Section 4: Systems of Linear Equations

Systems of Two Linear Equations

A pair of equations $\begin{cases} y = -3x + 4 \\ y = -2x - 5 \end{cases}$ is called **a system of linear equations**.

Determining Whether an Ordered Pair is a Solution

A solution of a system of two equations in two variables is an ordered pair (x, y) that makes __the two__ equations true.

Example 1: Determine whether the ordered pair $\boxed{\overset{x\ \ y}{(2,-1)}}$ is a solution of the system $\begin{cases} 3x + 2y = 4 \\ x - y = 3 \end{cases}$

$2 \cancel{-} + \cancel{-} 1 = 3$

$3 = 3$

$3(2) + 2(-1) = 4$

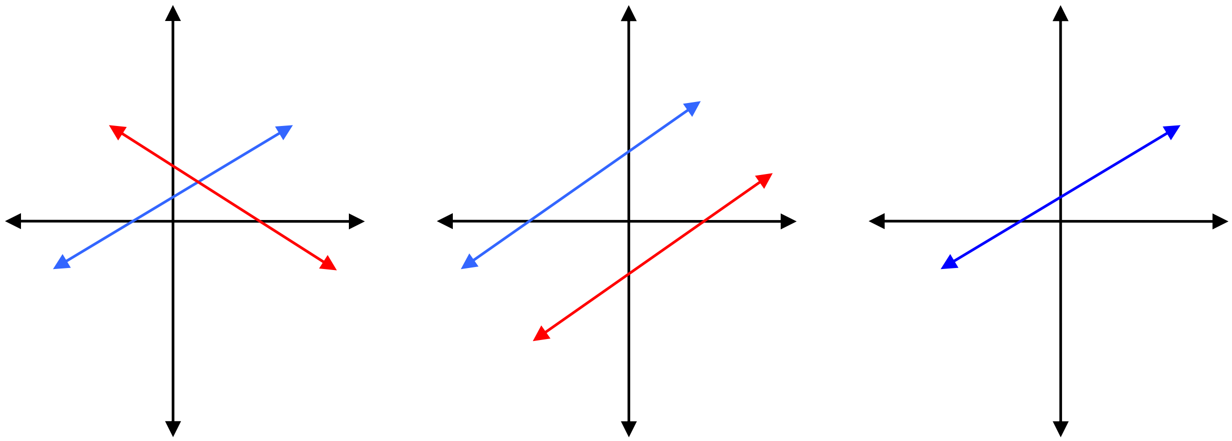
$6 + -2 = 4$

$4 = 4$ ✓

Example 2: Determine whether the ordered pair $(-3,4)$ is a solution of the system $\begin{cases} 2x + 3y = 6 \\ x - y = 1 \end{cases}$

Methods of Solving Two Linear Equations

Possible Solutions to Systems of Two Linear Equations



one solution | No solution | infinite solutions

I. The Graphing Method

Solving a System of Two Equations Using the Graphing Method

Example 5: Solve the system of linear equations by graphing $\begin{cases} 3x + y = 2 \\ 6x + 2y = 4 \end{cases}$

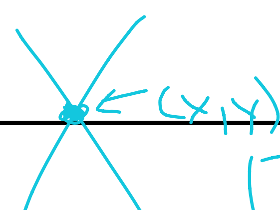
II. The Substitution Method

Solving a System of Two Equations Using the Substitution Method

Example 6: Solve the system of linear equations using the substitution method $\begin{cases} 2x + 3y = 24 \\ y - 2x = 0 \end{cases}$

$$y - \cancel{2x} = 0$$
$$+\cancel{2x} \quad +2x$$
$$y = 2x$$
$$y = 2(3)$$
$$\boxed{y = 6}$$

$$2x + 3(2x) = 24$$
$$2x + 6x = 24$$
$$\frac{8x}{8} = \frac{24}{8}$$
$$\boxed{x = 3}$$



$\boxed{\text{Solution: } (3,6)}$

III. The Elimination Method

Solving a System of Two Equations Using the Elimination Method

★ To eliminate a variable you need the opposite #

Example 9: Solve the system of linear equations using the elimination method $\begin{cases} -4x + 3y = 15 \\ 4x + 2y = 5 \end{cases}$

$ax + by = c$

$dx + ey = f$

$$\frac{5y}{5} = \frac{20}{5}$$

$$\boxed{y = 4}$$

$$4x + 2(4) = 5$$

$$4x + 8 = 5$$

$$-8 \quad -8$$

$$\frac{4x}{4} = \frac{-3}{4}$$

$$\boxed{x = \frac{-3}{4}}$$

Solution: $\left(\frac{-3}{4}, 4\right)$

$$-\frac{4}{1}\left(\frac{-3}{4}\right) + 3(4) = 15$$

$$\frac{12}{4} + 12$$

$$3 + 12 = 15 \checkmark$$

$$15 = 15$$

$$\cancel{4}\left(\frac{-3}{\cancel{4}}\right) + 2(4) = 5$$

$$-3 + 8 = 5 \checkmark$$

Example 12: Solve the system of linear equations using the elimination method $\boxed{\begin{cases} 2(-6x+2y=-3) \\ 3x-4y=3 \end{cases}}$

