

### Perfect Square Trinomial

$$a^2 + 2ab + b^2 = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)^2$$

**Exercise 11:** Factor each trinomial below. Check that the first and the third terms are perfect square, and the middle term is  $2ab$ , then apply the perfect square formula

a)  $t^2 - 8t + 16$

$$= t^2 - 2(4)t + (4)^2 = (t - 4)^2$$

" $a^2 \pm 2ab + b^2$ "

b)  $p^2 + 10p + 25 = p^2 + 2(5)p + (5)^2$

$$= (p + 5)^2$$

c)  $16a^2 - 40a + 25 = (4a)^2 - 2(5)(4a) + (5)^2$

$$= (4a - 5)^2$$

d)  $9b^2 + 42b + 49$

$$= (3b)^2 + 2(3b)(7) + (7)^2$$
$$= (3b + 7)^2$$

e)  $16y^2 - 72y + 81 = (4y)^2 - 2(9)(4y) + (9)^2$

$$= (4y - 9)^2$$

### Mixed Factoring/Factoring Completely

To factor a polynomial, **first factor the greatest common factor**, then consider the number of terms in the polynomial.

**I. Two terms:** Determine if the binomial is a difference of two squares.

If it is a difference of two squares, then  $a^2 - b^2 = (a + b)(a - b)$

**II. Three Terms:** Determine if the trinomial is a perfect square trinomial.

a) If the trinomial is a perfect square, then

$$a^2 + 2ab + b^2 = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)^2$$

b) If the trinomial  $ax^2 + bx + c$  is not a perfect square, then

i) if the leading coefficient  $a = 1$ , then check the product  $= c$  and sum  $= b$  to determine the correct combination.

ii) if the leading coefficient  $a > 1$ , then use trial and error or the ac-method.

**III. Four terms:** Try to factor by grouping.

**Exercise 12:** Factor each polynomial **completely**. This may mean factoring GCF and/or factoring in two or more steps.

a) Factor completely  $3x^4 - 3x^3 - 36x^2$

$$3x^2(x^2 - x - 12)$$

$$3x^2(x-4)(x+3)$$

b) Factor completely  $20a - 5a^3$

$$5a(4 - a^2)$$

$$5a(2-a)(2+a)$$

$$\cancel{-5a}(a-2)(a+2)$$

$$\cancel{b-a} = -(a-b)$$

c) Factor completely  $16a^5b - ab$

$$ab(16a^4 - 1)$$

$$ab(4a^2 + 1)(4a^2 - 1)$$

$$ab(4a^2 + 1)(2a + 1)(2a - 1)$$

d) Factor completely  $8x^2 - 24x + 18$

$$2(4x^2 - 12x + 9)$$

$$2((2x)^2 - 2(2x)(3) + (3)^2)$$

$$2(2x-3)^2$$

$$\begin{array}{r} \cancel{ac=36} \\ b=-12 \end{array} \rightarrow \begin{array}{l} 4x^2 - 6x - 6x + 9 \\ (4x^2 - 6x) + (-6x + 9) \end{array}$$

