 + 25 + 3a + 15$
   $= a^2 + 10a + 25 + 3a + 15$
   $= a^2 + 13a + 40$
9. Let $f(x) = x^2 - x + 7$. Find $f(b-2)$

1. Let $f(x) = \frac{x+3}{x+5}$. Find $f(7)$.
   $f(7) = \frac{7+3}{7+5} = \frac{10}{12} = \frac{5}{6}$
2. Let $f(x) = \sqrt{x^2 + 2x}$. Find $f(4)$
   $f(4) = \sqrt{4^2 + 2 \cdot 4} = \sqrt{16 + 8}$
   $= \sqrt{24} = 2\sqrt{6}$
3. Let $f(x) = x^2 + 3x$. Find $f(5)$
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Functions by formulas

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10. Let $f(x) = \sqrt{x^2-3}$. Find $f(x+h)$.
    $f(x+h) = \sqrt{(x+h)^2 - 3}$
    $= \sqrt{x^2 + 2xh + h^2 - 3}$

Difference quotient

Definition

For a function $y = f(x)$, the *difference quotient* is defined as

$$\frac{f(x+h)-f(x)}{h}$$

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For a function $y = f(x)$, the *difference quotient* is defined as

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2. Find the difference quotient of $f(x) = x^2 + 8x$

Difference quotient (quadratic polynomials) - exercises

Find the difference quotients of the following functions.

2. $f(x) = x^2 + 8x$
3. $f(x) = x^2 - 3x + 7$
4. $f(x) = 3x^2 - 2x - 5$
5. $f(x) = -5x^2 - x + 9$

Find the difference quotients of the following functions.

2. $f(x) = x^2 + 8x$

$$\begin{aligned} f(x+h) &= (x+h)^2 + 8(x+h) \\ &= x^2 + 2xh + h^2 + 8x + 8h \end{aligned}$$

$$\begin{aligned} f(x+h) - f(x) &= \\ &= (x^2 + 2xh + h^2 + 8x + 8h) - (x^2 + 8x) \\ &= x^2 + 2xh + h^2 + 8x + 8h - x^2 - 8x \\ &= 2xh + h^2 + 8h \end{aligned}$$

$$\frac{f(x+h)-f(x)}{h} = \frac{2xh+h^2+8h}{h} = \frac{h\cdot(2x+h+8)}{h} = 2x + h + 8$$

3. $f(x) = x^2 - 3x + 7$

4. $f(x) = 3x^2 - 2x - 5$

5. $f(x) = -5x^2 - x + 9$

Difference quotient (quadratic polynomials) - exercises

Find the difference quotients of the following functions.

2. $f(x) = x^2 + 8x$

$$f(x+h) = (x+h)^2 + 8(x+h) = x^2 + 2xh + h^2 + 8x + 8h$$

$$f(x+h) - f(x) = (x^2 + 2xh + h^2 + 8x + 8h) - (x^2 + 8x) = x^2 + 2xh + h^2 + 8x + 8h - x^2 - 8x = 2xh + h^2 + 8h$$

$$\frac{f(x+h)-f(x)}{h} = \frac{2xh+h^2+8h}{h} = \frac{h\cdot(2x+h+8)}{h} = 2x + h + 8$$

3. $f(x) = x^2 - 3x + 7$

$$f(x+h) = (x+h)^2 - 3(x+h) + 7 = x^2 + 2xh + h^2 - 3x - 3h + 7$$

$$f(x+h) - f(x) = (x^2 + 2xh + h^2 - 3x - 3h + 7) - (x^2 - 3x + 7) = 2xh + h^2 - 3h$$

$$\frac{f(x+h)-f(x)}{h} = \frac{2xh+h^2-3h}{h} = \frac{h\cdot(2x+h-3)}{h} = 2x + h - 3$$

4. $f(x) = 3x^2 - 2x - 5$

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Difference quotient (quadratic polynomials) - exercises

Find the difference quotients of the following functions.

2. $f(x) = x^2 + 8x$

$$f(x+h) = (x+h)^2 + 8(x+h)$$
$$= x^2 + 2xh + h^2 + 8x + 8h$$
$$f(x+h) - f(x) =$$
$$= (x^2 + 2xh + h^2 + 8x + 8h) - (x^2 + 8x)$$
$$= x^2 + 2xh + h^2 + 8x + 8h - x^2 - 8x$$
$$= 2xh + h^2 + 8h$$

$$\frac{f(x+h)-f(x)}{h} = \frac{2xh+h^2+8h}{h} = \frac{h\cdot(2x+h+8)}{h}$$
$$= 2x + h + 8$$