Solution: $\left(\frac{1}{3}, -\frac{1}{2}\right)$

$$\begin{cases} -12x+4y=-6 \\ 3x-4y=3 \end{cases}$$

$$\frac{-9x}{-9} = \frac{-3}{-9}$$

$$x = \frac{3}{9} = \frac{1}{3}$$

$$\boxed{x=\frac{1}{3}}$$

$$\cancel{3}\left(\frac{1}{\cancel{3}}\right) - 4y = 3$$

$$1-4y=3$$
$$-1 \qquad -1$$

$$\frac{-4y}{-4} = \frac{2}{-4}$$

$$y = \frac{-2}{4}\begin{matrix}\div 2\\ \div 2\end{matrix}$$

$$\boxed{y=-\frac{1}{2}}$$

Example 13: Solve the system of linear equations using the elimination method $\begin{cases} -2(3x+2y=12) \\ 6x-3y=24 \end{cases}$

$6y$
$-6y$

$$3(3x+2y=12)$$
$$2(6x-3y=24)$$

$$9x+6y=36$$
$$12x-6y=48$$

.
.
.

$$\boxed{\text{Solution: } (4,0)}$$

$$-6x-4y=-24$$
$$6x-3y=24$$

$$\frac{-7y}{-7}=\frac{0}{-7}$$

$$\boxed{y=0}$$

$$6x-3(0)=24$$

$$6x-0=24$$
$$+0 \qquad +0$$

$$\frac{6x}{6}=\frac{24}{6}$$

$$\boxed{x=4}$$

Section 7: Factoring Polynomials

The Greatest Common Factor

The **greatest common factor** for a polynomial is the largest monomial that divides each term of the polynomial.

Example 1: Factor $5p^3 + 15p^2 - 30p$

$5p(p^2 + 3p - 6)$

Example 2: Factor $8x^4 - 4x^3 + 16x^2 - 4x + 24$

$4(2x^4 - x^3 + 4x^2 - x + 6)$

Example 3: Factor $9a^3b^4 - 3a^2b^3 + 6ab^2$

$3ab^2(3a^2b^2 - ab + 2)$

Factoring by Grouping

Steps in factoring by grouping
1. Factor out any monomial that is common to all four terms.
2. Group together pairs of terms and factor each pair.
3. If there is a common binomial factor, then factor it out.
4. If there is no common binomial factor, then interchange the middle two terms and repeat the process over. If there is still no common binomial, then the polynomial cannot be factored.

Example 4: Factor $56 + 21k + 8h + 3hk$

$7(8 + 3k) + h(8 + 3k)$

$(8 + 3k)(7 + h)$

Example 5: Factor $5x^2 + 40x - xy - 8y$

$5x(x+8) \; -y(x+8)$

$(x+8)(5x-y)$

Factoring Trinomials

Factoring Trinomials with Lead Coefficients of 1

Since the product of two binomials is often a trinomial, it is expected that many trinomials will factor as two binomials. For example, to factor $x^2 + 9x + 18$, we must find two binomials $x + a$ and $x + b$ such that

$$x^2 + 9x + 18 = (x + a)(x + b)$$
where $ab = 18$ (product is 18)
and
$ax + bx = 9x$ (sum of the two numbers is 9)

To find the numbers a and b, we first list the possible factorizations of $+18$ and find the one where the sum of the factors is $+9x$.

The possible factorizations of $+18$ with their respective sums are:

Products of $+18$	**Sums**
$(+1)(+18)$	$(+1) + (+18) = +19$
$(-1)(-18)$	$(-1) + (-18) = -19$
$(+2)(+9)$	$(+2) + (+9) = +11$
$(-2)(-9)$	$(-2) + (-9) = -11$
$(+3)(+6)$	$(+3) + (+6) = +9$
$(-3)(-6)$	$(-3) + (-6) = -9$

Thus, $a = 3$ and $b = 6$, and

$$x^2 + 9x + 18 = (x + a)(x + b)$$
$$x^2 + 9x + 18 = (x + 3)(x + 6)$$

Steps to factoring trinomials with lead coefficient of 1

1. Write the trinomial in descending powers.
2. List the factorizations of the third term of the trinomial.
3. Pick the factorization where the sum of the factors is the coefficient of the middle term.
4. Check by multiplying the binomials.

Quadratic

Example 6: Factor $x^2 - 8x + 12$

$\boxed{(x-6)(x-2)}$

Example 7: Factor $a^2 + 5a + 6$

$\boxed{(a+3)(a+2)}$

Example 8: Factor $y^2 - 9y - 36$

$\boxed{(y-12)(y+3)}$ ✓

Example 9: Factor $x^2 + 14xy + 45y^2$

$ax^2 + bx + c$

Example 12: Factor $2h^2 - 5h - 12$

$a \cdot c = 2(-12)$

$= -24$

1	24
2	12
3	-8
4	6

$2h^2 + 3h - 8h - 12$

$h(2h+3) - 4(2h+3)$

$\boxed{(2h+3)(h-4)}$

Example 13: Factor $3k^2 - 14k - 5$