Exercise 1: Multiply the monomials $(5x^3y^6)(-3xy^4)$

$-15x^4y^{10}$

Exercise 2: Multiply using the Distributive Property

a) $2p^3(4p^7-5)$

$8p^{10}-10p^3$

b) $9m^3n^6(-6m^2n^2+8m^2n+1)$

$-54m^5n^8+72m^5n^7+9m^3n^6$

The Greatest Common Factor

The greatest common factor (GCF) for a polynomial is the largest monomial that divides each term of the polynomial.

Factoring the greatest common factor of a polynomial:

1. **Determine the greatest common factor**
2. **Write the answer in factored form.**

The GCF factoring process is the reverse of the Distributive Property

Multiply

The Distributive Property: $\boldsymbol{a}(\boldsymbol{b}+\boldsymbol{c})=\boldsymbol{ab}+\boldsymbol{ac}$

Factor/Divide

The GCF factoring: $\boldsymbol{ab}+\boldsymbol{ac}=\boldsymbol{a}(\boldsymbol{b}+\boldsymbol{c})$

Exercise 3: Factor the greatest common factor and express the answer in factored form. Check by multiplying using the Distributive Law.

a) Factor $5p^3 + 15p^2 - 30p$

GCF: $5p$

Answer in factored form: $5p(p^2 + 3p - 6)$

Check: $5p^3 + 15p^2 - 30p$

b) Factor $9a^3b^4 - 6a^2b^3 + 3ab^2$

$\frac{b^4}{b^2} = b^2 \quad \frac{b^3}{b^2} = b$

* b^4 and b^3 are divisible by b^2

GCF: $3ab^2$

Answer in factored form: $3ab^2(3a^2b^2 - 2ab + 1)$

Check: $9a^3b^4 - 6a^2b^3 + 3ab^2$ ✓

$3ab^2\left(\frac{9a^3b^4}{3ab^2} - \frac{6a^2b^3}{3ab^2} + \frac{3ab^2}{3ab^2}\right)$

c) Factor $5(x + y) - 6x(x + y)$

GCF: $(x+y)$

Answer in factored form: $(x+y)(5-6x)$

Check: $5(x+y) - 6x(x+y)$

Factoring by Grouping

Before factoring by grouping, first we factor out GCF from all four terms.

Steps in factoring by grouping (Assume there is no GCF)

1. Group pairs of terms and factor each pair.
2. If there is a common binomial factor, then factor it out.
3. If there is no common binomial factor, then interchange the middle two terms and repeat the process over. If there is still no common binomial, then the polynomial cannot be factored.

Factor $4x + 6y + 2xy + 3y^2$ by grouping

Step 1: Group the pairs of terms	$(\mathbf{4x + 6y}) + (\mathbf{2xy + 3y^2})$
Step 2: Factor the GCF from each pair	$2(\mathbf{2x + 3y}) + y(\mathbf{2x + 3y})$
Step 3: Factor the common binomial factor	$(\mathbf{2x + 3y})(\mathbf{2 + y})$

Exercise 4: Factor by grouping. Follow the steps in the table

a) Factor by grouping: $56 + 21k + 8h + 3hk$

	Show work here
Step 1: Group the pairs of terms	$(56 + 21k) + (8h + 3hk)$
Step 2: Factor the GCF from each pair	$7(8+3k) + h(8+3k)$
Step 3: Factor the common binomial factor	$(8+3k)(7+h)$

b) Factor by grouping: $5x^2 + 40x - xy - 8y$

	Show work here
Step 1: Group the pairs of terms. Be careful with the signs.	$(5x^2 + 40x) + (-xy - 8y)$
Step 2: Factor the GCF from each pair. Be careful with the signs.	$5x(x+8) - y(x+8)$
Step 3: Factor the common binomial factor	$(x+8)(5x-y)$

or $(5x^2 + 40x) - (xy + 8y)$

$5x(x+8) - y(x+8)$

Factoring Trinomials

Multiplying Two Binomials

The **FOIL** (Front-Outer-Inner-Last) Method: $(a+b)(c+d) = ac + ad + bc + bd$

$ac \quad bd$

$$(\boldsymbol{a}+\boldsymbol{b})(\boldsymbol{c}+\boldsymbol{d}) = \boldsymbol{ac}+\boldsymbol{ad}+\boldsymbol{bc}+\boldsymbol{bd}$$

bc

ad

Exercise 5: Multiply the binomials and combine like terms

a) $(x+2)(x+3)$

	x	$+2$
x	x^2	$+2x$
$+3$	$+3x$	6

$= x^2+2x+3x+6$

$= x^2+5x+6$

b) $(x+2)(x-3)$

$x^2-3x+2x-6$

x^2-x-6

Factoring Trinomials with Lead Coefficients of 1

Since the product of two binomials is often a trinomial, it is expected that many trinomials will factor as two binomials. For example, to factor x^2+5x+6, we must find two binomials $x+a$ and $x+b$ such that

$$\begin{aligned} x^2+5x+6 &= (x+a)(x+b) \\ &= x^2+bx+ax+ab \\ &= x^2+ax+bx+ab \\ &= x^2+(a+b)x+ab \end{aligned}$$

Matching the coefficient of each term, one can see the binomials must satisfy product $ab=6$ and the sum $a+b=5$. Since $2\cdot 3=6$ and $2+3=5$, we find our binomials $x^2+5x+6=(x+2)(x+3)$.

