NEW YORK CITY COLLEGE OF TECHNOLOGY
CITY UNIVERSITY OF NEW YORK

An Introduction to Intermediate Algebra

Preparation for
MAT 1175: Fundamentals of Mathematics

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An Introduction to Intermediate Algebra

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Section 1: Solving Linear Equations

An **equation** is a ____ .

A **linear equation** in one variable x is an equation that can be written in the standard form ______________________ where a and b are real numbers with $a \neq 0$.

Solving Linear Equations

To solve an equation means to find all values of the variable that make the equation a true statement.

Steps in solving linear equations:

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Example 1: If $4x+1=5$ find x and check your solution.

Example 2: If $7y-2=2y+8$ find y and check your solution.

Example 3: If $6(t-1)=2t-3$ find t and check your solution.

Example 4: If $5p-(p+4)=4+4(p+2)$ find p.

Solving Linear Equations Containing Fractions

Steps in solving linear equations containing fractions:

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Example 5: Solve for b: $\frac{4b}{5}-7=\frac{b}{10}$

Example 6: Solve for y: $\frac{4-2y}{3}=\frac{3}{4}-\frac{5y}{6}$

Example 7: Solve for k: $\frac{k}{3}+\frac{3k-1}{9}=\frac{1-3k}{6}+4$

Recognizing Identities and Equations with No Solutions

Identities

Any true statement such as $0=0,\ 4=4,$ or $-7=-7$ informs us that the original equation is an identity.

Example 8: Solve for x: $2(2x-4)=4(x-2)$

Equations with No Solutions

For linear equations, any false statements such as $2=5,\ 0=1,$ or $-3=3$ informs us that the original equation has no solutions.

Example 9: Solve for y: $5y-6=2+5y$

SECTION 1 SUPPLEMENTARY EXERCISES:

Solve each equation.

1. $9x - 25 = 4x$
2. $7y + 2 = 11y - 3$
3. $2 + 4w = -2(3 + 7w)$
4. $3(1 + 2a) = 7a - 3$
5. $2(w + 3) - 14 = 6(32 - 3w)$
6. $5(a + 2) - 4(a + 1) = 3$
7. $6(p - 4) = 8(p - 3)$
8. $5 - 4y = 7 - 2(3y - 1)$
9. $6m - 2(m - 3) = 4 + 4(m + 1)$
10. $-3(x - 1) + 8(x - 3) = 11x + 7$
11. $2 - 5(a - 1) - (3a + 2) = -(2a - 6) - (3a - 5)$
12. $-2(x - 3) + (4 + x) = 5 - 3(x - 1) - (5 - x)$
13. $\frac{5}{2} + \frac{3}{4}p = -4$
14. $\frac{1}{6}(5 - b) = \frac{1}{4}(2 + b)$
15. $\frac{4}{5}y - \frac{y}{4} = \frac{1}{2}$
16. $-\frac{3}{7} - \frac{d}{3} = -\frac{2d - 3}{7}$
17. $\frac{w}{4} + \frac{11}{12} = \frac{1}{2} - \frac{w}{6}$
18. $\frac{7 + m}{6} - \frac{3m - 1}{2} = \frac{5 - 2m}{5}$
19. $\frac{1 - 3t}{3} - t = \frac{3t - 1}{9} + \frac{1}{4}$
20. $\frac{29 + 9x}{18} = -\frac{5}{3}(x + \frac{1}{3})$
21. City A's public school population is growing much faster than the schools can handle. With an increase of new students each year, overcrowding has become a major problem. The growth in student population can be approximated by the equation

    $$P = 25{,}140x + 956{,}100$$

    where P is the city's school population and x is the year starting in 2010. In the equation substitute x=1 for 2010, x=2 for 2011, and so on.

    a) Find the number of students in current year.
    b) Predict the number of students in 2020.
    c) In what year did the student population equal 1,308,060?

22. The mathematical model

    $$S = .55x - 34.5$$

    approximates the Broadway tickets sold from 1999 – 2005. In the model, $x = 99$ corresponds to the 1999 – 2000 season, $x = 100$ corresponds to the 2000 – 2001 season, and so on. S is the number of tickets sold in millions.

    a) Based on this model, how many tickets were sold in the 2003 – 2004 season?
    b) In what season did the number of tickets sold reach 20.5 million

23. A salesperson receives a base salary of $30,000 and a commission of 10% of the total sales for the year.

a. If the salesperson sells $150,000 worth of merchandise, what is his total income for the year, including his base salary?
b. In the second year, he received a 5% of promotion of his base salary. At the end of the year, he received total income of $52,500, including his base salary. How much merchandise did he sell?

24. Jane works as a waitress. Today, she worked an 8 hour shift, and was paid $200, including $108 gratuity.

a. How much is Jane's base paid per hour?
b. At Saturday, Jane worked 10 hours and received $247 gratuity. How much did Jane received in total, including gratuity?

Section 2: Graphing Linear Equations

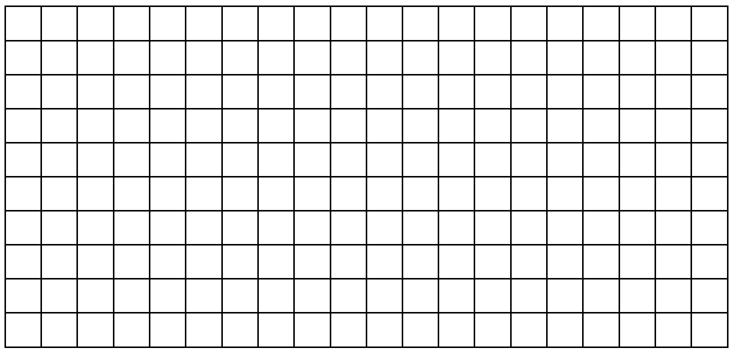
Ordered Pairs

Ordered pairs are represented by the notation (x, y)

Example 1: Plot each ordered pair on a rectangular coordinate system and identity the location of each ordered pair.

a) $(6,3)$ b) $(-4,2)$ c) $(-1,-2)$ d) $(5,-3)$ e) $(0,0)$ f) $(0,4)$

g) $(-5,0)$ h) $(0,-1)$ i) $(2,0)$



Linear Equation in Two Variables

A linear equation in two variables is an equation of the form $Ax + By = C$

Graphing Linear Equations Using Intercepts

x- and y-Intercepts

The x-intercept is where the graph crosses the ______________________ .

