Completing the square:

Example: Solve $x^2 = 4$.

$\Rightarrow x = 2$ or $-2 = \pm 2$

Example: Solve $x^2 = 8$.

$\Rightarrow x = \pm\sqrt{8} = \pm 2\sqrt{2}$.

Example: Solve $(x-2)^2 = 4$

$x - 2 = \pm 2$

$\Rightarrow x = 2 \pm 2$

$= 4$ or 0.

Example: Solve $(x-2)^2 = 27$

$x - 2 = \pm\sqrt{27}$

$= \pm 3\sqrt{3}$

$\Rightarrow x = 2 \pm 3\sqrt{3}$

Example: Solve: $(x-2)^2 = -18$

$x - 2 = \pm\sqrt{-18}$

$= \pm i 3\sqrt{2}$

$\Rightarrow x = 2 \pm i 3\sqrt{2}$.

Example: Solve:

$x^2 + 4x + 4 = 20$

$\Rightarrow (x+2)^2 = 20$

$\Rightarrow x + 2 = \pm 2\sqrt{5}$

$\Rightarrow x = -2 \pm 2\sqrt{5}$.

Example: Solve

$x^2 + 6x - 54 = 0$

$\Rightarrow (x^2 + 6x + 9) - 9 - 54 = 0$

$\Rightarrow (x+3)^2 - 63 = 0$

$\Rightarrow (x+3)^2 = 63$

$\Rightarrow x + 3 = \pm\sqrt{63} = \pm 3\sqrt{7}$

$\Rightarrow x = -3 \pm 3\sqrt{7}$.

Try

Solve $x^2 = 9$.

Solve $x^2 = 300$

Solve $(x+3)^2 = 9$

Solve: $(x-7)^2 = 18$

Solve $(x+3)^2 = -20$

Solve

$x^2 + 6x + 9 = 5$

Solve

$x^2 + 10x + 5 = 0$

Example: Solve

$$2x^2+10x-5=0$$

$$\Rightarrow 2(x^2+5x)-5=0$$

$$\Rightarrow 2\left(x^2+5x+\left(\tfrac{5}{2}\right)^2-\left(\tfrac{5}{2}\right)^2\right)-5=0$$

$$\Rightarrow 2\left[\left(x+\tfrac{5}{2}\right)^2-\tfrac{25}{4}\right]-5=0$$

$$\Rightarrow 2\left(x+\tfrac{5}{2}\right)^2-\tfrac{25}{2}-5=0$$

$$\Rightarrow 2\left(x+\tfrac{5}{2}\right)^2-\tfrac{35}{2}=0$$

$$\Rightarrow 2\left(x+\tfrac{5}{2}\right)^2=\tfrac{35}{2}$$

$$\Rightarrow \left(x+5/2\right)^2=\tfrac{35}{4}$$

$$\Rightarrow x+5/2=\pm\sqrt{35/4}$$

$$\Rightarrow x=-\tfrac{5}{2}\pm\tfrac{\sqrt{35}}{2}.$$

$$\left(x+\tfrac{5}{2}\right)^2=x^2+5x+\tfrac{25}{4}$$

OR

$$\frac{-5\pm\sqrt{35}}{2}$$

Solve

$$3x^3+6x+4=0$$

☆☆ Solve: $ax^2+bx+c=0,\ a\neq 0.$

$$\Rightarrow a\left(x^2+\tfrac{b}{a}x+\left(\tfrac{b}{2a}\right)^2-\left(\tfrac{b}{2a}\right)^2\right)+c=0$$

$$\Rightarrow a\left(\left(x+\tfrac{b}{2a}\right)^2-\left(\tfrac{b}{2a}\right)^2\right)+c=0$$

$$\Rightarrow a\left(x+\tfrac{b}{2a}\right)^2-\frac{b^2}{4a}+\frac{c\cdot 4a}{4a}=0$$

$$\Rightarrow a\left(x+\tfrac{b}{2a}\right)^2=\frac{b^2-4ac}{4a}$$

$$\Rightarrow \left(x+\tfrac{b}{2a}\right)^2=\frac{b^2-4ac}{4a^2}$$

$$\sqrt{4a^2}=2a$$

$$\Rightarrow x+\tfrac{b}{2a}=\pm\sqrt{b^2-4ac}\,/\,2a$$

$$\Rightarrow x = -\frac{b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}$$

$$\Rightarrow x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}.$$

The quadratic formula:

If $ax^2+bx+c=0$, $a \neq 0$, then

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}.$$

The process of writing ax^2+bx as $a\left(x+\frac{b}{2a}\right)^2 - a\left(\frac{b}{2a}\right)^2$ is called completing the square.

We will see this again in dealing with circles and parabolas.