MAT 1372 Stat w/ Prob classwk 19 Spring 2012

**7.3 SAMPLE MEAN**

Before we jump to the new material, let’s review some important definitions:

**Expectation:**

**Variance:**

and properties:





 : provided that X and Y are independent!



Suppose that we sample n times from a population with mean μ and std dev σ. The **sample mean** is defined to be



Exercise: use the properties above to prove

1. 
2. 

For result 2 above, if we take the square root of both sides, we get



If theare the standard normal distribution, some of the sample mean distributions are plotted below:



Surprisingly, the sample means for non-normal distributions become more and more normal as n gets large, a result known as the CENTRAL LIMIT THEOREM.

**1.** Consider the population consisting of equal numbers of 1’s and 2’s. Plot the possible values along with their probabilities of the sample mean of a sample of specified size

*n* = 2 is done in the book as Example 7.1



**(a)** *n* = 3 (done in class, below)

**(b)** *n* = 4 (for homework)

Also derive the variance and standard deviation of the sample mean.

Are your answers consistent with the formulas presented in this section?

Clearly, μ=1.5. To find variance, we use 



If flip a coin 3 times, there are 8 possible outcomes, each with equal probability:

111,112,121,122,211,212,221,222

We can group them base on their sums:

3 111

4 112,121, 211

5 122, 212,221

6 222

Remember, that in addition to summing, we want to divide by n, which is 3

The result is



which is consistent with the result 

To find the variance, we use the property



(the book uses the definition)





So 

which is consistent with the result 

**2.** Suppose that *X*1 and *X*2 constitute a sample of size 2 from a population

in which a typical value *X* is equal to either 1 or 2 with respective probabilities

*P*{*X* = 1} = 0.7 *P*{*X* = 2} = 0.3

**(a)** Compute *E*[*X*].

**(b)** Compute Var(*X*).

**(c)** What are the possible values of = (*X*1 + *X*2) /2?

**(d)** Determine the probabilities that assumes the values in (c).

**(e)** Using (d), directly compute and 

**(f)** Are your answers to (a), (b), and (e) consistent with the formulas

presented in this section?

Solution:

**(a)** 

**(b)** 

**(c), (d)**



**(e), (f)** 

which are in agreement with and 

**4.** The amount of money withdrawn in each transaction at an automatic

teller of a branch of the Bank of America has mean $80 and standard

deviation $40. What are the mean and standard deviation of the average

amount withdrawn in the next 20 transactions?

**7.** The weight of a randomly chosen person riding a ferry has expected

value 155 pounds and standard deviation 28 pounds. The ferry has

the capacity to carry 100 riders. Find the expected value and standard

deviation of the total passenger weight load of a ferry at capacity.