The y-intercept is where the graph crosses the ______________________ .

Steps to find the x-intercept

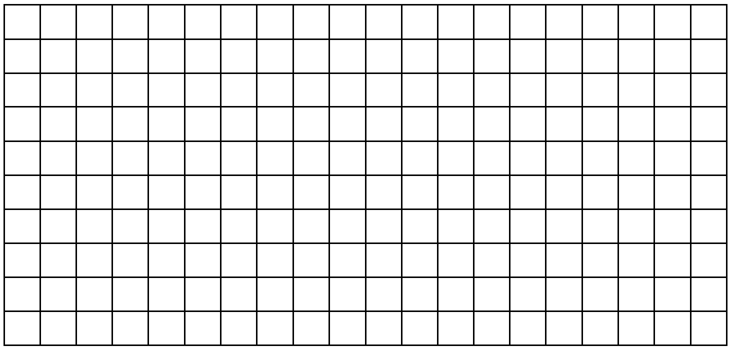
____

____

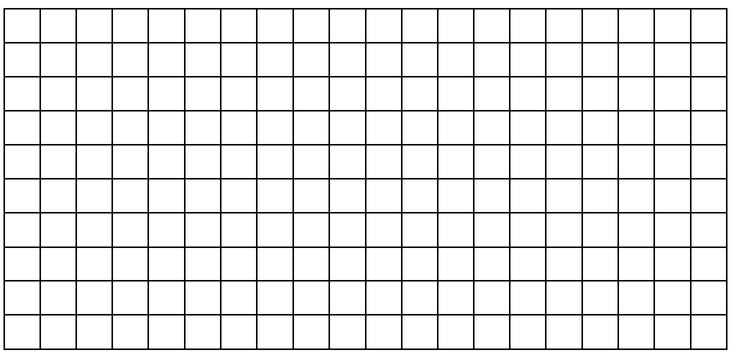
Steps to find the y-intercept

____

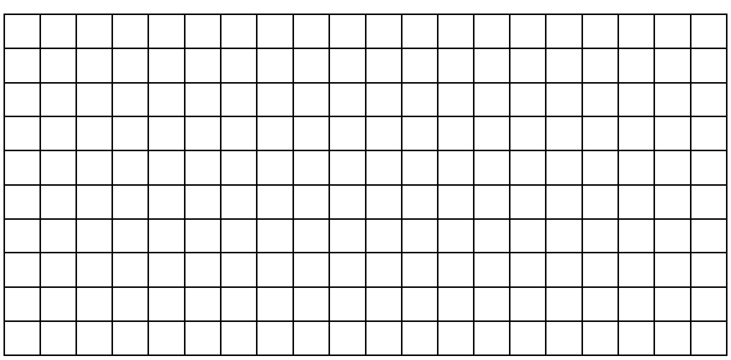
Example 2: Find the intercepts and graph $4x-2y=8$



Example 3: Find the intercepts and graph $3x+2y=12$



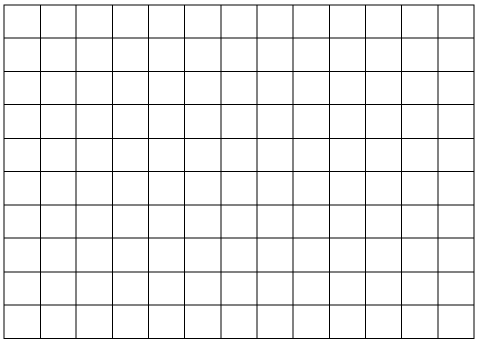
Example 4: Find the intercepts and graph $\frac{x}{3}-2y=3$



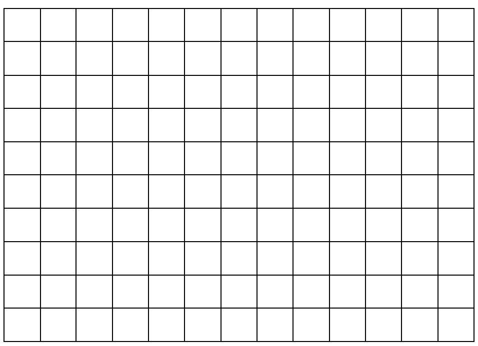
Graphing Vertical and Horizontal Lines

Graphing Vertical Lines

Example 5: Graph: $x = 3$



Example 6: Graph: $y = -1$



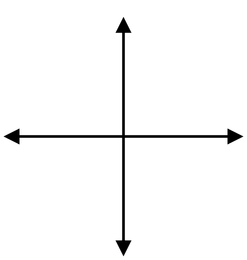
The Slope of a Line

Definition of Slope

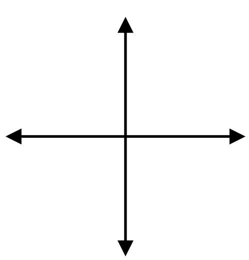
The slope, m, of the line between two points (x_1, y_1) and (x_2, y_2) is given by

Slope = m =

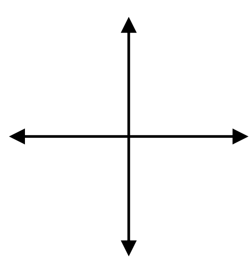
Graph a line for each given slope.



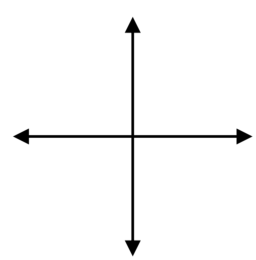
Negative Slope



Positive Slope



Zero Slope



Slope Undefined

Example 7: Find the slope of the line through $(-2,5)$ and $(-8,1)$. Sketch the graph.

Example 8: Find the slope of the line through $(-4,2)$ and $(-1,1)$. Sketch the graph.

Example 9: Find the slope of the line through $(-3,-6)$ and $(2,-6)$. Sketch the graph.

Example 10: Find the slope of the line through $(-5,5)$ and $(-5,1)$. Sketch the graph.

Example 11: If the slope of a line passing through the points $(2, x)$ and $(3,4)$ is 1, find x and sketch the graph.

The Equation of a Line

The Standard Form for the Equation of a Line

If a, b, and c are integers, then the equation of a line is in standard form when it has the

form ____________________ .