3. $f(x) = x^2 - 3x + 7$

$$f(x+h) = (x+h)^2 - 3(x+h) + 7$$
$$= x^2 + 2xh + h^2 - 3x - 3h + 7$$
$$f(x+h) - f(x) =$$
$$= (x^2 + 2xh + h^2 - 3x - 3h + 7) - (x^2 - 3x + 7)$$
$$= 2xh + h^2 - 3h$$
$$\frac{f(x+h)-f(x)}{h} = \frac{2xh+h^2-3h}{h} = \frac{h\cdot(2x+h-3)}{h}$$
$$= 2x + h - 3$$

4. $f(x) = 3x^2 - 2x - 5$

$$f(x+h) = 3(x+h)^2 - 2(x+h) - 5$$
$$= 3(x^2 + 2xh + h^2) - 2x - 2h - 5$$
$$= 3x^2 + 6xh + 3h^2 - 2x - 2h - 5$$
$$f(x+h) - f(x) = 6xh + 3h^2 - 2h$$
$$\frac{f(x+h)-f(x)}{h} = \frac{6xh+3h^2-2h}{h}$$
$$= \frac{h\cdot(6x+3h-2)}{h} = 6x + 3h - 2$$

5. $f(x) = -5x^2 - x + 9$

Find the difference quotients of the following functions.

2. $f(x) = x^2 + 8x$

$$f(x+h) = (x+h)^2 + 8(x+h)$$
$$= x^2 + 2xh + h^2 + 8x + 8h$$
$$f(x+h) - f(x) =$$
$$= (x^2 + 2xh + h^2 + 8x + 8h) - (x^2 + 8x)$$
$$= x^2 + 2xh + h^2 + 8x + 8h - x^2 - 8x$$
$$= 2xh + h^2 + 8h$$
$$\frac{f(x+h)-f(x)}{h} = \frac{2xh+h^2+8h}{h} = \frac{h\cdot(2x+h+8)}{h}$$
$$= 2x + h + 8$$

3. $f(x) = x^2 - 3x + 7$

$$f(x+h) = (x+h)^2 - 3(x+h) + 7$$
$$= x^2 + 2xh + h^2 - 3x - 3h + 7$$
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$$\frac{f(x+h)-f(x)}{h} = \frac{2xh+h^2-3h}{h} = \frac{h\cdot(2x+h-3)}{h}$$
$$= 2x + h - 3$$

4. $f(x) = 3x^2 - 2x - 5$

$$f(x+h) = 3(x+h)^2 - 2(x+h) - 5$$
$$= 3(x^2 + 2xh + h^2) - 2x - 2h - 5$$
$$= 3x^2 + 6xh + 3h^2 - 2x - 2h - 5$$
$$f(x+h) - f(x) = 6xh + 3h^2 - 2h$$
$$\frac{f(x+h)-f(x)}{h} = \frac{6xh+3h^2-2h}{h}$$
$$= \frac{h\cdot(6x+3h-2)}{h} = 6x + 3h - 2$$

5. $f(x) = -5x^2 - x + 9$

$$f(x+h) = -5(x+h)^2 - (x+h) + 9$$
$$= -5(x^2 + 2xh + h^2) - x - h + 9$$
$$= -5x^2 - 10xh - 5h^2 - x - h + 9$$
$$f(x+h) - f(x) = -10xh - 5h^2 - h$$
$$\frac{f(x+h)-f(x)}{h} = \frac{-10xh-5h^2-h}{h}$$
$$= \frac{h\cdot(-10x-5h-1)}{h} = -10x - 5h - 1$$

Find the difference quotients of the following functions.

6. $f(x) = x^3$
7. $f(x) = \frac{2}{3x-4}$
8. $f(x) = \frac{1}{7x+2}$
9. $f(x) = \sqrt{2x+3}$

Difference quotient (polynomial, rational, square root functions) - exercises

Find the difference quotients of the following functions.

6. $f(x) = x^3$

$$= (x+h)(x+h)(x+h)$$
$$= (x^2 + 2xh + h^2)(x+h)$$
$$= x^3 + x^2h + 2x^2h + 2xh^2 + xh^2 + h^3$$
$$= x^3 + 3x^2h + 3xh^2 + h^3$$
$$f(x+h) - f(x) = 3x^2h + 3xh^2 + h^3$$
$$\frac{f(x+h)-f(x)}{h} = \frac{3x^2h+3xh^2+h^3}{h}$$
$$= \frac{h\cdot(3x^2+3xh+h^2)}{h} = 3x^2 + 3xh + h^2$$

7. $f(x) = \frac{2}{3x-4}$

8. $f(x) = \frac{1}{7x+2}$

9. $f(x) = \sqrt{2x+3}$

Find the difference quotients of the following functions.