$$\begin{array}{l} 2x(2x-3) - 3(2x-3) \\ (2x-3)(2x-3) \end{array}$$

e) Factor completely  $16x^3y - 40x^2y^2 + 25xy^3$

$$\text{GCF: } xy$$

$$xy(16x^2 - 40xy + 25y^2)$$

$$xy((4x)^2 - 2(4x)(5y) + (5y)^2)$$

$$xy(4x - 5y)^2$$

f) Factor completely  $12x^3 + 11x^2 + 2x$

$$ac=24 = \underline{8 \cdot 3}$$

$$b=11 = \underline{8+3}$$

$$\times (12x^2 + 11x + 2)$$

$$\times (3x+2)(4x+1)$$

$$\times ((12x^2 + 8x + 3x + 2))$$

$$\times (4x(3x+2) + 1(3x+2))$$

g) Factor completely  $7z^2w^2 - 10zw^2 - 8w^2$

$$w^2(7z^2 - 10z - 8)$$

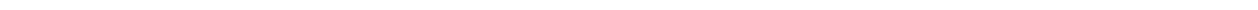
$$w^2(7z^2 - 14z + 4z - 8)$$

$$w^2(7z(z-2) + 2(z-2))$$

$$w^2(z-2)(7z+2)$$

## SECTION 1.1 SUPPLEMENTARY EXERCISES

- |     |                      |     |                           |
|-----|----------------------|-----|---------------------------|
| 1.  | $a^2 - 5a$           | 14. | $b^2 - 6b + 5$            |
| 2.  | $25y^3 - 15y^2$      | 15. | $x^2 + 7x + 10$           |
| 3.  | $5ab^2 - 15a^3b$     | 16. | $m^2 + 9mn + 18n^2$       |
| 4.  | $3p^6q^2 - 24pq^3$   | 17. | $x^2 + 9xy - 36y^2$       |
| 5.  | $2b^4c^5 - 14b^3c^3$ | 18. | $-6x^2 - 9x + 15$         |
| 6.  | $16x - 4x^3$         | 19. | $m^3 - 4m^2 - 21m$        |
| 7.  | $y^3 - 81y$          | 20. | $24 + 5n - n^2$           |
| 8.  | $49 - 9t^2$          | 21. | $2x^3 - 2x^2y - 12xy^2$   |
| 9.  | $64w^2 - 81$         | 22. | $2x^3y + x^2y^2 - 6xy^3$  |
| 10. | $36x^2y - 9y$        | 23. | $am - 5a + 2bm - 10b$     |
| 11. | $4a^2 - 121$         | 24. | $15x - 12ax + 10y - 8ay$  |
| 12. | $36x^2 - 25y^2$      | 25. | $7b - 2bd + 21c - 6cd$    |
| 13. | $a^2 - 4a - 12$      | 26. | $15ax + 4by + 10ay + 6by$ |



## Section 1.2 Solving Quadratic Equations by Factoring

### Solving a Quadratic Equation by Factoring

**Zero Factor Theorem:** If  $ab = 0$  then  $a = 0$  or  $b = 0$  or both.

#### Solving Quadratic Equations by Factoring

1. Put the equation in standard form:  $ax^2 + bx + c = 0$ .
2. Factor completely.
3. Use the zero-product rule, set each factor containing the variable equal to zero and solve for  $x$ . Note: Do not solve for the constant factor.

**Exercise 1:** Solve the equations by factoring

a) Solve:  $x^2 - 4x - 32 = 0$

$$(x-8)(x+4)=0$$

or

$$\begin{array}{r} x-8=0 \\ +8+8 \\ \hline x=8 \end{array}$$
$$\begin{array}{r} x+4=0 \\ -4-4 \\ \hline x=-4 \end{array}$$

$$\begin{aligned} *a &= 1 \\ ac &= -32 = \underline{-8} \cdot \underline{4} \\ b &= -4 = \underline{-8} + \underline{4} \end{aligned}$$

b) Solve:  $36 - 49y^2 = 0$

$$(6-7y)(6+7y)=0$$

or

$$\begin{array}{r} 6-7y=0 \\ -6 \\ \hline -7y=\underline{-6} \end{array} \quad \boxed{y=\frac{6}{7}}$$
$$\begin{array}{r} 6+7y=0 \\ -6 \\ \hline 7y=\underline{-6} \end{array} \quad \boxed{y=-\frac{6}{7}}$$

c) Solve:  $a^3 - 6a^2 + 5a = 0$

$$a(a^2 - 6a + 5) = 0$$
$$a(a-5)(a-1) = 0$$

or  $a-5=0$  or  $a-1=0$

$$\begin{array}{r} a=0 \\ +5+5 \\ \hline a=5 \end{array}$$
$$\begin{array}{r} a=1 \\ +1+1 \\ \hline a=1 \end{array}$$
$$\begin{aligned} *a &= 1 \\ ac &= 5 = \underline{-5} \cdot \underline{-1} \\ b &= -6 = \underline{-5} + \underline{-1} \end{aligned}$$

d) Solve:  $2x^3 + 9x^2 = 5x$

$$2x^3 + 9x^2 - 5x = 0$$
$$x(2x^2 + 9x - 5) = 0 \rightarrow a=2$$
$$x(2x^2 + 10x - 1x - 5) = 0 \quad ac = -10 = \underline{10} \cdot \underline{-1}$$
$$x(2x-1)(x+5) = 0 \quad b = 9 = \underline{10} + \underline{-1}$$