Example:

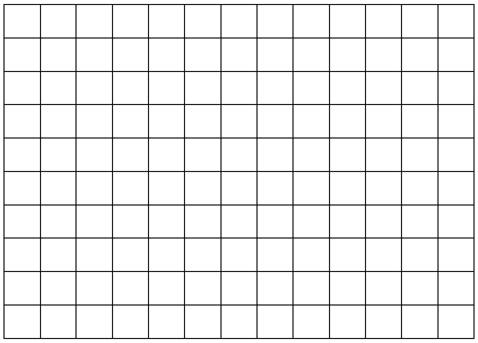
Slope-Intercept Form of the Equation of a Line

The equation of any line with slope m and y-intercept b is given by ________________________

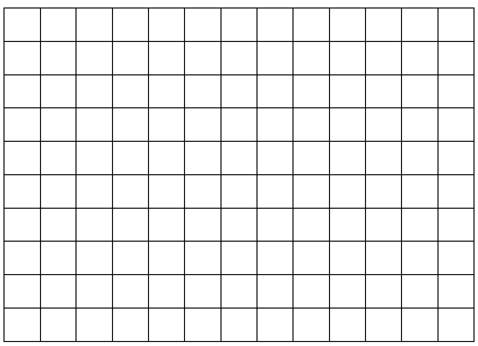
Example 12: Find the equation of the line with slope $-\frac{2}{3}$ and y-intercept $(0,4)$. Graph.

Example 13: Find the equation of the line with slope 4 and y-intercept $(0,-3)$. Graph.

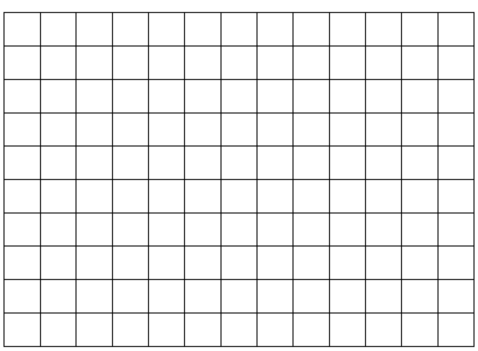
Example 14: Find the slope and y-intercept for the line $2x+3y=5$. Graph.



Example 15: Find the slope and y-intercept for the line $2x-2y=1$. Graph.



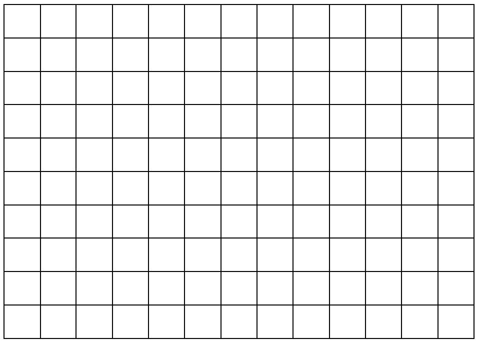
Example 16: Give the slope and y-intercept for the line $5x+y=0$. Graph.



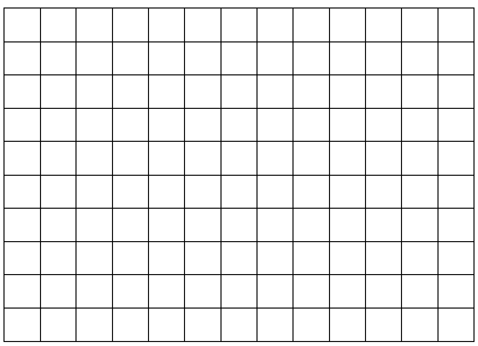
Point-Slope Form of the Equation of a Line

The equation of the line through (x_1, y_1) with slope m is given by ______________________________

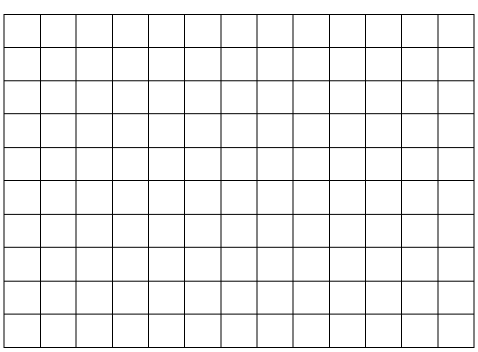
Example 17: Find the equation of the line that passes through the points $(-1,5)$ and $(0,-2)$ .Graph.



Example 18: Find the equation of the line that passes through the points $(-1,-6)$ and $(1,2)$. Graph.



Example 19: Find the equation of the line that passes through the points $(5,3)$ and $(-2,3)$. Graph.



SECTION 2 SUPPLEMENTARY EXERCISES:

1. Find the slope and the equation of the line through the given points
   a) $(8,-4)$ and $(-1,-6)$
   b) $(2,-4)$ and $(-5,3)$
   c) $(2,5)$ and $(2,-5)$
   d) $(3,6)$ and $(-1,6)$
   e) $(3,1)$ and $(5,6)$

2. a) If the slope passing through the points $(-4,-1)$ and $(x,2)$ is $-\frac{3}{5}$, find x.

   b) If the slope passing through the points $(-2,-3)$ and $(x,4)$ is $\frac{1}{3}$, find x.

   c) If the slope passing through the points $(-1,5)$ and $(0,y)$ is -7, find y.

   d) If the slope passing through the points $(-8,-4)$ and $(-4,y)$ is $-\frac{5}{4}$, find y.

3. Use the point-slope form to find the equation of a line:

   a) slope $=-\frac{2}{3}$ through $(-1,3)$

   b) slope $=4$ through $(3,-2)$

   c) slope $=-3$ through $(0,5)$

   d) through $(1,-3)$ and $(5,5)$

   e) through $(-3,2)$ and $(-5,-2)$

   f) through $(-1,0)$ and $(0,4)$

   g) through $(-2,4)$ and $(3,-6)$

4. Write in slope-intercept form equation of the following lines and sketch both lines in one graph. Can you point out the relationship of these two lines?

   a) The line pass the point $(6,-1)$ and has a slope of $-\frac{3}{2}$.

   b) The line pass the point $(3,-2)$ and has a slope of $\frac{2}{3}$.

