**Definitions**

The probability of the occurrence of an event *A*, given the occurrence of another event *B*, is called a ***conditional probability*** and is denoted by $P(B|A)$

For example:

*B*=adult has lung cancer

*A*=adult is a heavy smoker

Then $P(B|A)$ represents the probability of an adult having lung cancer, given that he/she is a heavy smoker.

**Motivational Example:**

Consider rolling a fair dice.

1. What is the probability of rolling a prime number?
2. What is the probability that a prime number has turned up if we are given the additional information that an odd number has turned up?

4

1

6

3

5

2

For events *A* and *B* in an arbitrary sample space *S*, we define the conditional probability of *B* given *A* by

$$P\left(A\right)=\frac{P(A∩B)}{P(A)} ;where P(A)\ne 0$$

Note that since we know that event *A* has occurred, it becomes our new sample space shown in figure below.



We obtain the following multiplication rule from conditional probability:

$$P\left(A∩B\right)=P(B)P\left(A\right)$$

Independent Events:

Two events A and B in a sample space S are said to be independent if and only if

$P\left(A∩B\right)=P(A)P(B)$.

If $P\left(A\right)=P(B)$, then the probability of *B* is not effected by occurrence of *A*.

**Examples:**

1. One of two urns is chosen at random with one as likely to be chosen as the other. Then a ball is withdrawn from the chosen urn. Urn 1 contains 1 white and 4 red balls, and urn 2 has 3 white and 2 red balls. If a white ball is drawn, what is the probability that it came from urn 2? (Hint: Draw a tree diagram)
2. A card is drawn at random from a standard 52-card deck. Events *A* and *B* are:

 *A* = the drawn card is a club.

 *B* = the drawn card is even (face cards are not valued).

1. Find $P(B|A)$.
2. Test *A* and *B* for independence.
3. Use the given tree diagram to answer the following questions.

 0.2 A

 M

 0.6 0.8 B

 0.4 0.7 A

 N

 0.3 B

(a) What is $P(A|M)$? (b) What is $P(A)$? (c) What is$ P(M|A)$?