or  $2x-1=0$  or  $x+5=0$

$$\boxed{x=\frac{1}{2}}$$
$$\boxed{x=-5}$$

## SECTION 1.2 SUPPLEMENTARY EXERCISES

1. Solve the following equations by factoring:

a.  $x^2 + 6x + 8 = 0$

b.  $2t^2 + 3t - 14 = 0$

c.  $x^2 - 10x + 25 = 0$

d.  $w^2 + 14w + 55 = 6$

e.  $t^2 - 5t = 24$

f.  $2y^3 + 12y^2 - 32y = 0$

g.  $k^2 - 25 = 0$

h.  $16x^2 = 49$

i.  $z^2 - 9z = 0$

j.  $-6x^2 - x + 12 = 0$

k.  $5m^2 + 20m = 6 - 9m$

l.  $(y + 5)^2 - 4 = 0$

m.  $(n - 3)(3n - 2) - 8n = 0$

n.  $10x^2 = 27x - 18$

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# Module II

## Radicals and the Square Root Property

Radicals, specifically square roots, date back as far as c.1650 B.C., from the time of Egypt's Middle Kingdom. The Rhind Papyrus makes references to square roots since they are tied to the diagonals of squares and rectangles; often applicable in the construction of a temple. The "Rhind Papyrus" is named after Henry Rhind, a Scottish lawyer, who purchased it in Egypt in 1858. It was placed in the British Museum in London, England, in 1864 and is still there today; a fragment is also in the collection of the Brooklyn Museum on Eastern Parkway. The Rhind Papyrus, according to the British Museum website, is a "list of practical problems encountered in administrative and building works. The text contains 84 problems concerned with numerical operations, practical problem-solving, and geometrical shapes."

Robins, G. & Shute, C. (1990). *The Rhind Mathematical Papyrus: An ancient Egyptian text*. New York, NY: Dover. The British Museum; downloaded on 8/1/ 2011, from [http://www.britishmuseum.org/explore/highlights/highlight\\_objects/aes/r/rhind\\_mathematical\\_papyrus.aspx](http://www.britishmuseum.org/explore/highlights/highlight_objects/aes/r/rhind_mathematical_papyrus.aspx)

## Section 2.1: Addition, Subtraction, Multiplication and Division of Radicals

### Simplifying Radicals

**Exercise 1:** Simplify each radical below

a.  $\sqrt{9} = 3$

c.  $\sqrt{25} = 5$

e.  $\sqrt{49} = 7$

g.  $\sqrt{100} = 10$

b.  $\sqrt{18} = \sqrt{9} \sqrt{2} = 3\sqrt{2}$

d.  $\sqrt{75} = \sqrt{25} \sqrt{3} = 5\sqrt{3}$

f.  $\sqrt{98} = \sqrt{49} \sqrt{2} = 7\sqrt{2}$

h.  $\sqrt{500} = \sqrt{100} \sqrt{5} = 10\sqrt{5}$

List at least 12 numbers that are perfect square:

**Exercise 2:** Simplify each radical below

,  $x \geq 0$

$$\sqrt[n]{a^m} = a^{\frac{m}{n}}$$

$$\sqrt[n]{a} = \sqrt[n]{a}$$

a.  $\sqrt{x^2} = x$

c.  $\sqrt{x^4} = x^2$

e.  $\sqrt{x^6} = x^3$

g.  $\sqrt{x^{98}} = x^{49}$

$|\sqrt{a} \rightarrow \text{positive square root of } a$   
 $|\sqrt{a^2} = |a| \quad , \sqrt{a^2} = a \text{ if } a \geq 0$

b.  $\sqrt{x^3} = \sqrt{x^2} \sqrt{x} = x\sqrt{x}$

d.  $\sqrt{x^5} = \sqrt{x^4} \sqrt{x} = x^2 \sqrt{x}$

f.  $\sqrt{x^7} = \sqrt{x^6} \sqrt{x} = x^3 \sqrt{x}$

h.  $\sqrt{x^{99}} = \sqrt{x^{98}} \sqrt{x} = x^{49} \sqrt{x}$

What pattern have you noticed? Write a rule for simplifying a radical of the form  $\sqrt{x^n}$

When  $n$  is even,  $\sqrt{x^n} = x^{\frac{n}{2}}$

When  $n$  is odd,  $\sqrt{x^n} = x^{\frac{n-1}{2}} \sqrt{x}$

**Exercise 3:** Simplify each radical

a)  $\sqrt{12x^6y^3}$

$$\sqrt{4x^6y^2} \sqrt{3y}$$

$4x^3y \sqrt{3y}$

b)  $3x\sqrt{20x^3y^6z^9}$

$$3x \sqrt{4x^2y^6z^8} \sqrt{5xz}$$

$$3x (2xy^3z^4) \sqrt{5xz}$$

$6x^2y^3z^4 \sqrt{5xz}$



Find the error in each problem. Justify your answers. Provide the correct answers.