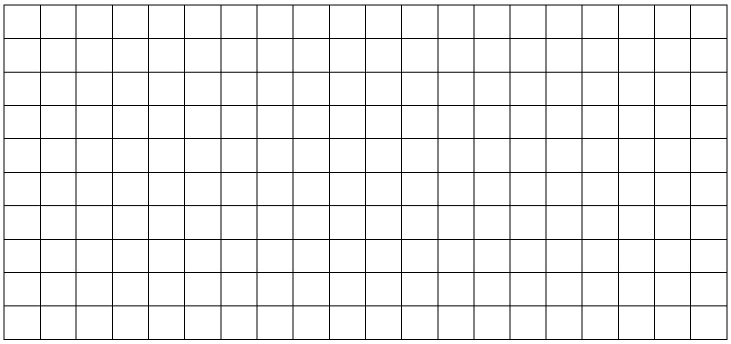
Section 3: Parallel and Perpendicular Lines

Parallel Lines

Non vertical parallel lines have the same ____________________

Example 1: State whether the lines are parallel or not. Graph.

$$y = -2x - 5$$
$$y = -2x + 7$$

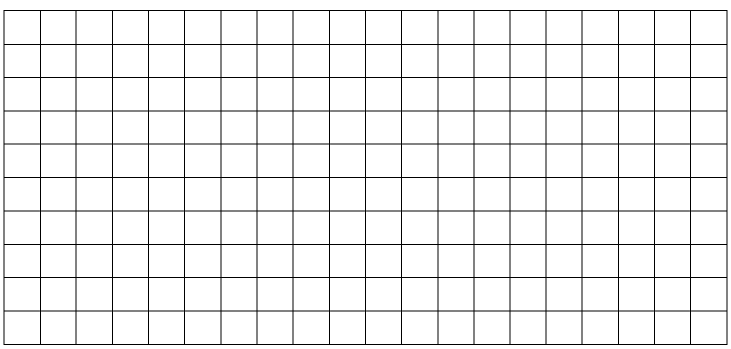


Perpendicular Lines

The slopes of perpendicular lines are ____

Example 2: State whether the lines are perpendicular or not. Graph.

$$y = \frac{4}{5}x - 2$$
$$y = \frac{4}{5}x$$



Example 3: Determine whether the lines are parallel, perpendicular, or neither.

a) $x - 2y = 3$
$-2y = -x - 4$

b) $$\begin{aligned} 6x + 3y &= 12 \\ -x + 2y &= 3 \end{aligned}$$

c) $$\begin{aligned} 5x + 3y &= 8 \\ 15x - 9y &= 5 \end{aligned}$$

Example 4: Prove that the following lines are parallel

$$l_1: \quad (3,-1) \quad and \quad (-2,-7)$$
$$l_2: \quad (-1,4) \quad and \quad (-6,-2)$$

Example 5: Prove that the following lines are perpendicular

$$l_1: \quad (-4,-3) \quad and \quad (2,5)$$
$$l_2: \quad (3,1) \quad and \quad (-5,2)$$

Example 6: Write an equation of a line passing through

a) the point $(3,6)$ and parallel to $y = 4x - 2$

b) the point $(3,6)$ and perpendicular to $y = 4x - 2$

c) the point $(-1,2)$ and perpendicular to $y = \frac{2}{3}x - 10$

d) the point $(-4,1)$ and parallel to $y = \frac{2}{5}x + 7$

e) the point $(6,-3)$ and perpendicular to $y = 8$

f) the point $(-5,-5)$ and perpendicular to $x = 5$

SECTION 3 SUPPLEMENTARY EXERCISES:

1. Determine whether the lines are parallel, perpendicular, or neither.

a) $y-3=4x$, $y=-\frac{1}{4}x+2$

b) $2x+5y=6$, $y-\frac{1}{4}x=2$

c) $6x+2y=7$, $3x=6-y$

d) $x+3y=-9$, $y=3x+4$

e) $14x+2y=1$, $-\frac{21}{2}x-\frac{3}{2}y=4$

f) $5x-6=y$, $y=-\frac{2}{3}x+3$

2. Write an equation of a line passing through

a) the point $(4,5)$ and parallel to $2x-y=4$

b) the point $(\frac{1}{2},2)$ and perpendicular to $y+5=2x$

c) the point $(3,-6)$ and parallel to $8x-y=4$

d) the point $(-1,-1)$ and perpendicular to $x+3y=9$

e) the point $(3,5)$ and parallel to $y=7$

f) the point $(3,5)$ and perpendicular to $y=7$

g) the point $(4,-5)$ and parallel to $x=5$

h) the point $(4,-5)$ and perpendicular to $x=5$

i) the point $(-2,4)$ and parallel to $7x+y=9$

j) the point $(-1,\frac{5}{2})$ and perpendicular to $y=3x+8$

Section 4: Systems of Linear Equations

Systems of Two Linear Equations

A pair of equations $\begin{cases} y = -3x + 4 \\ y = -2x - 5 \end{cases}$ is called **a system of linear equations**.

Determining Whether an Ordered Pair is a Solution

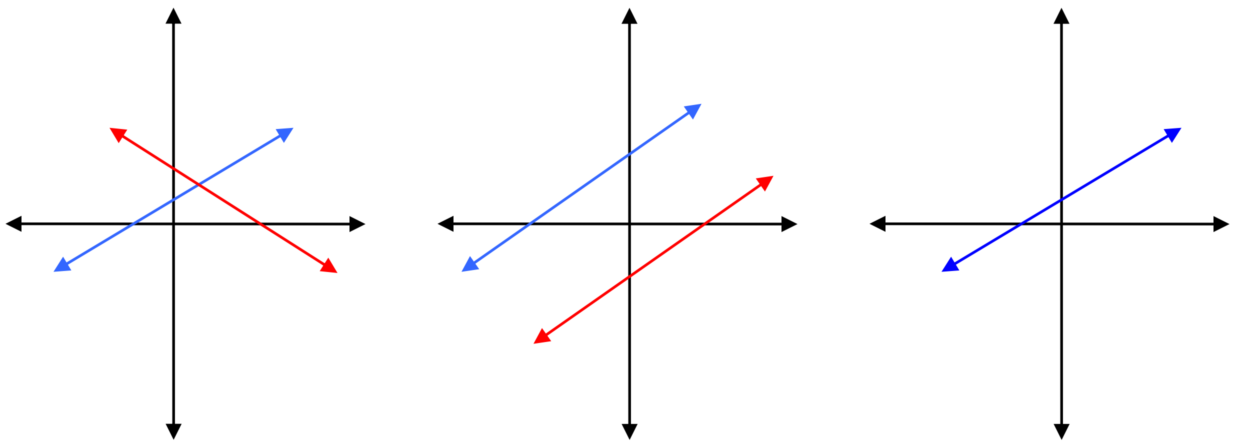
A solution of a system of two equations in two variables is an ordered pair (x, y) that makes ______________________ equations true.

