Definitions

The probability of the occurrence of an event *A*, given the occurrence of another event *B*, is called a ***conditional probability*** and is denoted by $P(B|A)$
For example:
B=adult has lung cancer
A=adult is a heavy smoker

Then $P(B|A)$ represents the probability of an adult having lung cancer, given that he/she is a heavy smoker.

Motivational Example:

Consider rolling a fair ~~dice~~. die.

a) What is the probability of rolling a prime number?
b) What is the probability that a prime number has turned up if we are given the additional information that an odd number has turned up?

1 3 4 6
2 5

a) Prime # = {2, 3, 5}
$P(\text{prime}) = \frac{3}{6} = .5$

b) Odd # = {1, 3, 5}

Consider now The entire Sample space

Now $P(\text{prime}|\text{odd}) = \frac{2}{3}$!!

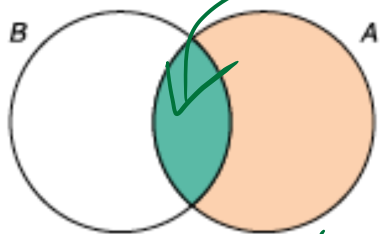
For events *A* and *B* in an arbitrary sample space *S*, we define the conditional probability of *B* given *A* by

In words $P(B|A) = \frac{P(A \text{ "and" } B)}{P(A)}$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} \text{ ; where } P(A) \neq 0$$

Note that since we know that event *A* has occurred, it becomes our new sample space shown in figure below.

P(B|A)



We obtain the following multiplication rule from conditional probability:

$$P(A \cap B) = P(B)P(B|A)$$

rearranging

Independent Events:

Two events A and B in a sample space S are said to be independent if and only if $P(A \cap B) = P(A)P(B)$.

If $P(B|A) = P(B)$, then the probability of *B* is not affected by occurrence of *A*.

Examples:

1) One of two urns is chosen at random with one as likely to be chosen as the other. Then a ball is withdrawn from the chosen urn. Urn 1 contains 1 white and 4 red balls, and urn 2 has 3 white and 2 red balls. If a white ball is drawn, what is the probability that it came from urn 2? (Hint: Draw a tree diagram)

Recall: $P(B|A) = \frac{P(B \text{ and } A)}{P(A)}$

.5 .5

Urn 1 Urn 2

$.2 = \frac{1}{5}$ White, $\frac{4}{5} = .8$ Red

$.6 = \frac{3}{5}$ White, $\frac{2}{5} = .4$ Red

$$P(W|\text{urn } 2) = \frac{P(W \text{ and urn } 2)}{P(\text{urn } 2)} = \frac{(\cancel{.5} \cdot .6)}{\cancel{.5}} = \boxed{.6}$$

2) A card is drawn at random from a standard 52-card deck. Events *A* and *B* are:

   A = the drawn card is a club. 13 clubs in deck

   B = the drawn card is even (face cards are not valued). 20 even cards

   (a) Find $P(B|A)$.

   (b) Test *A* and *B* for independence.

a) P(even | club) = $\frac{5}{13}$

Focus only on clubs

b) Are A + B independent?

$P(A|B) \stackrel{?}{=} P(A)$

$P(\text{club}|\text{even}) \stackrel{?}{=} P(\text{club})$

$\frac{5}{20} \stackrel{?}{=} \frac{13}{52}$ ✓

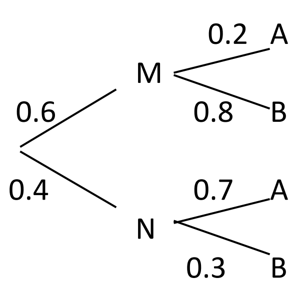
$P(B|A) \stackrel{?}{=} P(B)$

$P(\text{even}|\text{club}) \stackrel{?}{=} P(\text{even})$

$\frac{5}{13} \stackrel{?}{=} \frac{20}{52}$ ✓

A + B are independent!

3) Use the given tree diagram to answer the following questions.



a) $P(A|M) = \frac{P(A \text{ and } M)}{P(M)} = \frac{\cancel{.6} \cdot .2}{\cancel{.6}} = \boxed{.2}$

b) $P(A) = .6 \cdot .2 + .4 \cdot .7$

$= \boxed{.4}$

c) $P(M|A) = \frac{P(M \text{ and } A)}{P(A)} = \frac{.6 \cdot .2}{.4} = \boxed{.3}$

(a) What is $P(A|M)$?

(b) What is $P(A)$?

(c) What is $P(M|A)$?