Lesson #21

MAT 1372 Statistics with Probability

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Hypothesis Tests

Hypothesis test

- A process that uses sample statistics to test a claim about the value of a population parameter.
- **For example:** An automobile manufacturer advertises that its new hybrid car has a mean mileage of 50 miles per gallon. To test this claim, a sample would be taken. If the sample mean differs enough from the advertised mean, you can decide the advertisement is wrong.

Hypothesis Tests

Statistical hypothesis

- A statement, or claim, about a population parameter.
- Need a pair of hypotheses
  - one that represents the claim
  - the other, its complement
- When one of these hypotheses is false, the other must be true.

Stating a Hypothesis

Null hypothesis

- A statistical hypothesis that contains a statement of equality such as ≤, =, or ≥.
- Denoted $\boldsymbol{H_0}$ read "H sub-zero" or "H naught."

Alternative hypothesis

- A statement of strict inequality such as >, ≠, or <.
- Must be true if H_0 is false.
- Denoted $\boldsymbol{H_a}$ read "H sub-a."

Stating a Hypothesis

- To write the null and alternative hypotheses, translate the claim made about the population parameter from a verbal statement to a mathematical statement.
- Then write its complement.

$$H_0: \mu \leq k \qquad H_0: \mu \geq k \qquad H_0: \mu = k$$
$$H_a: \mu > k \qquad H_a: \mu < k \qquad H_a: \mu \neq k$$

- Regardless of which pair of hypotheses you use, you always assume $\mu = k$ and examine the sampling distribution on the basis of this assumption.

Example: Stating the Null and Alternative Hypotheses

Write the claim as a mathematical sentence. State the null and alternative hypotheses and identify which represents the claim.

1. A school publicizes that the proportion of its students who are involved in at least one extracurricular activity is 61%.

Solution:

H_0: $p = 0.61$ ← Equality condition (Claim)

H_a: $p \neq 0.61$ ← Complement of H_0

Example: Stating the Null and Alternative Hypotheses

Write the claim as a mathematical sentence. State the null and alternative hypotheses and identify which represents the claim.

2. A car dealership announces that the mean time for an oil change is less than 15 minutes.

Solution:

H_0: $\mu \geq 15$ minutes ← Complement of H_a

H_a: $\mu < 15$ minutes ← Inequality condition (Claim)

Example: Stating the Null and Alternative Hypotheses

Write the claim as a mathematical sentence. State the null and alternative hypotheses and identify which represents the claim.

3. A company advertises that the mean life of its furnaces is more than 18 years

Solution:

H_0: $\mu \leq 18$ years ⟵ Complement of H_a

H_a: $\mu > 18$ years ⟵ Inequality condition (Claim)

Types of Errors

- No matter which hypothesis represents the claim, always begin the hypothesis test **assuming that the equality condition in the null hypothesis is true**.
- At the end of the test, one of two decisions will be made:
  - reject the null hypothesis
  - fail to reject the null hypothesis
- Because your decision is based on a sample, there is the possibility of making the wrong decision.

Types of Errors

	Actual Truth of H_0	
Decision	H_0 is true	H_0 is false
Do not reject H_0	Correct Decision	**Type II Error**
Reject H_0	**Type I Error**	Correct Decision

- A **type I error** occurs if the null hypothesis is rejected when it is true.
- A **type II error** occurs if the null hypothesis is not rejected when it is false.

