MAT 1372 Stat w/ Prob classwk 24 Fall 2012

**9.2 HYPOTHESIS TESTS AND SIGNIFICANCE LEVELS**

**Definition** *A* statistical hypothesis *is a statement about the nature of a population. It is often stated in terms of a population parameter.*

The statistical hypothesis to be tested, the *null hypothesis, i*s denoted by H0.

Examples of H0:

1. In a criminal court of law, the innocence of a person accused of committing a crime.
2. The accepted distance to a star is 6.9 light years. μ=6.9
3. The FDA recommends a 15 mg dose of medicine for the cure of an ailment. μ=15
4. A telecommunications cable is rated to be effective for 2 km before its signal is too attenuated to be useful. μ≥2
5. The radiation level 20 km northwest of Fukishima is less than 50 microsieverts per hour. μ<50

The alternative hypothesis, H1, is opposite to the null hypothesis.

* Rejecting H0 is equivalent to adopting H1 as true.
* Continuing to accept H0 says nothing regarding H1.

H1 for the same examples are

1. In a criminal court of law, person is guilty of committing accused crime.
2. Accepted distance of 6.9 light years to a star is not its real distance. μ≠6.9
3. The recommended dose of 15 mg is not the most effective for the cure of an ailment. μ≠15
4. A telecommunications cable has its signal too attenuated to be useful before 2 km are reached. μ<2
5. The radiation level 20 km northwest of Fukishima is more than or equal to 50 microsieverts per hour. μ≥50

*The* critical region *are the values of the test statistic for which* H0 *is rejected.*

The critical region is one or both of the tails and has as probability the *significance level α*, usually 5%, but also commonly 10% or 1%.

The lower the significance level, the less chance H0 will be rejected. The significance level depends on the severity of the consequences for rejecting H0 when it is in fact true, called a *type I error*.

Example of different significance levels, in criminal court, it is considered highly undesirable to declare an innocent person guilty of a crime. So the significance level is set very low, whereas in civil court, it is not as harmful to ask an innocent person accused of a wrongdoing to pay the accuser a fine, so the significance level is higher. [OJ Simpson, who was not declared guilty in criminal court, had to pay large amounts of money in civil court.]

How sure β we need to be, to reject the null hypothesis, is complementary to the significance level α, i.e.,α *+* β=1.

A *type 2 error* results if H0 is not rejected when in fact it should have been.

The chart sums up the relation between H0, H1 and Types I and II errors:

|  |  |  |
| --- | --- | --- |
|  | H­0 is true  (truly not guilty) | H1 is true  (truly guilty) |
| Accept H0  (acquittal) | Right decision | Wrong decision:  Type II Error |
| Reject H0  (conviction) | Wrong decision:  Type I Error | Right decision |

Example from wikipedia - Radioactive suitcase:

Consider determining whether a suitcase contains some radioactive material. Placed under a [Geiger counter](http://en.wikipedia.org/wiki/Geiger_counter), it produces 10 counts per minute. The null hypothesis is that no radioactive material is in the suitcase and that all measured counts are due to ambient radioactivity typical of the surrounding air and harmless objects. We can then calculate how likely it is that we would observe 10 counts per minute if the null hypothesis were true. If the null hypothesis predicts on average 9 counts per minute and a [standard deviation](http://en.wikipedia.org/wiki/Standard_deviation) of 1 count per minute, then we say that the suitcase is compatible with the null hypothesis (this does not guarantee that there is no radioactive material, just that we don't have enough evidence to suggest there is). On the other hand, if the null hypothesis predicts 3 counts per minute and a standard deviation of 1 count per minute, then the suitcase is not compatible with the null hypothesis, and there are likely other factors responsible to produce the measurements.[like the presence of radioactive materials]

**9.3 P-values and CASE OF KNOWN VARIANCE: equality for Null Hypothesis**

For cases where the null hypothesis is an equality, we work with a 2 sided tail.

*The* p-value *is smallest significance level at which the data lead to rejection of* H0*.*

The p-value gives the probability that data as unsupportive of H0 as those observed will occur when H0 is true. A small p value (say, 0.05 or less) is a strong indicator that the null hypothesis is not true. The smaller the p value, the greater the evidence for the falsity of H0.

**9.3.2.** A previous sample of fish in Lake Michigan indicated that the mean

polychlorinated biphenyl (PCB) concentration per fish was 11.2 parts

per million with a standard deviation of 2 parts per million. Suppose a

new random sample of 10 fish has the following concentrations:

11.5, 12.0, 11.6, 11.8, 10.4, 10.8, 12.2, 11.9, 12.4, 12.6

Assume that the standard deviation has remained equal to 2 parts per

million, and test the hypothesis that the mean PCB concentration has

also remained unchanged at 11.2 parts per million. Use the 5 percent

level of significance.

This is an example of a 2-tailed problem and *H0* is μ=11.2. Referring to the accompanying Excel file we see, that the sample mean . To continue we need to standardize. Recall that . Since this is a 2-tale problem, we are not interested in the sign of the difference with the claimed population mean μ, so we find its absolute value.



For our problem,



The p-value is the complement of the probability of this z score.

To find it we use Excel:

= 2\*(1-NORMSDIST((SQRT(10)/2\*(11.72-11.2))))

which gives 41.1%. For α=5%, and in fact for α=10%, we do not reject *H0*. Our sample mean is too close to the claimed mean for us to question its validity.

**6.** It is known that the value received at a local receiving station is equal

to the value sent plus a random error that is normal with mean 0 and

standard deviation 2. If the same value is sent 7 times, compute the

*p* value for the test of the null hypothesis that the value sent is equal

to 14, if the values received are 14.6, 14.8, 15.1, 13.2, 12.4, 16.8, 16.3

**9.3.1 Inequality for Null Hypothesis**

For cases where the null hypothesis is an inequality, we work with a 1 sided tail.

The calculations are otherwise similar.

**9.3.1.2.** Consider a test of H­0: μ ≤ 100 versus H1: μ > 100. Suppose that a sample

of size 20 has a sample mean of *X* = 105. Determine the *p* value of

this outcome if the population standard deviation is known to equal

**(a)** 5

**(b)** 10

**(c)** 15

If the sample mean is within the interval of *H0*, then we continue to accept *H0*. Hence, we assume that the sample mean is outside the interval of *H0*. As before, we standardize; for part (a), we have

, and the Excel command is

1-NORMSDIST(SQRT(20)/5\*(105-100)), which gives essentially 0 (3.87211E-06). Note that since all the probability is in one tail, we do not multiply by 2.

For the other standard deviations, we get p-values of 1.27% and 6.8% respectively. If the significance level were set at 5%, we would reject *H0* with the first 2 σ’s, and continue to accept it for the 3rd σ.

We summarize with the chart:

|  |  |  |
| --- | --- | --- |
| σ | p-value | accept/reject *H0* , α=5% |
| 5 | 0 | reject |
| 10 | 1.27% | reject |
| 15 | 6.8% | accept |

**9.3.1.6.** A farmer claims to be able to produce larger tomatoes. To test this

claim, a tomato variety that has a mean diameter size of 8.2 centimeters

with a standard deviation of 2.4 centimeters is used. If a sample

of 36 tomatoes yielded a sample mean of 9.1 centimeters, does this

prove that the mean size is indeed larger? Assume that the population

standard deviation remains equal to 2.4, and use the 5 percent level of

significance.