Basic Definitions

As a general rule, to be able to draw valid inferences about a population from a sample, one needs to know how likely it is that certain events will occur under various circumstances.

Experiment:

An ***experiment*** is *any* process that produces an observation, or *outcome*.

Random Experiments:

Experiments that don't yield the same result s each time they are performed, no matter how carefully they are repeated under the same conditions, are called random experiments; i.e., the actual outcome cannot be determined in advance.
e.g., flipping a coin, rolling a dice, scoring on a statistics class etc.

Sample Space (S):

A sample space of a random experiment is a list of all possible outcomes of the experiment. Outcomes must be exhaustive and mutually exclusive.

Event (E):

An event is a subset of a sample space, $E \subset S$.

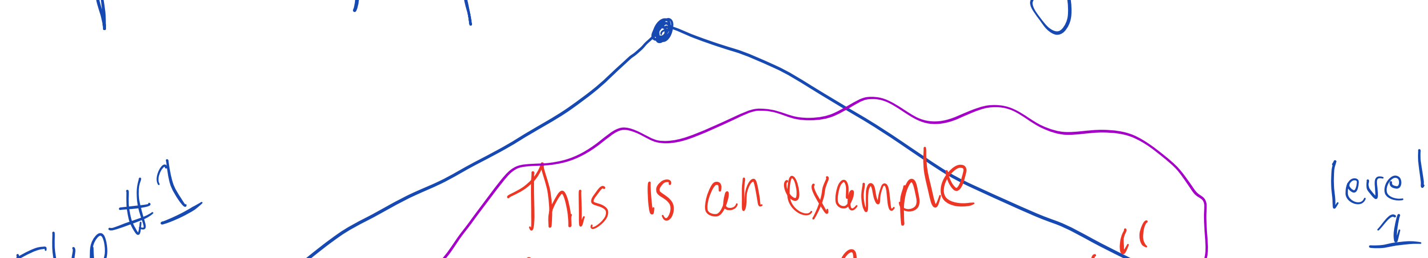
Simple Event:

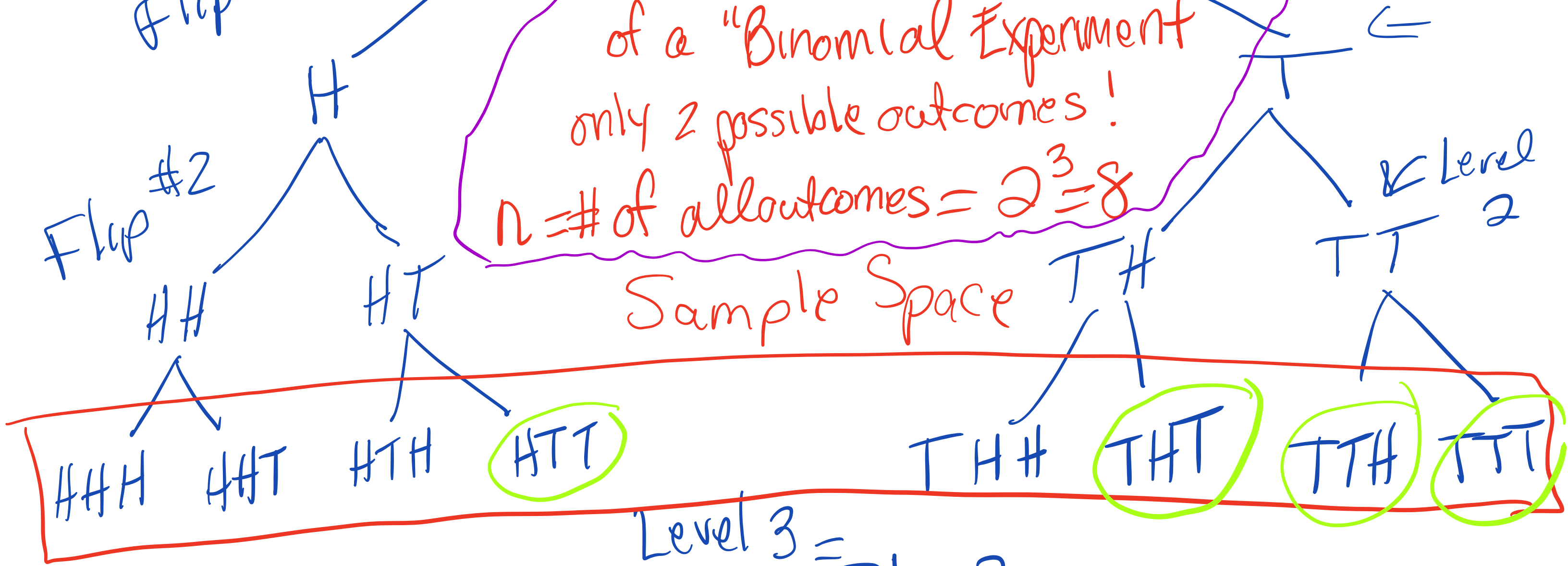
Simple event is an individual outcome of a sample space.