Example: Identifying Type I and Type II Errors

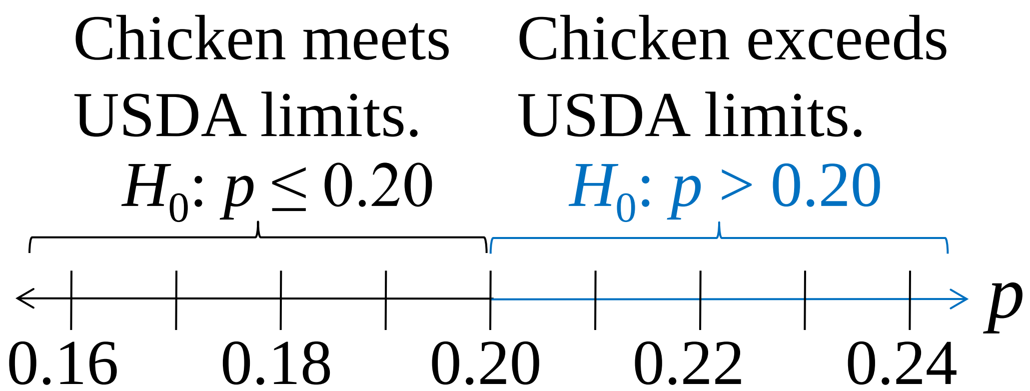
The USDA limit for salmonella contamination for chicken is 20%. A meat inspector reports that the chicken produced by a company exceeds the USDA limit. You perform a hypothesis test to determine whether the meat inspector's claim is true. When will a type I or type II error occur? Which is more serious? *(Source: United States Department of Agriculture)*

Solution: Identifying Type I and Type II Errors

Let p represent the proportion of chicken that is contaminated.

Hypotheses: H_0: $p \leq 0.2$

H_a: $p > 0.2$ (Claim)



Solution: Identifying Type I and Type II Errors

Hypotheses: H_0: $p \leq 0.2$
H_a: $p > 0.2$ (Claim)

A type I error is rejecting H_0 when it is true.

The actual proportion of contaminated chicken is less than or equal to 0.2, but you decide to reject H_0.

A type II error is failing to reject H_0 when it is false.

The actual proportion of contaminated chicken is greater than 0.2, but you do not reject H_0.

Solution: Identifying Type I and Type II Errors

Hypotheses: H_0: $p \leq 0.2$

H_a: $p > 0.2$ (Claim)

- With a type I error, you might create a health scare and hurt the sales of chicken producers who were actually meeting the USDA limits.
- With a type II error, you could be allowing chicken that exceeded the USDA contamination limit to be sold to consumers.
- A type II error could result in sickness or even death.

Level of Significance

Level of significance

- Your maximum allowable probability of making a type I error.
  - Denoted by α, the lowercase Greek letter alpha.
- By setting the level of significance at a small value, you are saying that you want the probability of rejecting a true null hypothesis to be small.
- Commonly used levels of significance:
  - $\alpha = 0.10$ $\alpha = 0.05$ $\alpha = 0.01$
- P(type II error) = β (beta)

Statistical Tests

- After stating the null and alternative hypotheses and specifying the level of significance, a random sample is taken from the population and sample statistics are calculated.
- The statistic that is compared with the parameter in the null hypothesis is called the **test statistic**.

Population parameter	Test statistic	Standardized test statistic
μ	$\bar{x}$	z (Section 7.2 $n \geq 30$) t (Section 7.3 $n < 30$)
p	$\hat{p}$	z (Section 7.4)
σ^2	s^2	χ^2 (Section 7.5)

***P*-values**

***P*-value** (or **probability value**)

- The probability, if the null hypothesis is true, of obtaining a sample statistic with a value as extreme or more extreme than the one determined from the sample data.
- Depends on the nature of the test.

