Steps to factoring trinomials with lead coefficient of 1

1. Write the trinomial in descending powers.
2. List the factorizations of the constant (third) term of the trinomial.
3. Pick the factorization where the sum of the factors is the coefficient of the middle term.
4. Check by multiplying the binomials.

Exercise 6: Factor the trinomial with lead coefficient 1 by checking for the correct pair of product and sum

ax^2+bx+c

$* \; a=1$

$b=-8=-6+-2$

$c=12=-6\cdot -2$

a) Factor $x^2-8x+12$

Answer in factored form: $(x-6)(x-2)$

Check: $(x-6)(x-2)=x^2-2x-6x+12=x^2-8x+12$

$a=1 \; *$

$b=5=2+3$

$c=6=2\cdot 3$

b) Factor a^2+5a+6

Answer in factored form: $(a+2)(a+3)$

Check: $(a+2)(a+3)=a^2+3a+2a+6=a^2+5a+6$

$a=1 \; *$

$b=-9=-12+3$

$c=-36=-12\cdot 3$

c) Factor $y^2-9y-36$ $\quad (y-12)(y+3)$

Answer in factored form: $(y+3)(y-12)$

Check:

ax^2+bx+c $\quad a=1 \; *$

$b=14y=9y+5y$

$c=45y^2=9y\cdot 5y$

d) Factor $x^2+14xy+45y^2$

Answer in factored form: $(x+9y)(x+5y)$

Check:

ax^2+bx+c $\quad * \; a=1$

$b=-9y=-3y+-6y$

$c=18y^2=-3y\cdot -6y$

e) Factor $x^2-9xy+18y^2$

Answer in factored form: $(x-3y)(x-6y)$

Check:

aa^2+ba+c $\quad * \; a=1$

$b=7b=-b+8b$

$c=-8b^2=-b\cdot 8b$

f) Factor $a^2+7ab-8b^2$

Answer in factored form: $(a-b)(a+8b)$

Check:

Factoring Trinomials with Lead Coefficients other than 1

Method 1: Trial and error

1. List the factorizations of the third term of the trinomial.
2. Write them as two binomials and determine the correct combination where the sum of the outer product, ad, and the inner product, bc, is equal to the middle term of the trinomial.

$$(a+b)(c+d) = ac + ad + bc + bd$$

bc

ad

Method 2: Factoring by grouping

1. Form the product ac.
2. Find a pair of numbers whose product is ac and whose sum is b.
3. Rewrite the polynomial to be factored so that the middle term bx is written as the sum of the two terms whose coefficients are the two numbers found in step
4. Factor by grouping.

Exercise 7: Factor each trinomial below

a) $2h^2 - 5h - 3$

b) $2h^2 - h - 3$

$a = 2$
$b = -1$ $\quad b = -1 = \underline{-3} + \underline{2}$
$c = -3$ $\quad ac = -6 = \underline{-3} \cdot \underline{2}$

$= 2h^2 + 2h - 3h - 3$
$= (2h^2 + 2h) + (-3h - 3)$
$= 2h(h+1) + (-3)(h+1)$
$= (2h-3)(h+1)$

$2h^2$	$-3h$	$\to h$
$2h$	-3	$\to 1$
$\downarrow 2h$	$\downarrow -3$	

$2h^2 - h - 3$
$= (h+1)(2h-3)$

c) $2h^2 - 5h - 12$

$a = 2$
$b = -5$ $\quad b = -5 = \underline{-8} + \underline{3}$
$c = -12$ $\quad ac = -24 = \underline{-8} \cdot \underline{3}$

$2h^2$	$3h$	$\to h$
$-8h$	-12	$\to -4$
$\downarrow 2h$	$\downarrow +3$	

$(h-4)(2h+3)$

Check
$(h-4)(2h+3)$
$2h^2 + 3h - 8h - 12$
$2h^2 - 5h - 12$ ✓

d) $3k^2 - 14k - 5$

$a = 3$
$b = -14$ $\quad b = -14 = \underline{-15} + \underline{1}$
$c = -5$ $\quad ac = -15 = \underline{-15} \cdot \underline{1}$

		GCF
$3k^2$	$1k$	$\to k$
$-15k$	-5	$\to -5$
$\downarrow 3k$	$\downarrow +1$	
GCF		

$(k-5)(3k+1)$

Check
$(k-5)(3k+1)$
$3k^2 + k - 15k - 5$
✓ $3k^2 - 14k - 5$

Factoring Special Products

Exercise 8: Multiply the binomials and combine like terms.
Can you come up with a formula for this type of multiplication?