Examples:

1) Gender of a child: $S = \{B, G\}$, simple event $E = \{G\}$.
2) Rolling a die: $S = \{1, 2, 3, 4, 5, 6\}$, event $E_1 = \{1, 3, 5\}$, simple event $E_2 = \{6\}$.
3) Flipping two coins: $S = \{HH, HT, TH, TT\}$. Create events:
   Let E_1 = flipping same = {HH, TT}, E_2 = flipping different = {TH, H,T}
4) An experiment consists of flipping a coin three times and each time noting whether it lands heads or tails.
   (a) What is the sample space of this experiment?
   (b) What is the event that tails occur more often than heads? {HTT, THT, TTH, TTT}

Systematic way of listing all possible outcomes for a probability experiment → tree diagrams!!





5) Rolling two dice:

(a) Determine the sample space.

{11, 12, 13, 14, 15, 16, 21, 22, 23, 24, 25, 26, 31, ... }

(b) Determine the event that produces a sum of 7.

$E_1 = \{34, 16, 25, 61, 43, 52\}$

(c) Determine the event of rolling the same number for both dice.

$E_2 = \{11, 22, 33, 44, 55, 66\}$

6) A cafeteria offers a three-course meal. One chooses a main course, a starch, and a dessert. The possible choices are as follows:

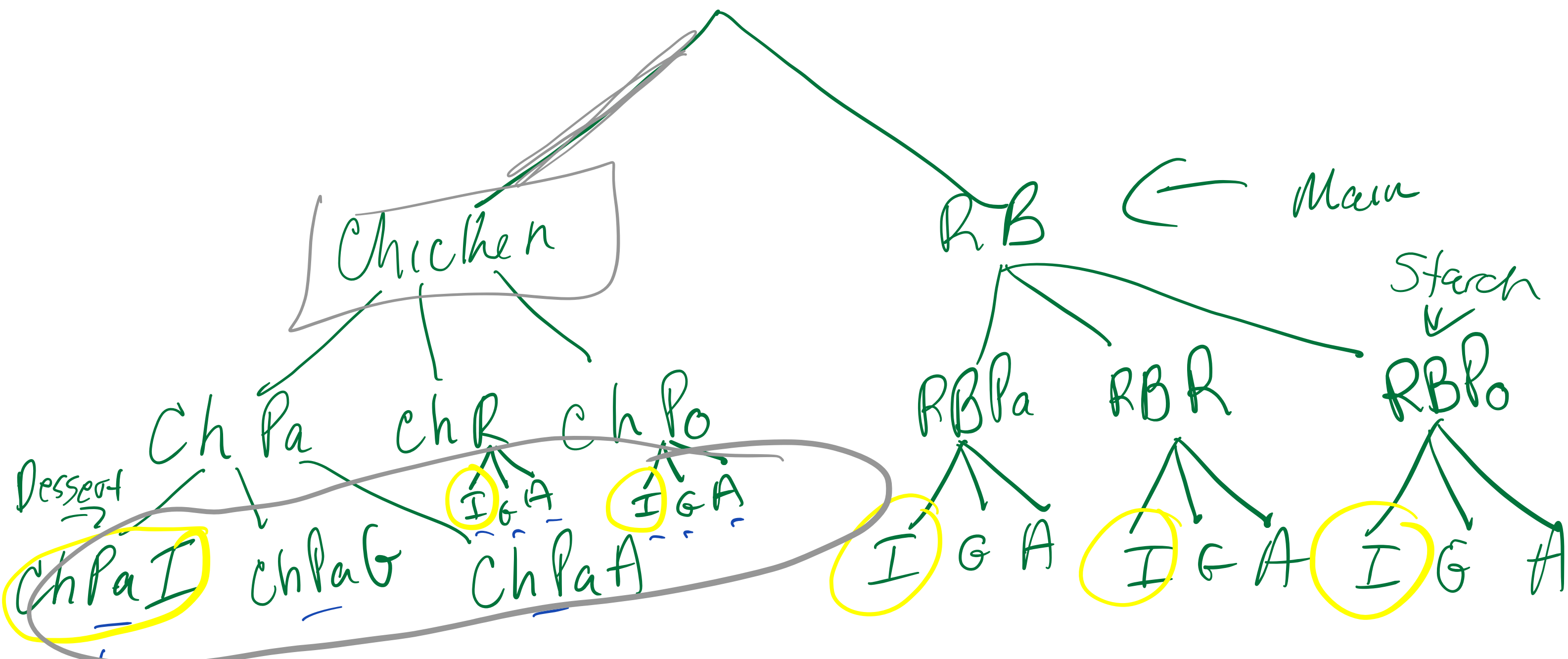
Meal	Choices
Main course	Chicken or roast beef
Starch course	Pasta or rice or potatoes
Dessert	Ice cream or gelatin or apple pie

An individual is to choose one course from each category.