Nature of the Test

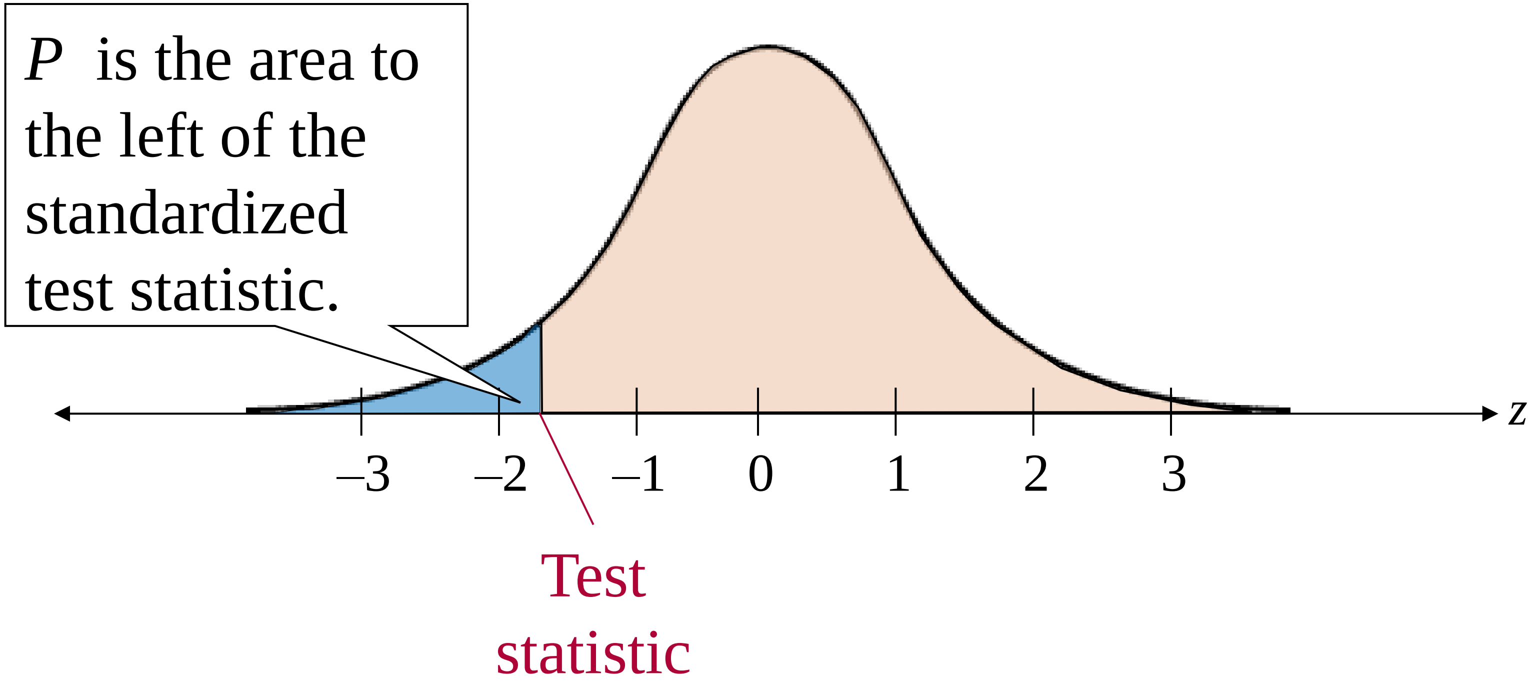
- Three types of hypothesis tests
  - left-tailed test
  - right-tailed test
  - two-tailed test
- The type of test depends on the region of the sampling distribution that favors a rejection of H_0.
- This region is indicated by the alternative hypothesis.

Left-tailed Test

- The alternative hypothesis H_a contains the less-than inequality symbol (<).

