MAT 1372 Stat w/ Prob classwk 10 Spring 2012

Review (of 5.2):

A **random variable** X is an experiment with a SINGLE numeric outcome

Some examples:

**8.** Suppose that 2 batteries are randomly chosen from a bin containing

10 batteries, of which 7 are good and 3 are defective. Let *X* denote the

number of defective batteries chosen. Give the possible values of *X*

along with their probabilities.

Answer:



**19.** A bakery has 3 special cakes at the beginning of the day. The daily

demand for this type of cake is

# probability

0 0.15

1 0.20

2 0.35

3 0.15

4 0.10

5+ 0.05

Let *X* denote the number of cakes that remain unsold at the end of the

day. Determine the probability distribution of *X*.

Answer:

0 1 2 3

0.3 0.35 0.2 0.15

**5.3 EXPECTED VALUE**

The expected value of a random variable X (also known as the **mean)** is the sum of the outcomes weighted by their probabilities:



**Examples**:

From 5.2.8, the number of bad batteries that will be selected on average:



In other words, just slightly more than half a bad battery.

From 5.2.9, the number of customers wanting a cake is



In other words, on average 2 customers will ask for a cake.

If instead we look at the expectation for the number of cakes that will be left unsold, we get



In other words, on average 1.2 cakes will be left unsold.

More examples and exercises:

**6.** Let *X* be a random variable that is equally likely to take on any of the

values 1, 2, . . . , *n*, i.e.,



(This is a generalization of a 6 sided die, which has expectation 3.5.)

Find E[X].

n E[X] formula

1 1 SUM(A$1:A1)/A1

2 1.5 SUM(A$1:A2)/A2

3 2 SUM(A$1:A3)/A3

4 2.5 SUM(A$1:A4)/A4

5 3 SUM(A$1:A5)/A5

6 3.5 SUM(A$1:A6)/A6

7 4 SUM(A$1:A7)/A7

8 4.5 SUM(A$1:A8)/A8

9 5 SUM(A$1:A9)/A9

10 5.5 SUM(A$1:A10)/A10

From the above table, we can see that the answer is (n+1)/2. To prove it, factor out 1/n from the sum and use the identity



**7.** A pair of fair dice is rolled. Find the expected value of the

**(a)** Smaller

**(b)** Larger

of the two upturned faces. (If both dice show the same number, then

take this to be the value of both the smaller and the larger of the

upturned faces.)

See Excel file. Answers are 91/36≈2.53 and 161/36≈4.47, respectively.

**12.** If the two teams in a World Series have the same chance of winning

each game, independent of the results of previously played games,

then the probabilities that the series will end in 4, 5, 6, or 7 games are,

respectively, 1/8, 1/4, 5/16, and 5/16. What is the expected number of

games played in such a series?

(Exercise: Show that these probabilities are indeed correct.)



**5.3.1 Properties of Expected Values**

Let *X* be a random variable with expected value *E*[*X*]. If *c* is a constant, then

*cX* and *X* + *c* are also random variables and have expected values:

*E*[*cX*] = *cE*[*X*]

*E*[*X* + *c*] = *E*[*X*] + *c*

Exercise: Put these properties into words.

Examples:

1. The government institutes an across the board 10% cut in salaries. How will the average salary be affected? (Ans: it will be cut by 10%: *E*[0*.9X*] = *0.9E*[*X*])
2. The government institutes an across the board raise of $1000 in salaries. How will average salary be affected? (Ans: it is $1000 more: *E*[*X* + *1000*] = *E*[*X*] + *1000*)

For any random variables *X* and *Y*, *E*[*X* + *Y*] = *E*[*X*] + *E*[*Y*]

Example: suppose that in a mythical country, there is one woman for every man and they are paired up to form household units. Suppose that on average, a woman makes a 45k salary (due to childrearing responsibilities) and a man makes a 63k salary. What is the average household salary?

*E*[*X* + *Y*] = *E*[*X*] + *E*[*Y*]=45k+63k=108k

Exercise:

**36.** Suppose that 2 batteries are randomly selected from a drawer containing

8 good and 2 defective batteries. Let *W* denote the number of defective batteries selected.

**(a)** Find *E*[*W*] by first determining the probability distribution of *W*.

Let *X* equal 1 if the first battery chosen is defective, and let *X* equal

0 otherwise. Also let *Y* equal 1 if the second battery is defective and

equal 0 otherwise.

**(b)** Give an equation relating *X*, *Y*, and *W*.

**(c)** Use the equation in (b) to obtain *E*[*W*].