MAT 1372 Stat w/ Prob classwk 13 Fall 2012

**5.7 POISSON RANDOM VARIABLES**

For outcomes i=0,1,2…, we define the Poisson Random Variable to be



(\*)Fact: 

**Exercise**: Show that the Poisson RV is a probability distribution:

1. 
2.  Hint: use (\*).

If we take a binomial distribution with a very large pool (n) and a small probability of success (p), we can approximate the distribution by the Poisson distribution. The Wikipedia entry has the following graph to show this:

“Comparison of the Poisson distribution (black dots) and the binomial distribution with n=10 (red line), n=20 (blue line), n=1000 (green line). All distributions have a mean of 5. The horizontal axis shows the number of events *k*. Notice that as n gets larger, the Poisson distribution becomes an increasingly good approximation for the binomial distribution with the same mean.”

Examples of random variables whose probabilities are approximately Poisson:

Number of

1. misprints on a page of a book
2. people in a community of a certain size who are at least 100 years old
3. people entering a post office on a given day
4. clicks in a Geiger counter in one second for a given radioactive substance
5. defective items in a shipment of a specified size
6. accidents on a stretch of highway over one day for a specified set of conditions (e.g., day of week, not raining or holiday weekend, raining)
7. time between arrivals of parties of customers at a store or restaurant (must be sparse enough so that there is no bunching)
8. telemarketing calls received by a household on a given day
9. defects in a 5 feet length of iron rod
10. cars, televisions, etc. sold at a car dealership, dept store, etc. during a 1-week period (**λ** would not be the same in the weeks before Christmas as it would be in March).

Non-examples of random variables whose probabilities are approximately Poisson:

Number of

1. arrival of patients at a physician’s office (reason: schedules)
2. arrival, departure of airplanes at an airport (reason: schedules)
3. arrival of buses, subways (reason: schedules)
4. arrival of buses (say on Broadway upper west side), subways (say on the Lexington Avenue line) during rush hour (reason: scheduling but more significantly bunching)
5. people in line at supermarket, coffee truck, etc. during peak time (reason: bunching)

For more information, research “queuing theory” which is important to many fields including business, traffic engineering, communications and computer science.

**Exercise**: Come up with 2 other examples that you think might be Poisson as well as 1 that you think may not be Poisson.

## What is the mean and variance?



The 3rd equal sign has change in the index of summation performed by making the substitution. The 4th equal sign is based on the Fact (\*) mentioned earlier.

Surprisingly,the **Variance** alsois **λ**.

Its proof is more elaborate if done as above using series manipulations.

**2. (modified)** Compare the Poisson approximation with the true binomial probability

in the following cases:

**(a)** *n* = 10, *p* = 0.3

**(b)** *n* = 10, *p* = 0.1

**(c)** *n* = 10, *p* = 0.05

**(d)** *n* = 10, *p* = 0.01

The book only asks for i=2. It is more illustrative to see the whole distribution. See the excel file. Also, look at the sheet with the mean fixed, which provides graphs similar to the Wikipedia graph given earlier.

**4.** If *X* is Poisson with mean λ = 144, find

**(a)** *E*[*X*]

**(b)** SD(*X*)

**Exercise**: At Junior’s on a Friday between 6 and 8PM, an average of 2 parties of customers will arrive every 5 minutes. During a 10 minute stretch, what is the chance that no parties arrive? At most one party arrives? Exactly 2 parties arrive? At least 4 parties arrive? At least 8 parties arrive?

no parties X=0 0.02 POISSON(0,4,FALSE)

at most one X<=1 0.09 POISSON(1,4,TRUE)

2 parties X=2 0.15 POISSON(2,4,FALSE)

at least 4 X>=4 0.57 1-POISSON(3,4,TRUE)

at least 8 X>=8 0.05 1-POISSON(7,4,TRUE)