$$H_0: \mu \geq k$$

$$H_a: \mu < k$$

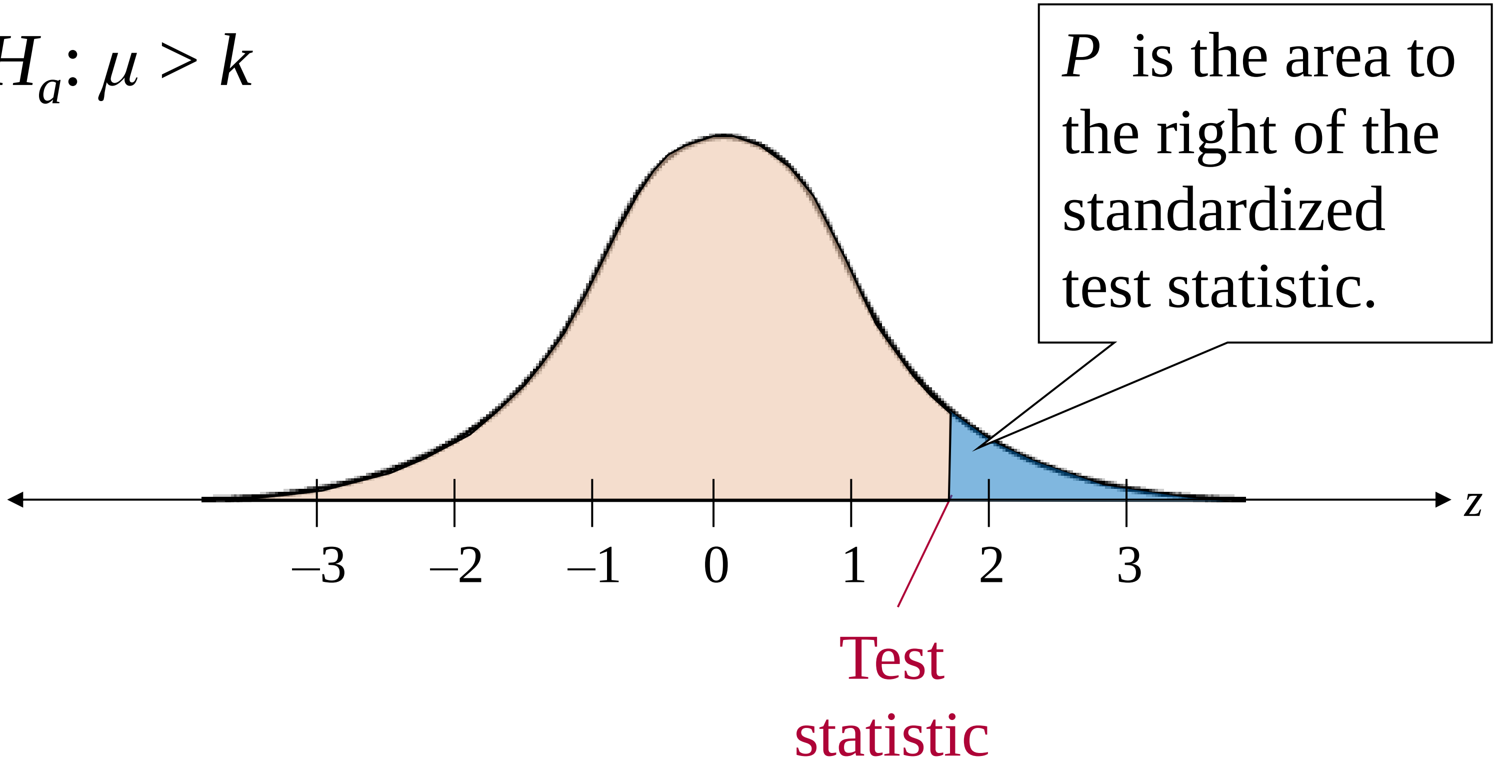


Right-tailed Test

- The alternative hypothesis H_a contains the greater-than inequality symbol (>).