Find the errors	What are the errors?	Corrections
$\sqrt{x^9} = x^3$	$\sqrt{9} = 3$	$\sqrt{x^9} = \sqrt{x^8} \sqrt{x}$ $= x^4 \sqrt{x}$
$\sqrt{48} = \sqrt{4} \cdot \sqrt{12} = 2\sqrt{12}$	$\sqrt{12}$ can be factored further	$\sqrt{4} \cdot \sqrt{4}\sqrt{3} =$ $2 \cdot 2\sqrt{3} = 4\sqrt{3}$
$\sqrt{16} = \sqrt{4} = 2$	$\sqrt{4} = \text{perfect square continue?}$	$\sqrt{16} = 4$
$5 + 2\sqrt{x} = 7\sqrt{x}$	not like terms	$5 + 2\sqrt{x}$
$\frac{2 + \sqrt{6}}{2} = 1 + \sqrt{3}$	$\frac{\sqrt{6}}{2}$ cannot divide unless 2 is in radic(s) or $\sqrt{6}$ is simplified	$\frac{2}{2} + \frac{\sqrt{6}}{2} = 1 + \frac{\sqrt{6}}{2}$

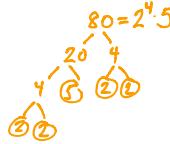


## Adding and Subtracting Radicals

**Exercise 4:** Simplify each radical term, then combine “like radicals” terms by adding or subtracting their coefficients.

a)  $2\sqrt{50} - 6\sqrt{125} + \sqrt{98} + 3\sqrt{80}$

$$\begin{aligned} & 2\sqrt{25}\sqrt{2} - 6\sqrt{25}\sqrt{5} + \sqrt{49}\sqrt{2} + 3\sqrt{16}\sqrt{5} \\ & 2\cdot 5\sqrt{2} - 6\cdot 5\sqrt{5} + 7\sqrt{2} + 3\cdot 4\sqrt{5} \\ & 10\sqrt{2} - 30\sqrt{5} + 7\sqrt{2} + 12\sqrt{5} \\ & \quad \quad \quad \boxed{17\sqrt{2}} \quad \quad \quad \boxed{-18\sqrt{5}} \end{aligned}$$



b)  $\sqrt{4x^7y^5} + 9x^2y^2\sqrt{x^3y} - 5xy\sqrt{x^5y^3}$

$$\begin{aligned} & \sqrt{4x^6y}\sqrt{xy} + 9x^2y^2\sqrt{x^2}\sqrt{xy} - 5xy\sqrt{x^4y^2}\sqrt{xy} \\ & 2x^3y^2\sqrt{xy} + 9x^2y^2 \cdot x\sqrt{xy} - 5xy \cdot x^2\sqrt{xy} \\ & 2x^3y^2\sqrt{xy} + \boxed{9x^3y^2\sqrt{xy}} - \boxed{-5x^3y^2\sqrt{xy}} \\ & \quad \quad \quad \boxed{6x^3y^2\sqrt{xy}} \end{aligned}$$

## Multiplying Radicals

**Exercise 5:** Multiplying two radicals of the form:  $(a\sqrt{x}) \cdot (b\sqrt{y}) = ab\sqrt{xy}$ . Simplify the answer.

a)  $(-2\sqrt{3})(-4\sqrt{6})$

$$\begin{aligned} & (-2)(-4)\sqrt{(3)(6)} \\ & 8\sqrt{18} \\ & 8\sqrt{9}\sqrt{2} \\ & 8(3)\sqrt{2} \\ & \boxed{24\sqrt{2}} \end{aligned}$$

b)  $(-2x\sqrt{6x^6y^5}) \cdot (5y\sqrt{8x^3y^5})$

$$\begin{aligned} & -10xy\sqrt{48x^9y^{10}} \\ & -10xy\sqrt{16x^8y^{10}}\sqrt{3y} \\ & -10xy \cdot 4x^4y^5\sqrt{3y} = \boxed{-40x^5y^6\sqrt{3y}} \end{aligned}$$