(a) List all the outcomes in the sample space.

(b) Let *A* be the event that ice cream is chosen. List all the outcomes in *A*.

(c) Let *B* be the event that chicken is chosen. List all the outcomes in *B*.



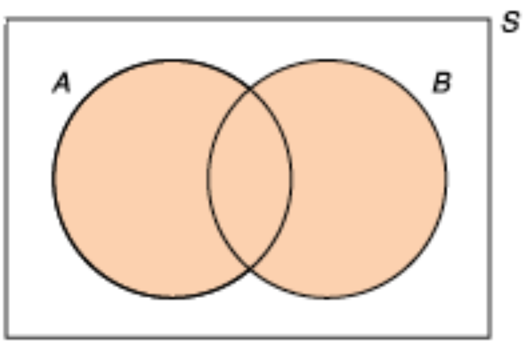
Outcomes in SS = $2 \cdot 3 \cdot 3 = 18$

E_I = { ChPaI, ChRI, ChPoI, RBPaI, RBRI, RBPoI }

E_C = { ChPaI, ChPaG, ChPaA, ChRI, ChRG, ChRA, ChPoI, ChPoG, ChPoA }

Set Operations for Events:

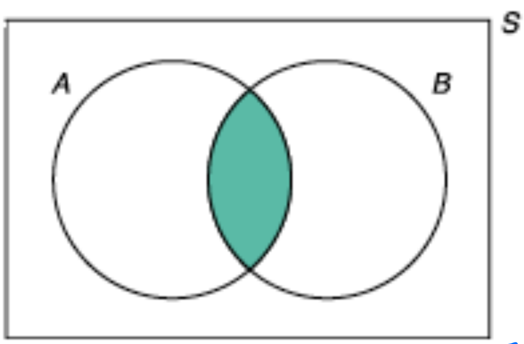
Venn diagram below shows the union of events A and B and it is represented by $A \cup B$; i.e., an element is in either A OR B.



Union = $A \cup B$

"or"

Venn diagram below shows the intersection of events A and B and it is represented by $A \cap B$; i.e., an element is in either A AND B.

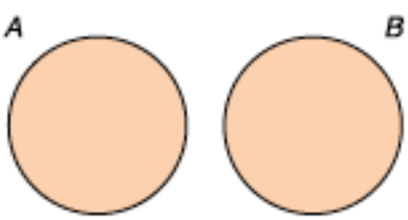


Intersection = $A \cap B$

"and"

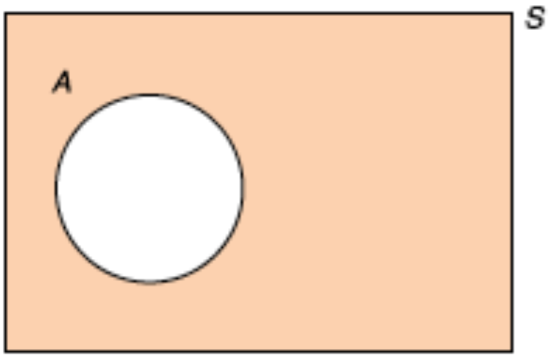
Venn diagram below shows two $\boxed{\text{disjoint}}$ events $\boxed{\text{(mutually exclusive)}}$ A and B. The intersection is a null set (empty set) and it is represented by $A \cap B = \emptyset$.

"null set"
"empty set"



For any event A we define the event A^c, called the *complement* of A, to consist of all outcomes in the sample space that are not in A. That is, A^c will occur when A does not, and vice versa. Venn diagram of this case is shown below.

"A^c" or "A'"



Example:

Let $S = \{1,2,3,4,5,6\}$, $A = \{1,3,5\}$, $B = \{4,6\}$, and $C = \{1,4\}$. Find

(a) $A \cup B$
(b) $B \cup C$
(c) $A \cup (B \cap C)$
(d) $(A \cup B)^c$

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a) $A \cup B$ (A or B) everything in A, everything in B if there are elements in BOTH, just list once

$= \boxed{\{1,3,4,5,6\}}$

b) $B \cup C = \{1,4,6\}$

c) $A \cup (B \cap C)$ First: $B \cap C = \{4\}$

$A \cup (B \cap C) = \{1,3,4,5\}$

d) $\{2\} = (A \cup B)^c$

Basic Counting Principles

We will use set operations to determine the number of elements in a set without actually enumerating the elements one by one.

