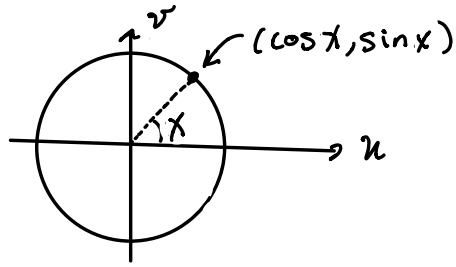
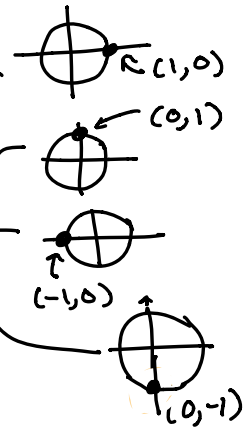
Graphs of equations involving sin and cos.



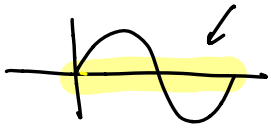
Graph $y = \sin x$.

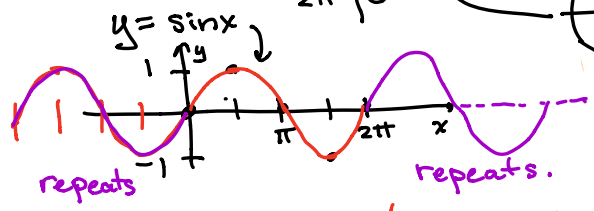
Note:

x	y
0	0
$\frac{\pi}{2}$	1
π	0
$\frac{3\pi}{2}$	-1
2π	0



This graph repeats itself after 2π. This length is called the period of sin.



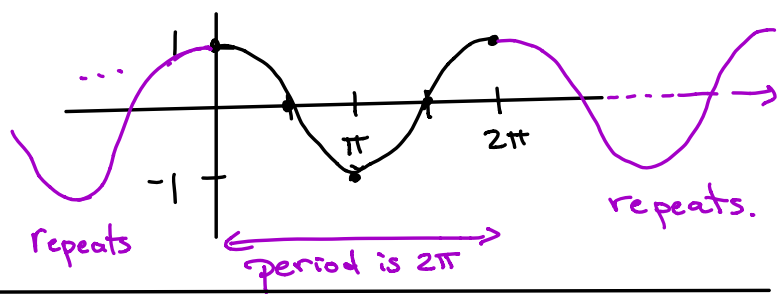


See how the second-coordinate of a point changes as it moves around the unit circle.

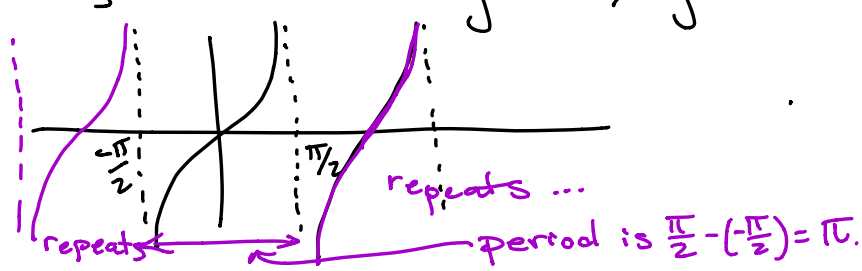
Similarly, looking at the pictures above but focusing on the first-coordinate gives:

x	$y = \cos x$
0	1
$\frac{\pi}{2}$	0
π	-1
$\frac{3\pi}{2}$	0
2π	1
$\vdots$	

So the graph of $y = \cos x$ is



Remark: Doing the same for $y = \tan x$ yields



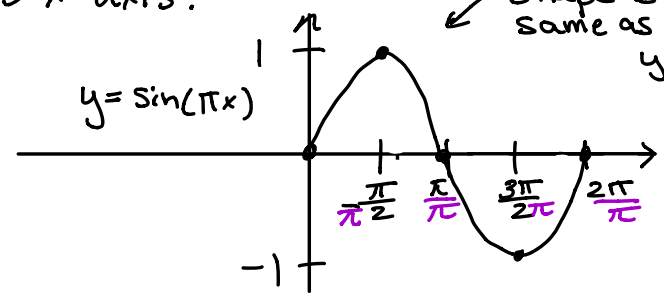
Transformed equations

Consider $y = \sin(\pi x)$.