6. $f(x) = x^3$

   $= (x+h)(x+h)(x+h)$

   $= (x^2 + 2xh + h^2)(x+h)$

   $= x^3 + x^2h + 2x^2h + 2xh^2 + xh^2 + h^3$

   $= x^3 + 3x^2h + 3xh^2 + h^3$

   $f(x+h) - f(x) = 3x^2h + 3xh^2 + h^3$

   $\frac{f(x+h)-f(x)}{h} = \frac{3x^2h+3xh^2+h^3}{h}$

   $= \frac{h\cdot(3x^2+3xh+h^2)}{h} = 3x^2 + 3xh + h^2$

7. $f(x) = \frac{2}{3x-4}$

   $f(x+h) = \frac{2}{3(x+h)-4} = \frac{2}{3x+3h-4}$

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   $= \frac{2(3x-4)-2(3x+3h-4)}{(3x+3h+4)(3x-4)}$

   $= \frac{6x-8-6x-6h+8}{(3x+3h+4)(3x-4)}$

   $= \frac{-6h}{(3x+3h+4)(3x-4)}$

   $\frac{f(x+h)-f(x)}{h} = \frac{-6h}{h\cdot(3x+3h+4)(3x-4)}$

   $= \frac{-6}{(3x+3h+4)(3x-4)}$

8. $f(x) = \frac{1}{7x+2}$

9. $f(x) = \sqrt{2x+3}$

Find the difference quotients of the following functions.

6. $f(x) = x^3$

$= (x+h)(x+h)(x+h)$

$= (x^2 + 2xh + h^2)(x+h)$

$= x^3 + x^2h + 2x^2h + 2xh^2 + xh^2 + h^3$

$= x^3 + 3x^2h + 3xh^2 + h^3$

$f(x+h) - f(x) = 3x^2h + 3xh^2 + h^3$

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$$= x^3+3x^2h+3xh^2+h^3$$
$$f(x+h)-f(x) = 3x^2h+3xh^2+h^3$$
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$$= \frac{(\sqrt{2x+2h+3}-\sqrt{2x+3})}{h} \cdot \frac{(\sqrt{2x+2h+3}+\sqrt{2x+3})}{(\sqrt{2x+2h+3}+\sqrt{2x+3})}$$
$$= \frac{\sqrt{2x+2h+3}^2-\sqrt{2x+3}^2}{h\cdot(\sqrt{2x+2h+3}+\sqrt{2x+3})}$$
$$= \frac{2x+2h+3-2x-3}{h\cdot(\sqrt{2x+2h+3}+\sqrt{2x+3})}$$
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$$= \frac{2}{\sqrt{2x+2h+3}+\sqrt{2x+3}}$$

Standard domain

- $f(x) = \frac{2}{x-3}$. Find $f(8) =$
  $f(3) =$

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- $f(x) = \frac{2}{x-3}$. Find $f(8) = \frac{2}{8-3} = \frac{2}{5}$ ✓

  ✗ $f(3) = \frac{2}{0}$ is undefined!

  $\Rightarrow$ 3 is not an allowed input!

  The domain is $D = (-\infty, 3) \cup (3, \infty)$.

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- $f(x) = \sqrt{x-5}$. Find $f(9) =$

  $f(1) =$

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- $f(x) = \sqrt{x-5}$. Find $f(9) = \sqrt{4} = 2$ ✓
  ✗ $f(1) = \sqrt{-4}$ is undefined!
  $\Rightarrow$ 1 is not an allowed input: we don't want complex numbers as outputs!
  Domain: $x - 5 \geq 0 \Rightarrow x \geq 5$
  $\Rightarrow D = [5, \infty)$

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Standard domain

If no domain is specified, then we assume it to be the *standard domain*, which is the largest possible domain for which our outputs are real numbers.

- fractions $\frac{g(x)}{h(x)}$ must have non-zero denominator $h(x) \neq 0$
- square roots $\sqrt{g(x)}$ must have positive or zero arguments $g(x) \geq 0$
- polynomials are defined for all real numbers

Standard domain - exercises

Find the domain of the given function:

1. $f(x) = \frac{x+4}{x+5}$

2. $f(x) = \frac{7x+3}{6x-4}$

3. $f(x) = \frac{2}{x^2+5x-14}$

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2. $f(x) = \frac{7x+3}{6x-4}$
   When is the denominator $= 0$?
   $$\begin{aligned} 6x - 4 = 0 \quad &\Longrightarrow \quad 6x = 4 \\ &\Longrightarrow \quad x = \frac{4}{6} = \frac{2}{3} \end{aligned}$$
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5. $f(x) = 5 \cdot \sqrt{7 - 3x}$

6. $f(x) = x^3 + 4x^2 - 2x + 7$

7. $f(x) = \begin{cases} 3x & \text{for } 2 < x \leq 5 \\ x^2 & \text{for } 7 \leq x < 9 \\ x + 5 & \text{for } 12 \leq x \end{cases}$

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