

Numbers and functions

Lesson #1

MAT 1375 Precalculus

New York City College of Technology CUNY



Number systems

Natural numbers \mathbb{N}

1, 2, 3, 4, 5, ...

Examples: 7, 2, 5, 1003

Integers \mathbb{Z}

..., -3, -2, -1, 0, 1, 2, 3, ...

Examples: -3, 7, 2, 5, 0, 1

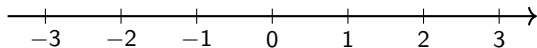
Rational numbers \mathbb{Q}

fractions $\frac{a}{b}$ for integers a, b , and $b \neq 0$

Examples: $\frac{2}{3}$, $7 = \frac{7}{1}$, -3, 0, $\frac{-36}{17}$

Real numbers \mathbb{R}

numbers on the number line



Examples: $\frac{2}{3}$, 7, -3, 0, $\frac{-36}{17}$
Examples of **irrational** numbers:
 π , $\sqrt{2}$, $\sqrt[3]{7}$

Complex numbers \mathbb{C}

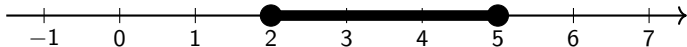
$a + bi$, for real numbers a, b

Examples: $2+3i$, $i=0+1i$

Interval notation - review

3 ways to express an interval

1 On the number line:



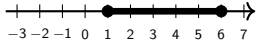
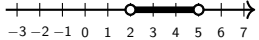
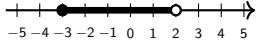
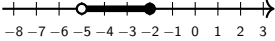
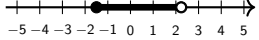
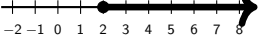
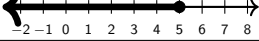
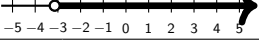
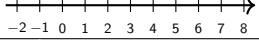
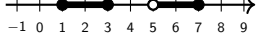
2 Inequality notation:

$$2 \leq x \leq 5$$

3 Interval notation:

$$[2, 5]$$

Interval notation - exercises

	Inequality notation	Number line	Interval notation
(a)	$1 \leq x \leq 6$		$[1, 6]$
(b)	$2 < x < 5$		$(2, 5)$
(c)	$-3 \leq x < 2$		$[-3, 2)$
(d)	$-5 < x \leq -2$		$(-5, -2]$
(e)	$-1.5 \leq x < \sqrt{5}$		$[-1.5, \sqrt{5})$
(f)	$2 \leq x$		$[2, \infty)$
(g)	$x \leq 5$		$(-\infty, 5]$
(h)	$-\pi < x$		$(-\pi, \infty)$
(i)	$5 \leq x \leq 2$		no such number
(j)	$1 \leq x \leq 3$ or $5 < x \leq 7$		$[1, 3] \cup (5, 7]$

Functions - definition

Definition

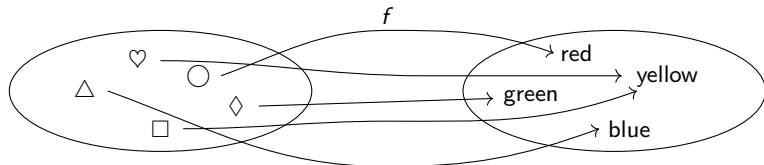
A *function* is an assignment, which assigns to each input x exactly one output y .

The set of all inputs is called the *domain*, denoted by D .

The set of all outputs is called the *range*, denoted by R .

We write $f(x) = y$ or $f : x \mapsto y$, if f assigns to the input x the output y .

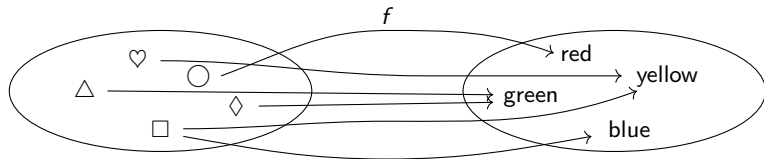
1



Function values: $f(\square) = \text{yellow}$ $f(\diamond) = \text{green}$ $f(\heartsuit) = \text{yellow}$...
Domain $D = \{\triangle, \diamond, \circ, \square, \heartsuit\}$ Range $R = \{\text{red}, \text{green}, \text{blue}, \text{yellow}\}$

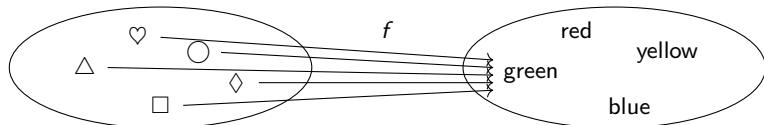
Functions - examples

2



Function values: $f(\triangle) = \text{green}$ $f(\square) = ???$ blue or yellow ???
 \Rightarrow f is not a function \times , since \square has more than one output! (f is a *relation*.)

3



Is f a function? Yes, f is a function! \checkmark

Domain $D = \{\triangle, \diamond, \circ, \square, \heartsuit\}$

Range $R = \{\text{green}\}$

Codomain $C = \{\text{green, yellow, red, blue}\}$

• Notation: we write $f : D \rightarrow C$

Function values: $f(\square) = \text{green}$ $f(\diamond) = \text{green}$ $f(\heartsuit) = \text{green}$...

Functions - word problems

- 1 Consider the assignment which associates to each student the chair on which the student sits.
 - **Question:** What is the domain?
Answer: The domain is the set of all students in the classroom.
 - **Question:** What is the range?
Answer: The range is the set of all chairs in the classroom.
 - **Question:** Does this assignment constitute a function?
Answer: It is a function as long as each student sits on a chair.
 - Pick a specific student.
Describe what the function assigns to this student in your own words!
- 2 A gift card has been preloaded with a value of \$30. You want to use the gift card at a coffee shop, where your favorite cup of coffee costs \$2. The function $f(x) = 30 - 2x$ models the amount of money left on the card after purchasing x many cups of coffee.
 - **Question:** Interpret the meaning of $f(8) = 14$.
Answer: After buying 8 cups of coffee, there is \$14 left on the card.
 - **Question:** Interpret the meaning of $f(x) > 0$.
Answer: $f(x) > 0$ means that after purchasing x cups of coffee there is still some money left on the card.