This corresponds to a scaling of the x-axis or, equivalently a compression of the graph along the x-axis:

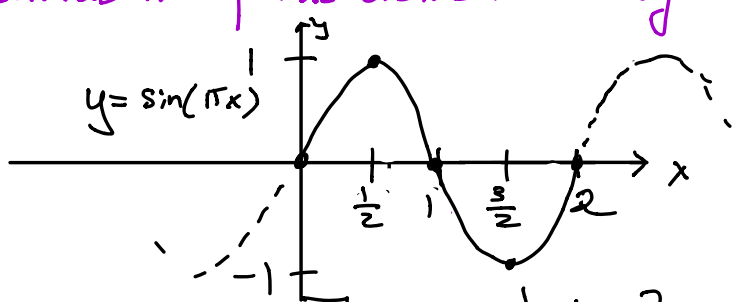
shape is the same as for $y = \sin x$.

The



But divide the points on the x-axis by π.

Simplifying gives:



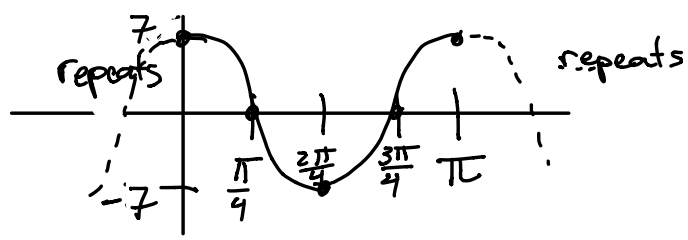
The period is 2.

Alternatively, we could first find the period T by solving $\underbrace{2\pi}_{\text{the period of } \sin x} = \pi T \Rightarrow T = 2$.

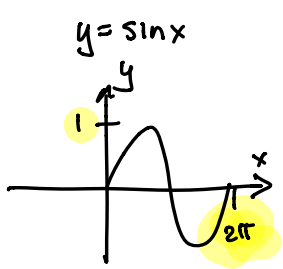
Then form the 4 analogous points on the x-axis: $\frac{T}{4}, \frac{2T}{4}, \frac{3T}{4}, \frac{4T}{4}$, which in this case is $\frac{1}{2}, \frac{2}{2}, \frac{3}{2}, \frac{4}{2}$ or $\frac{1}{2}, 1, \frac{3}{2}, 2$.

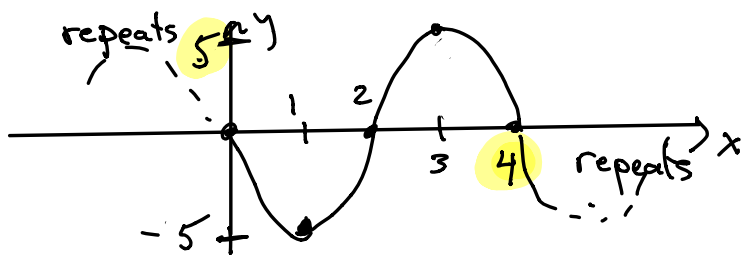
Example Graph $y = 7\cos(2x)$.

We can note the period T satisfies $2T = 2\pi$ so $T = \pi$. Also (perhaps thinking of the equation as $\frac{1}{7}y = \cos(2x)$) the y axis is scaled also, stretching the graph by a factor of .



Example Write an equation for:





This looks like a the graph of $y = \sin x$ with modifications:
- reflection in y
- Scaling of the y axis
- Scaling of the x axis

$2\pi ? = 4$

$\frac{4}{2\pi} = \frac{2}{\pi}$

x-scaling factor

4 is the period. $\longrightarrow$

So $-\frac{1}{5}y = \sin(ax)$

The period is 4

So $a \cdot 4 = 2\pi$

$\Rightarrow a = \frac{\pi}{2}$.

So this is a graph of $-\frac{1}{5}y = \sin\left(\frac{\pi x}{2}\right)$

or, more conventionally, multiplying both sides by -5 gives

$$y = -5\sin\left(\frac{\pi x}{2}\right).$$

Here, $|-5| = 5$ is called the amplitude (it is the height of the peak).

For $y = 7\cos(2x)$, 7 is the amplitude.