Example 1: Determine whether the ordered pair $(2,-1)$ is a solution of the system $\begin{cases} 3x + 2y = 4 \\ x - y = 3 \end{cases}$

Example 2: Determine whether the ordered pair $(-3,4)$ is a solution of the system $\begin{cases} 2x + 3y = 6 \\ x - y = 1 \end{cases}$

Methods of Solving Two Linear Equations

Possible Solutions to Systems of Two Linear Equations



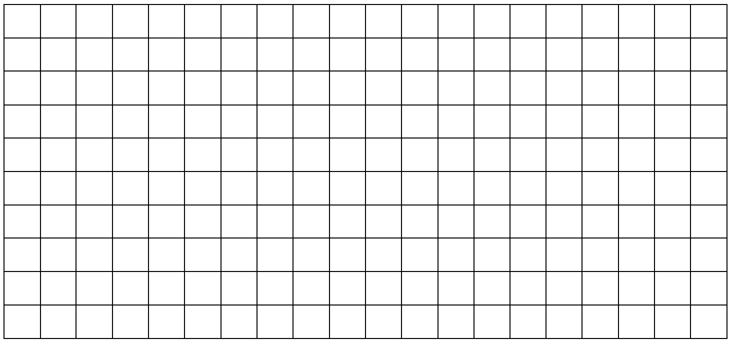
I. The Graphing Method

Solving a System of Two Equations Using the Graphing Method

Example 3: Solve the system of linear equations by graphing $\begin{cases} 3x + y = 4 \\ x - y = 3 \end{cases}$

Example 4: Solve the system of linear equations by graphing $\begin{cases} x + 2y = 4 \\ 2x + 4y = 6 \end{cases}$

Example 5: Solve the system of linear equations by graphing $\begin{cases} 3x + y = 2 \\ 6x + 2y = 4 \end{cases}$



II. The Substitution Method

Solving a System of Two Equations Using the Substitution Method

Example 6: Solve the system of linear equations using the substitution method $\begin{cases} 2x + 3y = 24 \\ y - 2x = 0 \end{cases}$

Example 7: Solve the system of linear equations using the substitution method $\begin{cases} 2x+2y=-5 \\ x-y=-1 \end{cases}$

Example 8: Solve the system of linear equations using the substitution method $\begin{cases} \frac{3}{4}x-2y=-\frac{7}{4} \\ x+\frac{1}{3}y=\frac{2}{3} \end{cases}$

III. The Elimination Method

Solving a System of Two Equations Using the Elimination Method

Example 9: Solve the system of linear equations using the elimination method $\begin{cases} -4x+3y=15 \\ 4x+2y=5 \end{cases}$

Example 10: Solve the system of linear equations using the elimination method $\begin{cases} 3x - 2y = 12 \\ 4x + 2y = 23 \end{cases}$

Example 11: Solve the system of linear equations using the elimination method $\begin{cases} 2x - 3y = 8 \\ -3x + y = -5 \end{cases}$

Example 12: Solve the system of linear equations using the elimination method $\begin{cases} -6x+2y=-3 \\ 3x-4y=3 \end{cases}$

Example 13: Solve the system of linear equations using the elimination method $\begin{cases} 3x+2y=12 \\ 6x-3y=24 \end{cases}$

Example 14: Solve the system of linear equations using the elimination method $\begin{cases} 4x - 4y = 16 \\ -x + y = 6 \end{cases}$

Example 15: Solve the system of linear equations using the elimination method $\begin{cases} 2x - y = \dfrac{3}{2} \\ 8x - 4y = 6 \end{cases}$

SECTION 4 SUPPLEMENTARY EXERCISES:

1. a) Determine whether the ordered pair $(-5,1)$ is a solution of the system $\begin{cases} x+2y=-3 \\ x-y=-4 \end{cases}$

   b) Determine whether the ordered pair $(3,-1)$ is a solution of the system $\begin{cases} \frac{3}{2}x+\frac{1}{2}y=4 \\ \frac{2}{3}x-y=-8 \end{cases}$

2. Solve the system of linear equations by the graphing, substitution and elimination methods.

   a) $\begin{cases} x+y=3 \\ 2x-2y=10 \end{cases}$ b) $\begin{cases} x+y=5 \\ 3x-y=3 \end{cases}$ c) $\begin{cases} 3x-2y=8 \\ x+4y=12 \end{cases}$

3. Solver the system of linear equations by any method.

   a) $\begin{cases} 2x+y=5 \\ 3x-y=5 \end{cases}$ b) $\begin{cases} 4x+3y=-7 \\ 2x-y=-1 \end{cases}$ c) $\begin{cases} 2x+2y=-2 \\ 2x-y=4 \end{cases}$

   d) $\begin{cases} 3x-4y=5 \\ 6x-8y=8 \end{cases}$ e) $\begin{cases} 4x-3y=9 \\ 3x+2y=11 \end{cases}$ f) $\begin{cases} x+y=4 \\ 3x+3y=12 \end{cases}$

   g) $\begin{cases} 2x+3y=23 \\ y=x+1 \end{cases}$ h) $\begin{cases} y=-\frac{1}{3}x+8 \\ x=3y \end{cases}$ i) $\begin{cases} \frac{1}{2}x+\frac{1}{3}y=6 \\ \frac{1}{5}x+\frac{1}{8}y=1 \end{cases}$

   j) $\begin{cases} x=y+5 \\ 3x+7y=115 \end{cases}$ k) $\begin{cases} 5x-6y=18 \\ 10x+12y=-6 \end{cases}$ l) $\begin{cases} \frac{1}{2}x-\frac{1}{3}y=1 \\ \frac{1}{3}x-\frac{1}{4}y=1 \end{cases}$

Section 5: Exponents

Properties of Exponents

Let x be a real number and m and n are integers.