Similarly, if we want to factor x^2-x-6, we will consider $x^2-x-6=(x+a)(x+b)$ where the product $ab=-6$ and the sum $a+b=-1$. Considering all possible factorizations of -6, we find

$$(2)(-3) = -6 \text{ and } 2+(-3) = -1\text{, thus,}$$

$$x^2-x-6=(x+2)(x-3).$$

Steps to factoring trinomials with lead coefficient of 1

1. Write the trinomial in descending powers.
2. List the factorizations of the constant (third) term of the trinomial.
3. Pick the factorization where the sum of the factors is the coefficient of the middle term.
4. Check by multiplying the binomials.

Exercise 6: Factor the trinomial with lead coefficient 1 by checking for the correct pair of product and sum

Factors of 12

$+6+2$

$+4+3$

$+12+1$

$-6-2=-8$

$-4-3$

$-12-1$

a) Factor $x^2-8x+12$

Answer in factored form: $(x-6)(x-2)$

Check: $x^2-6x-2x+12 = x^2-8x+12$ ✓

b) Factor a^2+5a+6

Answer in factored form: $(a+2)(a+3)$

Check: $a^2+2a+3a+6 = a^2+5a+6$

c) Factor $y^2-9y-36$

Answer in factored form: $(y-12)(y+3)$

Check: $y^2+3y-12y-36 = y^2-9y-36$

Factors of $45y^2$

$+9y+5y=14y$

$+3y+15y$

$+1y+45y$

$-1y-45y$

$-3y-15y$

$-9y-5y$

d) Factor $x^2+14xy+45y^2$

Answer in factored form: $(x+9y)(x+5y)$

Check: $x^2+5xy+9xy+45y^2 = x^2+14xy+45y^2$

e) Factor $x^2-9xy+18y^2$

Answer in factored form: $(x-3y)(x-6y)$

Check:

f) Factor $a^2+7ab-8b^2$

Answer in factored form: $(a+8b)(a-b)$

Check:

Factoring Trinomials with Lead Coefficients other than 1

Method 1: Trial and error

1. List the factorizations of the third term of the trinomial.
2. Write them as two binomials and determine the correct combination where the sum of the outer product, ad, and the inner product, bc, is equal to the middle term of the trinomial.

$$(a+b)(c+d) = ac + ad + bc + bd$$

bc

ad

Method 2: Factoring by grouping

1. Form the product ac.
2. Find a pair of numbers whose product is ac and whose sum is b.
3. Rewrite the polynomial to be factored so that the middle term bx is written as the sum of the two terms whose coefficients are the two numbers found in step
4. Factor by grouping.

Exercise 7: Factor each trinomial below

a) $2h^2 - 5h - 3$ $= 2h^2 - 6h + 1h - 3$

$a = 2$, $b = -5$, $c = -3$

$ac = -6 = -6 \cdot 1$

$b = -5 = -6 + 1$

$= (2h^2 - 6h) + (h - 3)$

$= 2h(h-3) + 1(h-3)$

$= (2h+1)(h-3)$

$2h^2 - 5h - 3$

$ac = (2)(-3) = -6$

$\to h^2 - 5h - 6$

Factor

① $h^2 + bh + ac$

$(h-6)(h+1)$

$\to \left(h - \frac{6}{2}\right)\left(h + \frac{1}{2}\right)$

② Divide Factors by a

$(h-3)\left(h+\frac{1}{2}\right)$

$\to (h-3)(2h+1)$

b) $2h^2 - h - 3$ $= 2h^2 - 3h + 2h - 3$

$a = 2$, $b = -1$, $c = -3$

$ac = -6 = -3 \cdot 2$

$b = -1 = -3 + 2$

$= (2h^2 - 3h) + (2h - 3)$

$= h(2h-3) + 1(2h-3)$

$= (h+1)(2h-3)$

$\to h^2 - h - 6$

$ac = -6$

$(h-3)(h+2)$

$a = 2$

$\to \left(h - \frac{3}{2}\right)\left(h + \frac{2}{2}\right)$

divide everything by 2

$\to \left(h - \frac{3}{2}\right)(h+1)$

$\to (2h-3)(h+1)$

c) $2h^2 - 5h - 12$

d) $3k^2 - 14k - 5$

Factoring Special Products

Exercise 8: Multiply the binomials and combine like terms.
Can you come up with a formula for this type of multiplication?

$(a+b)(a-b)=a^2-b^2$

a) $(x+3)(x-3)$

$x^2 \cancel{-3x} + \cancel{3x} - 9$

x^2-9

b) $(5x+8)(5x-8)$

$25x^2-64$

Exercise 9: Multiply the binomials and combine like terms.
Can you come up with a formula for this type of multiplication?

$(a-b)^2 = a^2-2ab+b^2$ $\quad$ $(a+b)^2 = a^2+2ab+b^2$

a) $(x-3)^2 = (x-3)(x-3)$

$= x^2-3x-3x+9$

$= x^2-6x+9$

$(x-3)^2 \neq x^2-3^2$ don't do this

b) $(2x+5)^2 = (2x+5)(2x+5)$

$= 4x^2+10x+10x+25$

$= 4x^2+20x+25$

Many trinomials can be factored by using special product formulas.

Difference of Two Squares (DOTS): $\boldsymbol{a^2-b^2=(a+b)(a-b)}$

Exercise 10: Factor each binomial below. Check if it is a difference of two squares.

a) x^2-49

b) $25-b^2$

c) $36x^2-16y^2$

d) $100t^2-49r^2$

e) Factor completely $16x^4-81$

f) x^2+36

What have you noticed about the sum of two squares? ______________________________