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a) $(x+3)(x-3)$

b) $(5x+8)(5x-8)$

Exercise 9: Multiply the binomials and combine like terms.
Can you come up with a formula for this type of multiplication?

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a) $(x-3)^2$

b) $(2x+5)^2$

Many trinomials can be factored by using special product formulas.

Difference of Two Squares (DOTS): $\boldsymbol{a^2 - b^2 = (a+b)(a-b)}$

Exercise 10: Factor each binomial below. Check if it is a difference of two squares.

a) $x^2 - 49 = (x)^2 - (7)^2$
$= (x-7)(x+7)$

b) $25 - b^2 = (5+b)(5-b)$

c) $36x^2 - 16y^2 = (6x)^2 - (4y)^2$
$= (6x-4y)(6x+4y)$

d) $100t^2 - 49r^2 = (10t+7r)(10t-7r)$

e) Factor completely $16x^4 - 81 = (4x^2)^2 - (9)^2$
$= (4x^2+9)(4x^2-9)$
$= (4x^2+9)(2x+3)(2x-3)$

f) $x^2 + 36$

What have you noticed about the sum of two squares? not factorable
... for now...

Perfect Square Trinomial $\boldsymbol{a^2+2ab+b^2=(a+b)^2}$
$\boldsymbol{a^2-2ab+b^2=(a-b)^2}$

Exercise 11: Factor each trinomial below. Check that the first and the third terms are perfect square, and the middle term is $2ab$, then apply the perfect square formula

a) $t^2-8t+16 = (t)^2 - 2(t)(4) + (4)^2$

$= (t-4)^2$

* the $-$ is because 2nd term is negative.

b) $p^2+10t+25$

c) $16a^2-40a+25$

d) $9b^2+42b+49$

$(3b)^2 + 2(3b)(7) + (7)^2$

the key

$= (3b+7)^2$

e) $16y^2-72y+81$

Mixed Factoring/Factoring Completely

To factor a polynomial, **first factor the greatest common factor**, then consider the number of terms in the polynomial.

I. Two terms: Determine if the binomial is a difference of two squares.
If it is a difference of two squares, then $a^2-b^2=(a+b)(a-b)$

II. Three Terms: Determine if the trinomial is a perfect square trinomial.
a) If the trinomial is a perfect square, then
$$a^2+2ab+b^2=(a+b)^2$$
$$a^2-2ab+b^2=(a-b)^2$$
b) If the trinomial ax^2+bx+c is not a perfect square, then
   i) if the leading coefficient $a=1$, then check the product = c and sum = b to determine the correct combination.
   ii) if the leading coefficient $a>1$, then use trial and error or the ac-method.

III. Four terms: Try to factor by grouping.

Exercise 12: Factor each polynomial **completely**. This may mean factoring GCF and/or factoring in two or more steps.

a) Factor completely $3x^4 - 3x^3 - 36x^2$

$\frac{3x^4}{3x^2}$ $\frac{3x^3}{3x^2}$ $\frac{36x^2}{3x^2}$

$3x^2(x^2 - x - 12)$

$3x^2(x-4)(x+3)$

*$a = 1$
$b = -1 = \underline{-4} + \underline{3}$
$c = -12 = \underline{-4} \cdot \underline{3}$

b) Factor completely $20a - 5a^3$

c) Factor completely $16a^5b - ab$

d) Factor completely $8x^2 - 24x + 18$

e) Factor completely $16x^3y - 40x^2y^2 + 25xy^3$

f) Factor completely $12x^3 + 11x^2 + 2x$

g) Factor completely $7z^2w^2 - 10zw^2 - 8w^2$