		Examples
Product Rule	$x^m \cdot x^n =$	$y^8 \cdot y^3 =$
Quotient Rule	$\frac{x^m}{x^n} =$ where $(x \neq 0)$	$\frac{a^7}{a} =$
Zero Exponent	$x^0 =$ where $(x \neq 0)$	$w^0 =$
Power Rule	$(x^m)^n =$	$(b^6)^2 =$
Power of a Product	$(x \cdot y)^n =$	$(r^4 t)^3 =$
Power of a Quotient	$\left(\frac{x}{y}\right)^n =$ where $(y \neq 0)$	$\left(\frac{p^9}{q^2}\right)^5 =$
Negative Exponent	$x^{-n} =$ where $(x \neq 0)$	$h^{-3} =$

Simplify the following exponents:

Example 1: $(5x^7)(-2x^8)$

Example 2: $\frac{-24a^3}{18a}$

Example 3: $\left(3s^4t^7\right)^2$

Example 4: $\left(\dfrac{32p^6}{24q^2}\right)^3$

Example 5: $\left(-9a^5b^3c\right)^0$

Example 6: $\dfrac{-27x^6y^4}{18xy^2}$

Example 7: $\dfrac{7b^4}{14b^{-7}}$

Example 8: $\left(\frac{12m^3n^{-3}}{8m^9n^3}\right)^2$

Example 9: $\frac{\left(-3x^4y^{-2}\right)^3}{9x^{-12}y^{-6}}$

Example 10: $\left(\frac{a^4b^{-5}c^6}{a^3b^{-3}c^{-2}}\right)^{-3}$

Example 11: $\left(\dfrac{4p^{-2}q^3r^9}{-6p^2qr^{-4}}\right)^2$

Example 12: $\dfrac{\left(-6m^{-3}n^7\right)^{-2}(3m^5n^{-1})^3}{(2m^3n)^{-2}}$

SECTION 5 SUPPLEMENTARY EXERCISES:

Simplify.

1. $(3x^6y)(-2xy^7)$

2. $(6a^3b^3)(-9ab^{-3})$

3. $\dfrac{p^6qr^7}{p^3q^4r}$

4. $\dfrac{-8m^4n^6}{-4m^4n^2}$

5. $\dfrac{-15u^6v^2}{(-12u^3v)^2}$

6. $(-3a^4b^3)^4$

7. $(6g^4h)^2$

8. $(2x^5y^2)^{-4}(4x^4y)^2$

9. $(5x^7y^0z^{-1})^2(2xy^{-5})^3(2y^3z^2)^3$

10. $\dfrac{(3a^3b^6)^{-2}}{(2a^6b)^{-3}}$

11. $\left(\dfrac{16x^3y^6z^6}{-4xy^{-5}z^{-2}}\right)^{-2}$

12. $\left(\dfrac{e^6f^{-3}}{e^{-3}f^4}\right)^8$

13. $\left(\dfrac{b^{-3}c}{a^2b^{-9}c^5}\right)^{-5}$

14. $\dfrac{(3x^8y^{-2})^{-3}(2x^6y^{-2})^2}{(2x^{-2}y^7)^{-5}}$

15. $\dfrac{(3a^3b^6)^{-2}}{(2a^6b)^{-3}}$

16. $\left(\dfrac{16x^3y^6z^6}{-4xy^{-5}z^{-2}}\right)^{-2}$

17. $\left(\dfrac{e^6f^{-3}}{e^{-3}f^4}\right)^8$

18. $\left(\dfrac{b^{-3}c}{a^2b^{-9}c^5}\right)^{-5}$

19. $\dfrac{(3x^8y^{-2})^{-3}(2x^6y^{-2})^2}{(2x^{-2}y^7)^{-5}}$

20. $\dfrac{(5a^{-2}b^6c^3)^3(25a^3b^4)^{-4}}{15a^{12}c^5}$

Section 6: Addition, Subtraction, and Multiplication of Polynomials

A **polynomial** is any finite sum of terms.

The **degree of the polynomial** with one variable is the highest power to which the variable is raised in any one term.

Like terms have identical variable parts, but differ only in their numerical coefficients.

Addition and Subtraction of Polynomials

Add or subtract like terms.

Example 1: Add and subtract $5x^4 - 2x^3$ and $3x^4 + 6x^3$

Example 2: Find the sum of $4a^2 + 6a - 1$ and $a^4 - 7a^2 + 9a + 4$

Example 3: $3(y^2 + 2y - 5) - 6(y^2 - y + 3)$

Example 4: $(5r^3t^2 + 6r^3t - r^2t) + (9r^3t - 3r^2t + 2rt)$

Example 5: $(4x + 6y - 9z) - (-2x - 4y - 3z)$

Example 6: $(-3x - 4y + 2z) + (2x + 2y + 2z)$

Example 7: $2(4a^2 - a + 9) - 5(a^2 + 2a - 6)$

Multiplication of Polynomials

Multiplying a Monomial by a Monomial

Remember the rules of exponents.

Example 8: Multiply $(5x^3y^6)(-3xy^4)$

Example 9: Multiply $(4a^3b^6)(5a^2b)^{-2}$

Multiplying a Monomial by a Binomial

Use the **Distributive Law**: $a(b+c)=ab+ac$

Example 10: Multiply: $2p^3(4p^7-5)$

Example 11: Multiply: $-5s^6t(3s^8-4st)$

Example 12: Multiply: $9m^3n^6(-6m^2n^{-5}+8m^2n)$

Multiplying a Binomial by a Binomial

The **FOIL** (Front-Outer-Inner-Last) Method: $(a+b)(c+d) = ac + ad + bc + bd$

Example 13: Multiply: $(x+4)(x+6)$

Example 14: Multiply: $(p-5)(p-3)$

Example 15: Multiply: $(t-8)(t+7)$

Example 16: Multiply: $(4+b)(4-b)$

Example 17: Multiply: $(a-8)(2a+3)$

Example 18: Multiply: $(m-2)(m-2)$

Example 19: Multiply: $(3a+2b)(6a-b)$

Example 20: Multiply: $(7m-3n)(m-5n)$

Example 21: Multiply: $(3m+2n)(7n-5m)$

SECTION 6 SUPPLEMENTARY EXERCISES:

Perform the indicted operation:

1. $(3x-9y)+(5x-2y)$

2. $(12xy^4+15x^2y^2-2xy+3)+(3xy^4-6x^2y^2+9)$

3. $(7a-3b+2c)+(3c-4b-6a)$

4. $(5m^4+12n^3)-(-6m^4-7n^3)$

5. Subtract $(3a+2b)$ from $(9a-5b)$

6. From $(2y^3-15y)$ subtract $(4y^3+8y)$

7. $(9-5t^2)-(5-2t^2)$

8. $(9m^3n^4-2m^{-3}n^{-4})-(-4m^3n^4+8m^{-3}n^{-4})$

9. $(5p^3q)^2(-8p^{-2}q^5)$

10. $(7xy)(-3xy)$

11. $-5a^3(6a^2+4a-5)$

12. $6x^3y(3x^2-5y)$

13. $-10(a^2b^3)^2(b^2-6b-5)$

14. $(x+5)(x-2)$

15. $(y-3)(y+8)$

16. $(5-m)(2-2m)$

17. $(2b+3a)(6b-9a)$

18. $(3y-1)(6y-5)$

19. $(n-3)(n-3)$

20. $(n+7)(n-7)$

Section 7: Factoring Polynomials

The Greatest Common Factor

The **greatest common factor** for a polynomial is the largest monomial that divides each term of the polynomial.

Example 1: Factor $5p^3 + 15p^2 - 30p$

Example 2: Factor $8x^4 - 4x^3 + 16x^2 - 4x + 24$

Example 3: Factor $9a^3b^4 - 3a^2b^3 + 6ab^2$

Factoring by Grouping

Steps in factoring by grouping

1. Factor out any monomial that is common to all four terms.
2. Group together pairs of terms and factor each pair.
3. If there is a common binomial factor, then factor it out.
4. If there is no common binomial factor, then interchange the middle two terms and repeat the process over. If there is still no common binomial, then the polynomial cannot be factored.

Example 4: Factor $56 + 21k + 8h + 3hk$

Example 5: Factor $5x^2 + 40x - xy - 8y$

Factoring Trinomials

Factoring Trinomials with Lead Coefficients of 1

Since the product of two binomials is often a trinomial, it is expected that many trinomials will factor as two binomials. For example, to factor $x^2 + 9x + 18$, we must find two binomials $x + a$ and $x + b$ such that

$$x^2 + 9x + 18 = (x + a)(x + b)$$

where $ab = 18$ (product is 18)

and

$ax + bx = 9x$ (sum of the two numbers is 9)

To find the numbers a and b, we first list the possible factorizations of $+18$ and find the one where the sum of the factors is $+9x$.

The possible factorizations of $+18$ with their respective sums are:

Products of $+18$	**Sums**
$(+1)(+18)$	$(+1) + (+18) = +19$
$(-1)(-18)$	$(-1) + (-18) = -19$
$(+2)(+9)$	$(+2) + (+9) = +11$
$(-2)(-9)$	$(-2) + (-9) = -11$
$(+3)(+6)$	$(+3) + (+6) = +9$ ←
$(-3)(-6)$	$(-3) + (-6) = -9$

Thus, $a = 3$ and $b = 6$, and

$$x^2 + 9x + 18 = (x + a)(x + b)$$
$$x^2 + 9x + 18 = (x + 3)(x + 6)$$

Steps to factoring trinomials with lead coefficient of 1

1. Write the trinomial in descending powers.
2. List the factorizations of the third term of the trinomial.
3. Pick the factorization where the sum of the factors is the coefficient of the middle term.
4. Check by multiplying the binomials.

Example 6: Factor $x^2 - 8x + 12$

Example 7: Factor $a^2 + 5a + 6$

Example 8: Factor $y^2 - 9y - 36$

Example 9: Factor $x^2 + 14xy + 45y^2$

Example 10: Factor $x^2 - 9xy + 18y^2$

Example 11: Factor $a^2 + 7ab - 8b^2$

Factoring Trinomials with Lead Coefficients other than 1

Two methods in factoring trinomials

1. Trial-and-error
2. Factoring by grouping

Steps to factoring trinomial with lead coefficients other than 1

1. Form the product ac.
2. Find a pair of numbers whose product is ac and whose sum is b.
3. Rewrite the polynomial to be factored so that the middle term bx is written as the sum of the two terms whose coefficients are the two numbers found in step #2.
4. Factor by grouping.

Example 12: Factor $2h^2 - 5h - 12$

Example 13: Factor $3k^2 - 14k - 5$

Factoring Special Products

Many trinomials can be factored by using the following special product formulas:

Factoring Perfect Square Trinomial $a^2 + 2ab + b^2 = (a+b)^2$

$a^2 - 2ab + b^2 = (a-b)^2$

Example 14: Factor $t^2 - 8t + 16$

Example 15: Factor $p^2 + 10t + 25$

Example 16: Factor $16a^2 - 40a + 25$

Example 17: Factor $9b^2 + 42b + 49$

Example 18: Factor $16y^2 - 72y + 81$

Factoring a Difference of Squares

Factoring a Difference of Squares $a^2 - b^2 = (a+b)(a-b)$

Example 19: Factor $x^2 - 49$

Example 20: Factor $25 - b^2$

Example 21: Factor $36x^2 - 16y^2$

Example 22: Factor $100t^2 - 49r^2$

Example 23: Factor $x^2 + 36$

Example 24: Factor completely $16x^4 - 81$

Example 25: Factor completely $16x^4 - y^4$

Factoring a Polynomial

To factor a polynomial, first factor the greatest common factor, then consider the number of terms in the polynomial.

I. Two terms: Determine if the binomial is a difference of squares.
   If it is a difference of squares, then $a^2 - b^2 = (a+b)(a-b)$

II. Three Terms: Determine if the trinomial is a perfect square trinomial.
   a) If the trinomial is a perfect square then

$$a^2 + 2ab + b^2 = (a+b)^2$$
$$a^2 - 2ab + b^2 = (a-b)^2$$

   b) If the trinomial is not a perfect square, then
      i) If it is $x^2 + bx + c$, then
      find two factors (x + first number)(x + second number)
      ii) If it is $ax^2 + bx + c$, then use trial and error or the factoring method.