H_0: $\mu \leq k$

H_a: $\mu > k$

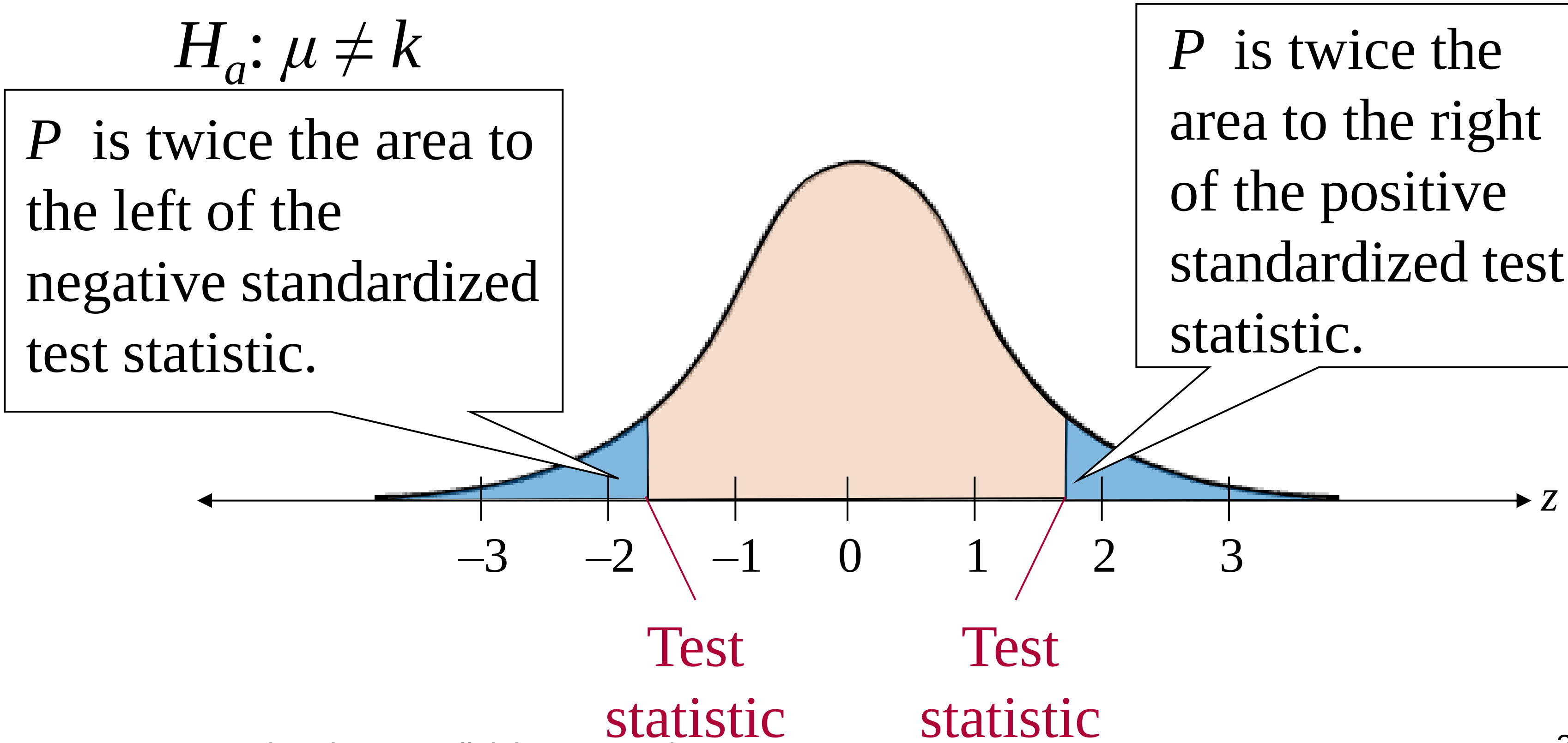


Two-tailed Test

- The alternative hypothesis H_a contains the not-equal-to symbol ($\neq$). Each tail has an area of ½P.

H_0: $\mu = k$

H_a: $\mu \neq k$



Example: Identifying The Nature of a Test

For each claim, state H_0 and H_a. Then determine whether the hypothesis test is a left-tailed, right-tailed, or two-tailed test. Sketch a normal sampling distribution and shade the area for the *P*-value.

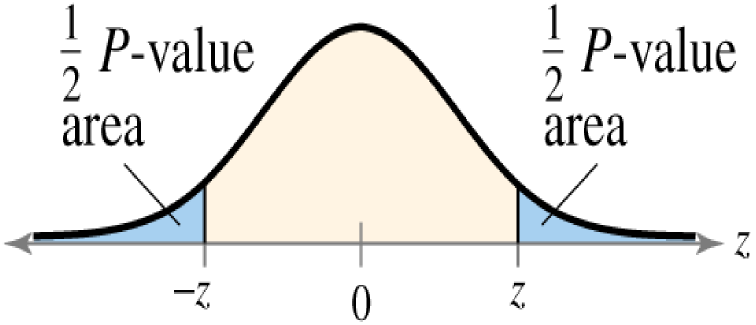
1. A school publicizes that the proportion of its students who are involved in at least one extracurricular activity is 61%.

