Lesson 13: Exponential and Logarithmic Functions

$f(x) = 2^x$ ← exponent
↑ base

Domain $(-\infty, \infty)$ $\mathbb{R}$
Range $(0, \infty)$
Asymptotes $y = 0$
Intercepts $(0, 1)$

What happens when we multiply by a constant?

$y = 2^x$

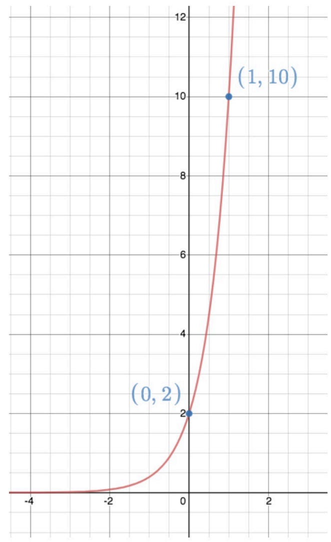
$y = 5 \cdot 2^x$

Exponential functions

$y = c \cdot b^x$, $c, b \in \mathbb{R}$
$b > 0$

y-intercept $(0, c)$

Example 3: The graph below shows an exponential function $f(x)$.
Find a formula for $f(x)$.



$f(x) = c \cdot b^x$ $c = 2$

y-intercept $(0,2)$

$f(x) = 2 \cdot b^x$

$f(1) = 10$

$10 = 2 \cdot b^1$

$\frac{10}{2} = \frac{2 \cdot b}{2}$

$5 = b$

$\boxed{f(x) = 2 \cdot 5^x}$

Logarithmic Functions

If $b > 0$, $b \neq 1$ then we define the logorithm in base b as:

$y = \log_b x$ if and only if $b^y = x$

Example 13.9. Rewrite the equation as a logarithmic equation.

a) $3^4 = 81$, b) $10^3 = 1000$,
c) $e^x = 17$, d) $2^{7 \cdot a} = 53$.

a) $4 = \log_3 81$ b) $3 = \log_{10}(1000)$

c) $x = \log_e 17$ d) $7 \cdot a = \log_2 53$

special bases: $\boxed{\log x}$ means $\log_{10} x$

$e = 2.718\ldots$ $\boxed{\ln x}$ means $\log_e x$

a) $\log_2(16) = y$
$2^y = 16$
$y = 4$
$\log_2(16) = 4$

b $\log_5 125 = y$
$5^y = 125$
$y = 3$
$\log_5 125 = 3$

c) $\log_{13} 1 = y$
$13^y = 1$
$y = 0$
$\log_{13} 1 = 0$

d) $\log_4(4) = y$
$4^y = 4$
$y = 1$
$\log_4 4 = 1$

Example 13.10. Evaluate the expression by rewriting it as an exponential expression.

a) $\log_2(16)$, b) $\log_5(125)$, c) $\log_{13}(1)$, d) $\log_4(4)$
e) $\log(10,000)$, f) $\log(0.001)$, g) $\ln(e^7)$, h) $\log_b(b^x)$

e) $\log(10,000) = \log_{10}(10\,000)$
$= 4$

f) $\log(0.001)$
$\log_{10}(0.001) = y$
$10^y = 0.001$
$y = -3$
$\log(0.001) = -3$

g) $\ln(e^7)$
$\log_e(e^7) = y$
$e^y = e^7$
$y = 7$

h) $\log_b(b^x) = y$

$$b^y = b^x$$
$$y = x$$
$$\log_b(b^x) = x$$

Logarithm as inverse function

$y = 2^x$

Is it one-to-one? Yes.
It has an inverse function.

$y = 2^x$

x	y
1	2
0	1
2	4
-1	1/2

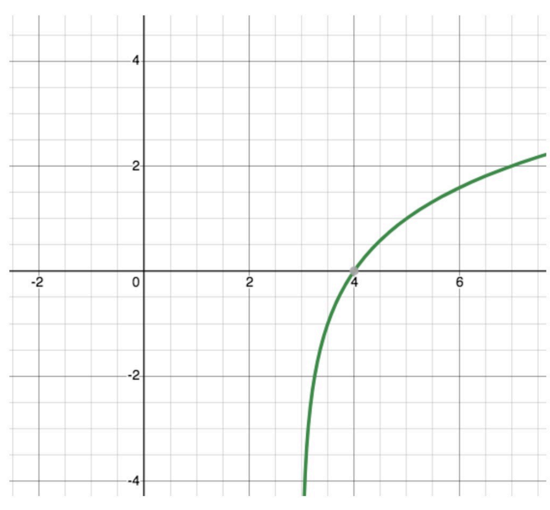
$y = \log_2 x$

x	y
2	1
1	0
4	2
$\frac{1}{2}$	-1

Domain: $(0, \infty)$
Range: $(-\infty, \infty)$
$\mathbb{R}$

vertical asymptote
$x = 0$

Example 4. The graph below shows the function $y = \log_2(x)$ but shifted to the right 3 units. Find a formula for the function in the graph.



replace x
with
$x - 3$

$y = \log_2 x$

$$\boxed{y = \log_2(x - 3)}$$

Example 5: Sketch the graph of the function $y = \log_3(2x + 5)$.