Addition Principle

For any sets *A* and *B*,

$$\boxed{n(A \cup B) = n(A) + n(B) - n(A \cap B)}$$

If sets *A* and *B* are disjoint, then

$n(A \cup B) = n(A) + n(B)$.

Here $n(A)$ represents the number of elements in set *A*.

Example:

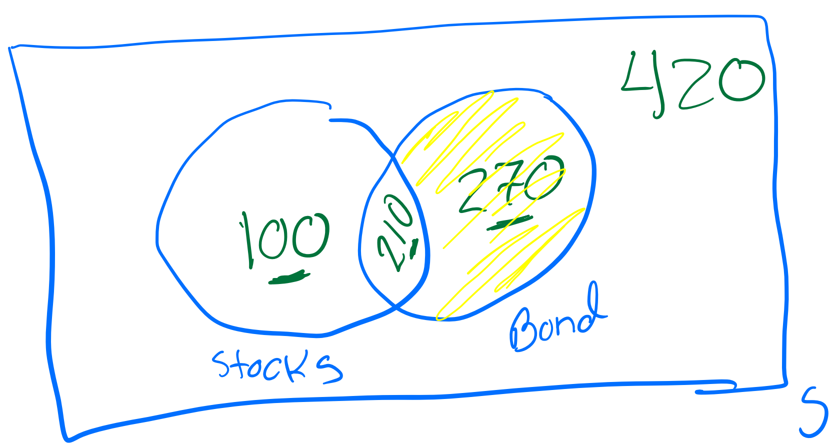
$n(A)+n(B)-n(A\cap B) = 310+480-210$

A survey of 1,000 people indicates that 310 have invested in stocks, 480 have invested in bonds, and 210 have invested in stocks and bonds.

(a) How many people in the survey have invested in stocks or bonds? STOCKS $\cup$ BONDS $= 580$

(b) How many have invested in neither stocks nor bonds? 420

(c) How many have invested in bonds but not in stocks? 270



STOCKS: 310 total

BONDS: 480

BOTH: 210

1000 TOTAL = 100 + 210 + 270 + 420

Properties of Probability

The determination of the likelihood, or chance, that an event will occur is the subject matter of ***probability***.

Consider an experiment whose sample space is *S*. We suppose that for each event *A* there is a number, denoted *P*(*A*), called the ***probability*** of event *A*, that has the following three properties.

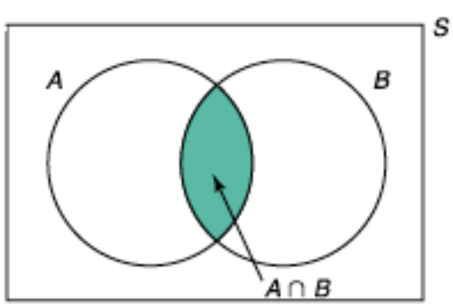
(i) The probability of a simple event is a number between 0 and 1. That is $0 \leq P(A) \leq 1$.
(ii) The probability of sample space is S is 1. That is $P(S) = 1$.
(iii) The probability of the union of disjoint events is equal to the sum of the probabilities of these events. For instance, if *A* and *B* are disjoint, then $P(A \cup B) = P(A) + P(B)$

Above properties can be used to establish some general results:

1) $S = A \cup A^c$
2) $1 = P(S) = P(A \cup A^c) = P(A) + P(A^c)$, that is $P(A^c) = 1 - P(A)$.

Addition Rule

For any events *A* and *B*, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. Note that events *A* and *B* are not necessarily disjoint.



Examples:

1) Anita has a 40 percent chance of receiving an *A* grade in statistics, a 60 percent chance of receiving an *A* in physics, and an 86 percent chance of receiving an *A* in either statistics or physics. Find the probability that she

a) Does not receive an *A* in either statistics or physics

b) Receives *A*'s in both statistics and physics

a) $P((S \cup P)^c) = 1 - .86 = .14$

$P(S) = .4$

$P(P) = .6$

$P(S \cup P) = .86$

$P(S \cup P) = P(S) + P(P) - P(S \cap P)$

can solve for it

b) $P(S \cap P) = P(S) + P(P) - P(S \cup P)$

intersection

$= .4 + .6 - .86$

$= .14$

1) Consider the sample space $S = \{X, Y, Z\}$. Determine if the following probability assignment is acceptable: $P(X) = 0.2$, $P(Y) = 0.3$, $P(Z) = 0.8$; give reasons.

BONUS QUESTION: Consider three sets A, B, and C. What does $n(A \cup B \cup C)$ look like?