**Exercise 6:** Multiply the radical expressions using the Distributive Law  $a(b + c) = ab + ac$ . Simplify the answer.

c)  $\sqrt{5}(4 + \sqrt{5})$

$$\begin{aligned} & \text{brace over } 4 \quad \text{brace over } \sqrt{5} \\ & 4\sqrt{5} + \sqrt{25} \\ & \boxed{4\sqrt{5} + 5} \end{aligned}$$

d)  $\sqrt{6}(\sqrt{12} + \sqrt{24})$

**Exercise 7:** Multiply the radical expressions using FOIL. Simplify the answer.

FOIL (Front-Outer-Inner-Last) or  $(a + b)(c + d) = ac + ad + bc + bd$ .

e)  $(2 + \sqrt{3})(4 + \sqrt{3})$

	2	$+\sqrt{3}$
4	8	$+4\sqrt{3}$
$+\sqrt{3}$	$+2\sqrt{3}$	3

$$= 6\sqrt{3} + 11$$

f)  $(2\sqrt{5} - 9)^2$

g)  $(\sqrt{7} - 5)(\sqrt{7} + 5)$

h)  $(\sqrt{7} + \sqrt{5})(\sqrt{7} - \sqrt{5})$

**Helpful Radical Multiplication Rule:**

$$\text{Square of a Radical: } (\sqrt{a})^2 = (\sqrt{a})(\sqrt{a}) = a$$

$$\text{Product of radical conjugates: } (\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = (\sqrt{a})^2 - (\sqrt{b})^2 = a - b$$

$$\text{Or } (\sqrt{a} + b)(\sqrt{a} - b) = (\sqrt{a})^2 - (b)^2 = a - b^2$$

## Dividing Radicals by Rationalizing the Denominators

**Strategy for rationalizing denominators with radicals:**

If the denominator has one term, multiply the numerator and the denominator by the radical:

$$\text{Examples a)} \frac{2}{\sqrt{a}} = \frac{2}{\sqrt{a}} \cdot \frac{\sqrt{a}}{\sqrt{a}} = \frac{2\sqrt{a}}{a}$$

$$\cancel{\sqrt{x}} = \frac{y\sqrt{x}}{x}$$

$$\text{b)} \frac{4}{3\sqrt{a}} = \frac{4}{3\sqrt{a}} \cdot \frac{\sqrt{a}}{\sqrt{a}} = \frac{4\sqrt{a}}{3a}$$

If the denominator has two terms, multiply the numerator and the denominator by the conjugate of the denominator:

$$\text{Examples a)} \frac{3}{\sqrt{a}+\sqrt{b}} = \frac{3}{\sqrt{a}+\sqrt{b}} \cdot \frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{3(\sqrt{a}-\sqrt{b})}{a-b}$$

*x+y and x-y  
are conjugates*

$$\text{b)} \frac{5}{\sqrt{a}-b} = \frac{5}{\sqrt{a}-b} \cdot \frac{\sqrt{a}+b}{\sqrt{a}+b} = \frac{5(\sqrt{a}+b)}{a-b^2}$$

$$(x+y)(x-y) = x^2 - y^2$$

**Exercise 8:** Rationalize denominators. Simplify the answer.

$$\text{a)} \frac{9}{\sqrt{7}} : \frac{\sqrt{7}}{\sqrt{7}} = \frac{9\sqrt{7}}{\sqrt{49}} = \frac{9\sqrt{7}}{7}$$

$$\text{b)} \frac{3}{2\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}} = \frac{3\sqrt{x}}{2\sqrt{x^2}} = \boxed{\frac{3\sqrt{x}}{2x}}$$

$$\text{c)} \frac{-12}{\sqrt{10}-\sqrt{6}} \cdot \frac{\sqrt{10}+\sqrt{6}}{\sqrt{10}+\sqrt{6}}$$

$$\text{d)} \frac{\sqrt{6}}{5-\sqrt{2}}$$

$$\begin{aligned} & \frac{-12(\sqrt{10}+\sqrt{6})}{(\sqrt{10}-\sqrt{6})(\sqrt{10}+\sqrt{6})} \\ & \frac{-12(\sqrt{10}+\sqrt{6})}{(\sqrt{10})^2 - (\sqrt{6})^2} \\ & \frac{-12(\sqrt{10}+\sqrt{6})}{10-6} \\ & = \frac{-12(\sqrt{10}+\sqrt{6})}{4} \\ & = \boxed{-3(\sqrt{10}+\sqrt{6})} \end{aligned}$$