III. Four terms: Try to factor by grouping.

Example 26: Factor completely $4x^3 - 12x^2 + 9x$

Example 27: Factor completely $10y^3 + 2y^2 - 36y$

Example 28: Factor completely $20a - 5a^3$

Example 29: Factor completely $16a^5b - ab$

Example 30: Factor completely $3x^4 - 3x^3 - 36x^2$

SECTION 7 SUPPLEMENTARY EXERCISES:

1. $a^2 - 5a$
2. $25y^3 - 15y^2$
3. $5ab^2 - 15a^3b$
4. $3p^6q^2 - 24pq^3$
5. $2b^4c^5 - 14b^3c^3$
6. $16x - 4x^3$
7. $y^3 - 81y$
8. $49 - 9t^2$
9. $64w^2 - 81$
10. $36x^2y - 9y$
11. $4a^2 - 121$
12. $36x^2 - 25y^2$
13. $a^2 - 4a - 12$
14. $b^2 - 6b + 5$
15. $x^2 + 7x + 10$
16. $m^2 + 9mn + 18n^2$
17. $x^2 + 9xy - 36y^2$
18. $-6x^2 - 9x + 15$
19. $m^3 - 4m^2 - 21m$
20. $24 + 5n - n^2$
21. $2x^3 - 2x^2y - 12xy^2$
22. $2x^3y + x^2y^2 - 6xy^3$
23. $am - 5a + 2bm - 10b$
24. $15x - 12ax + 10y - 8ay$
25. $7b - 2bd + 21c - 6cd$
26. $15ax + 4by + 10ay + 6by$

Section 8: Quadratic Equations

The quadratic $ax^2 + bx + c = 0$ can be solved in many ways:

I. Factor the quadratic and then use the Zero Factor Theorem.
2. Square root principle
3. Completing the square
4. Quadratic Formula

Solving a Quadratic Equation by Factoring

Zero Factor Theorem: If ab=0 then a = 0 or b = 0 or both.

Example 1: Solve: $x^2 - 4x - 32 = 0$

Example 2: Solve: $36 - y^2 = 0$

Example 3: Solve: $a^3 - 6a^2 + 5a = 0$

Solving a Quadratic Equation by The Square Root Principle

If $x^2 = a$ and $a \geq 0$, then $x = \sqrt{a}$ or $x = -\sqrt{a}$. Sometime it is indicated as $x = \pm\sqrt{a}$

Isolate the square and then apply the Square Root Principle

Example 4: Solve using the square root principle: $x^2 = 40$

Example 5: Solve using the square root principle: $(a-5)^2 = 16$

Example 6: Solve using the square root principle: $(4t-3)^2 - 18 = 0$

Solving Quadratics Equations by Completing the Square

Example 7: Solve by completing the square. $x^2 + 8x + 4 = 0$

Example 8: Solve by completing the square. $y^2 - 6y - 8 = 0$

Example 9: Solve by completing the square. $9a^2 - 6a - 4 = 0$

Example 10: Solve by completing the square. $4k^2 - 20k - 6 = 0$

Example 11: Solve by completing the square. $2w^2 + 12w - 14 = 0$

Example 12: Solve by completing the square. $a^2 - 10a + 9 = 0$

Solving Quadratics Equations by Using the Quadratic Formula

To solve a quadratic equation in the form $ax^2 + bx + c = 0$, where $a \neq 0$, use the **quadratic formula**:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The **discriminant** is the radicand $b^2 - 4ac$ in the quadratic formula.

If the discriminant is positive, then the equation has two real number solutions.
If the discriminant is 0, then the equation has one real solution.
If the discriminant is negative, then the equation has two non real complex solutions.

Example 11: Solve using the quadratic formula $x^2 - 5x - 8 = 0$

Example 12: Solve using the quadratic formula $-3x^2+2x+4=0$

Example 13: Solve using the quadratic formula $5x^2-3x-1=0$

Example 14: Solve using the quadratic formula $4x^2 - 4x = 0$

Example 15: Solve using the quadratic formula $x^2 - 6x - 9 = 0$

SECTION 8 SUPPLEMENTARY EXERCISES:

1. Solve the following equations by factoring:

   a. $x^2 + 6x + 8 = 0$
   b. $2t^2 + 3t - 14 = 0$
   c. $x^2 - 10x + 25 = 0$
   d. $w^2 + 14w + 55 = 6$
   e. $t^2 - 5t = 24$
   f. $2y^3 + 12y^2 - 32y = 0$
   g. $k^2 - 25 = 0$
   h. $16x^2 = 49$
   i. $z^2 - 9z = 0$
   j. $-6x^2 - x + 12 = 0$
   k. $5m^2 + 20m = 6 - 9m$
   l. $(y + 5)^2 - 4 = 0$
   m. $(n - 3)(3n - 2) - 8n = 0$
   n. $10x^2 = 27x - 18$

2. Solve the following equations by using the square root principle:

   a. $x^2 = 100$
   b. $s^2 - 12 = 0$
   c. $y^2 + 7 = 88$
   d. $(y - 3)^2 - 16 = 0$
   e. $(y + 7)^2 - 25 = 0$
   f. $(y - 1)^2 - 12 = 0$
   g. $(2k - 1)^2 - 9 = 0$
   f. $9(2m - 3)^2 + 8 = 449$

3. Solve the following equations by completing the square:

   a. $p^2 - p = 6$
   b. $x^2 + 10x - 7 = 0$
   c. $y^2 - 3y - 6 = 0$
   d. $6k^2 + 17k + 5 = 0$
   e. $5m^2 - 10m + 3 = 0$
   f. $3x^2 + 4 = 8x$
   g. $6w^2 + 12x = 48$
   h. $x^2 - 10x + 26 = 8$

4. Solve the following equations by using the quadratic formula:

a. $y^2 + 10y + 9 = 0$

b. $s^2 - s = 30$

c. $m^2 - 81 = 0$

d. $6x^2 + 2x + 3 = 0$

e. $10x^2 - 13x - 3 = 0$

f. $4x^2 + 6x + 1 = 0$

g. $m^2 + 5m = 2$

h. $2k^2 + 9k = -7$

i. $(z-2)(z+4) + 6 = 0$

j. $(3x-7)(x+5) = -31$

k. $11n^2 - 4n(n-2) = 6(n+3)$

l. $x(x-3) = -10x - 7$