**Multiplication Principle**

If *n* operations $O\_{1}, O\_{2}, …, O\_{n}$ are performed in order, with possible number of outcomes $N\_{1}, N\_{2}, …, N\_{n}$ respectively, then there are $N\_{1}∙N\_{2}∙ \cdots ∙N\_{n}$ possible combined outcomes of the operations performed in the given order.

**Examples**

1. A retail store stocks windbreaker jackets in small, medium, large, and extra-large, and all are available in blue or red. We choose small, medium, large and xl jackets. How many choices do we have? What are they?

Use the fundamental counting principle to get: 2\*2\*2\*2=16

{rrrr, bbbb, rrbb, rbbr, rbrb, brbr,…etc.}

1. From the 26 letters in the alphabet, how many ways can 3 letters appear in a row on a license plate if no letter is repeated? If letters can be repeated? If adjacent letters cannot be alike?

Using the Fundamental Counting Principle

Letters can be repeated: 26\*26\*26= 17,576

Letters cannot be repeated: 26\*25\*24=15,600

Adjacent letters not alike: 26\*24\*26 = 16,224

1. How many 5-digit ZIP code numbers are possible if successive digits must be different?

Again, use the Fundamental Counting Principle

 10\*9\*9\*9\*9

**Factorials**

The product of the first $n$ natural numbers is called $n$ factorial and is denoted by $n!$.

For *n* a natural number,

$$n!=n∙(n-1)∙(n-2)∙\cdots ∙2∙1$$

$$0!=1$$

$$n!=n∙\left(n-1\right)!$$

**Permutations**

A permutation of a set of distinct objects is an arrangement of the objects in a specific order without repetition.

**Example**

Suppose that 4 pictures are to be arranged from left to right on one wall of an art gallery. How many permutations (ordered arrangements) are possible?

4!=4\*3\*2\*1=24

**Permutations of *n* Objects Taken *r* at a Time**

A permutation of a set of *n* distinct objects taken *r* at a time without repetition is an arrangement of *r* of the *n* objects in a specific order.

$$P\_{n,r}=\frac{n!}{(n-r)!}$$

where $0\leq r\leq n$.

Note that $P\_{n,n}=n!$

**Example**

Now suppose that the director of the art gallery decides to use only 2 of the 4 available paintings, and they will be arranged on the wall from left to right. How many permutations of 2 paintings can be formed from the 4?

P(4,2)=12

**Combinations**

A combination of a set of *n* distinct objects taken *r* at a time without repetition is an *r*-element subset of the set of *n* objects. The arrangement of the elements in the subset does NOT matter.

$$C\_{n,r}=\left(\begin{matrix}n\\r\end{matrix}\right)=\frac{n!}{r!\left(n-r\right)!}$$

where $0\leq r\leq n$.

**Example**

Now suppose that an art museum owns 8 paintings and another museum wishes to borrow 3 of these paintings for a special show. How many ways can the 3 paintings be selected for shipment out of 8 available? When shipping the paintings, is the order important?

Order does not matter, so use combinations.

C(8,3)=56

**Experiments with Equally Likely Outcomes**

Each outcome in the sample space *S* is equally likely to occur. That is, if sample space *S* consists of *n* outcomes, say, $S=\left\{E\_{1}, E\_{2}, E\_{3}, \cdots , E\_{n}\right\}$, then $P\left(E\_{1}\right)=P\left(E\_{2}\right)=\cdots =P\left(E\_{n}\right)$.

The probability of any event *A* is equal to the proportion of the outcomes in the sample space that is in *A*. That is,

$$P\left(A\right)=\frac{number of outcomes in S that are in A}{n}$$

**Examples:**

1. Rolling a die with $S=\left\{1, 2, 3, 4, 5, 6\right\}$. Compute the probability of the events $E\_{1}=\left\{1, 3, 5\right\}$ and $E\_{2}=\left\{6\right\}$.

P(E1)=n(E1)/n(S)=3/6=.5

P(E2)=1/6

1. Flipping two coins: $S=\left\{HH, HT, TH, TT\right\}$. Compute the probability of following events.
2. Getting at least one $H$.={HT,HH,TH}, P(at least one H)=3/4
3. Getting exactly one $H.$={HT, TH}, P(getting one H)=2/4=1/2
4. Getting at least one $H$ or at least one $T$.{HT,TH,HH,TT} P(Event)=4/4=1
5. Getting three $T's$. P(Event)=0
6. Consider an experiment of rolling two die and compute the probability of following events.
7. Producing a sum of 7. ={(1,6), (2,5),(3,4),(6,1),(5,2),(4,3)}, P(E)=6/36
8. Producing a sum of 11.={(5,6),(6,5)}, P(E)=2/36=1/18
9. Producing sum that is NOT 7 or 11.

First let’s find Event=sum 7 OR 11 ={(1,6),(2,5),(3,4),(6,1),(5,2),(4,3),(5,6),(6,5)}, P(E)=8/36

Now let’s find the complement NOT sum 7 or 11 = 1-P(sum 7 or 11)=

1-8/36=28/36=7/9

1. Producing a sum less than 4.

P(E)=3/36=1/12

1. Rolling the same number for both dice.

P(E)=6/36=1/6

1. Among 32 dieters following a similar routine, 18 lost weight, 5 gained weight, and 9 remained the same weight. If one of these dieters is randomly chosen, find the probability that he or she gained weight, lost weight, and neither lost nor gained weight.

P(Gain)=5/32

P(Lost)=18/32

P(No change)=9/32

**Exercises (Counting Principles)**

1. From a committee of 10 people
2. In how many ways can we choose a chairperson, a vice-chairperson, and a secretary, assuming that one person cannot hold more than one position?
3. In how many ways can we choose a subcommittee of 3 people?
4. Order matters! So use permutations: P(10,3)=720=10\*9\*8
5. Order does NOT matter! So use combinations=C(10,3)=120
6. A group of 100 people touring Europe includes 55 people who speak French, 42 who speak German, and 16 who speak neither language. How many people in the group speak both French and German?
7. From a standard 52-card deck, how many 7-card hands will have all diamonds?
8. How many 5-card hands will have 3 aces and 2 kings?
9. How many 5-person committees are possible from a group of 11 people if:
10. There are no restrictions.
11. Both Jim and Mary must be on the committee.
12. Either Jim or Mary (but not both) must be in the committee.

**Exercises (Probability)**

1. In a family with 3 children, excluding multiple births, what is the probability of having 3 girls? Assume that a girl is as likely as a boy at each birth.
2. Consider an experiment consists of drawing 5 cards from 52-card deck. Compute the probability of following hands:
3. 5 face cards.
4. 4 aces.
5. Royal flush (10, J, Q, K, A all in one suit).
6. 2 aces and 3 queens.
7. 2 kings and 3 aces.
8. From a standard 52-card deck, what is the probability of 5-card hand having at least one face card?
9. A sports club has 120 members, of whom 44 play tennis, 30 play squash, and 18 play both tennis and squash. If a member is chosen at random, find the probability that this person.
10. Does not play tennis.
11. Does not play squash.
12. Plays neither tennis nor squash.
13. A shipment of 55 precision parts, including 12 that are defective, is sent to an assembly plant. The quality control division selects 10 at random for testing and rejects the entire shipment if one or more in the sample are found defective. What is the probability that the shipment will be rejected?