Solution:

H_0: $p = 0.61$

H_a: $p \neq 0.61$

Two-tailed test



Example: Identifying The Nature of a Test

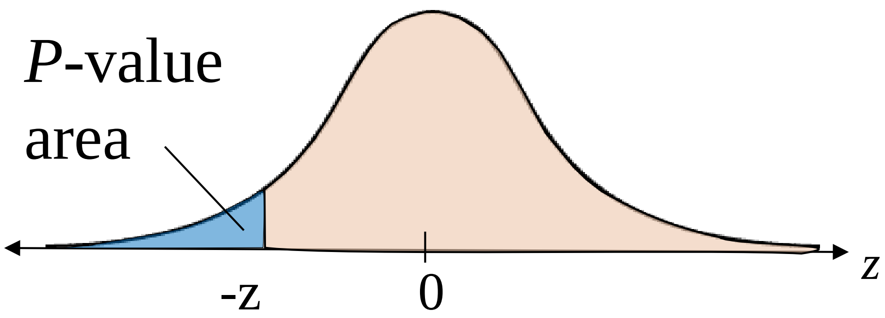
For each claim, state H_0 and H_a. Then determine whether the hypothesis test is a left-tailed, right-tailed, or two-tailed test. Sketch a normal sampling distribution and shade the area for the *P*-value.

2. A car dealership announces that the mean time for an oil change is less than 15 minutes.

Solution:

$$H_0: \mu \geq 15 \text{ min}$$
$$H_a: \mu < 15 \text{ min}$$

Left-tailed test



Example: Identifying The Nature of a Test

For each claim, state H_0 and H_a. Then determine whether the hypothesis test is a left-tailed, right-tailed, or two-tailed test. Sketch a normal sampling distribution and shade the area for the P-value.

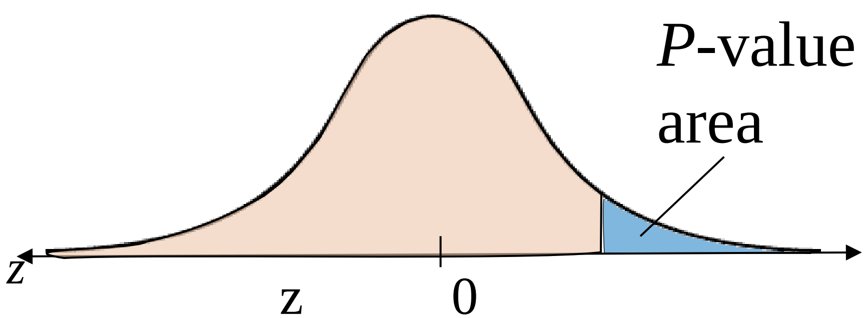
3. A company advertises that the mean life of its furnaces is more than 18 years.

Solution:

$H_0: \mu \leq 18$ yr

$H_a: \mu > 18$ yr

Right-tailed test



Making a Decision

Decision Rule Based on *P*-value

- Compare the *P*-value with α.
  - If $P \leq \alpha$, then reject H_0.
  - If $P > \alpha$, then fail to reject H_0.

	Claim	
Decision	Claim is H_0	Claim is H_a
Reject H_0	There is enough evidence to reject the claim	There is enough evidence to support the claim
Fail to reject H_0	There is not enough evidence to reject the claim	There is not enough evidence to support the claim

Example: Interpreting a Decision

You perform a hypothesis test for the following claim. How should you interpret your decision if you reject H_0? If you fail to reject H_0?

1. H_0 (Claim): A school publicizes that the proportion of its students who are involved in at least one extracurricular activity is 61%.

