MAT 1372 Stat w/ Prob classwk 8 Fall 2012

4.7 Counting Principles

**Basic Principle of Counting**

Suppose an experiment consists of two parts. If part 1 can result in any of *n* possible

outcomes and if for each outcome of part 1 there are *m* possible outcomes

of part 2, then there is a total of *nm* possible outcomes of the experiment.

Example: suppose that there are 20 students in a class and each student takes

3 exams over the course of semester,

then the instructor can look forward to grading 20\*3=60 exams.

**Generalized Basic Principle of Counting**

Suppose an experiment consists of *r* parts. Suppose there are *n*1 possible outcomes

of part 1 and then *n*2 possible outcomes of part 2 and then *n*3 possible outcomes

of part 3, and so on. Then there is a total of *n*1 · *n*2 · · · *nr* possible outcomes of the

experiment.

**6.** A well-known nursery tale starts as follows:

As I was going to St. Ives

I met a man with 7 wives.

Each wife had 7 sacks,

Each sack had 7 cats,

Each cat had 7 kittens.

How many kittens did our traveler meet?

**Permutations**

If a random, ordered selection of size m is made of n distinct objects without replacement

(n distinct balls in urn), then # of possibilities is .

**2.** How many different batting orders are possible for baseball team consisting of

9 players? 15 players? (Assume national league rules, i.e., no designated hitter)

**Example 4.20** (see Excel file)

If 25 people are in a room, what is the probability that no two of them

celebrate their birthday on the same day of the year (ignore the leap year complication)?

Solution: If we ask each person in some order what their birthday is, he or she must have

a different birthday from those asked earlier so 365\*364….346 is the number of 25-tuples

with distinct entries. The overall number of possible birthday 25-tuples is 365^25,

so probability is 365\*364….346/365^25 or 43%

**Combinations**

If a random, **un**ordered selection of size m is made of n distinct objects without replacement

(e.g. differently colored balls in an urn),

then the number of possibilities is “n choose m”: .

**Example** Consider a group of 10 people. A committee of 5 must be chosen,

with one member designated as chairperson. How many possible committees are there?

**Solution** First chose the chairperson, then the other 4 members (unordered):

 

**12.** A delivery company has 10 trucks, of which 3 have faulty brakes. If an

inspector randomly chooses 2 of the trucks for a brake check, what is

the probability that none of the trucks with faulty brakes are chosen?

(see Excel file)

**14.** In a state lottery, a player must choose 8 of the numbers from 1 to 40.

The Lottery Commission then performs an experiment that selects 8 of

these 40 numbers. Assuming that the choice of the Lottery Commission

is equally likely to be any of the  combinations, what is the

probability that a player has

**(a)** All 8 of the selected numbers?

**(b)** Seven of the selected numbers?

**(c)** At least 6 of the selected numbers?

(see Excel file)

**18.** Consider the grid of points shown here.



Suppose that starting at the point labeled *A* you can at each move

either go one step up or one step to the right. You keep doing this

until the point labeled *B* is reached. How many different paths from

*A* to *B* are possible? *Hint*: To go from *A* to *B* you have to go 4 steps

to the right and 3 steps up. Indeed, any path can be specified by an

arrangement of 4 *r*’s and 3 *u*’s. For instance, the arrangement

*r*, *r*, *r*, *r*, *u*, *u*, *u*

specifies the following path:



**19.** Suppose, in Prob. 18, that a path from *A* to *B* is randomly chosen. What

is the probability it goes through the point circled in the following grid?

(*Hint*: How many paths are there from *A* to the circled point? How

many from the circled point to *B*?) (see Excel file)