Solution:

- The claim is represented by H_0.

Solution: Interpreting a Decision

- If you reject H_0, then you should conclude "there is enough evidence to reject the school's claim that the proportion of students who are involved in at least one extracurricular activity is 61%."

- If you fail to reject H_0, then you should conclude "there is not enough evidence to reject the school's claim that proportion of students who are involved in at least one extracurricular activity is 61%."

Example: Interpreting a Decision

You perform a hypothesis test for the following claim. How should you interpret your decision if you reject H_0? If you fail to reject H_0?

2. H_a (Claim): A car dealership announces that the mean time for an oil change is less than 15 minutes.

Solution:

- The claim is represented by H_a.
- H_0 is "the mean time for an oil change is greater than or equal to 15 minutes."

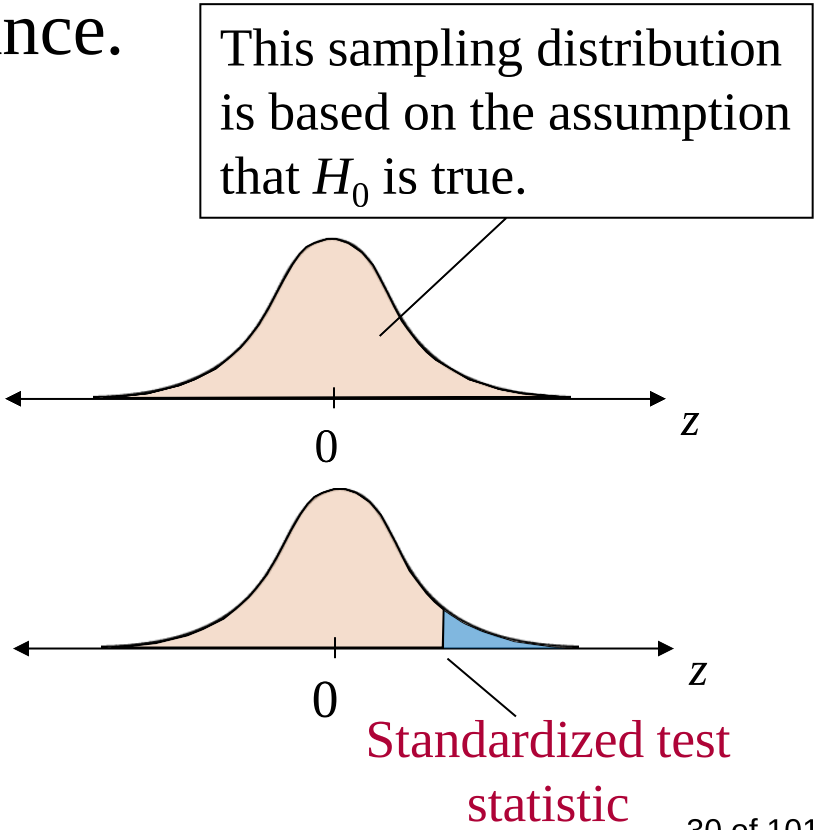
Solution: Interpreting a Decision

- If you reject H_0, then you should conclude "there is enough evidence to support *the dealership's* claim that the mean time for an oil change is less than 15 minutes."

- If you fail to reject H_0, then you should conclude "there is not enough evidence to support the *dealership's* claim that the mean time for an oil change is less than 15 minutes."

Steps for Hypothesis Testing

1. State the claim mathematically and verbally. Identify the null and alternative hypotheses.
   H_0: ? H_a: ?
2. Specify the level of significance.
   $\alpha =$?
3. Determine the standardized sampling distribution and sketch its graph.
4. Calculate the test statistic and its corresponding standardized test statistic. Add it to your sketch.



Steps for Hypothesis Testing

5. Find the *P*-value.
6. Use the following decision rule.

Is the *P*-value less than or equal to the level of significance?

No → Fail to reject H_0.

Yes → Reject H_0.

7. Write a statement to interpret the decision in the context of the original claim.